1. NAND Circuit

Let us consider a NAND logic gate. This circuit implements the boolean function \((A \cdot \overline{B})\). The \(\cdot\) stands for the AND operation, and the \(\overline{\text{}}\) stands for NOT; combining them, we get NAND!

![NAND gate transistor-level implementation.](image)

\(V_{tn}\) and \(V_{tp}\) are the threshold voltages for the NMOS and PMOS transistors, respectively. Assume that \(V_{DD} > V_{tn}, |V_{tp}| > 0\).

(a) Label the gate, source, and drain nodes for the NMOS and PMOS transistors above.

**Solution:** As a convention throughout the course, we will draw NMOS transistors with their source at the bottom (and drain at the top). On the other hand, PMOS transistors will have their source at the top. Therefore, the drains are at the top of \(N_A\) (connected to \(V_{out}\)) and the top of \(N_B\) (connected to \(N_A\)). The sources are at the bottom of \(N_A\) (connected to \(N_B\)) and the bottom of \(N_B\) (connected to ground). The gate terminal of \(N_A\) is connected to \(V_A\); the gate of \(N_B\) is connected to \(V_B\).

For the PMOS transistors, the source is at the top of \(P_A\) and \(P_B\) (connected to \(V_{DD}\)). The drain is at the bottom of \(P_A\) and \(P_B\) (connected to \(V_{out}\)). The gate terminal of \(P_A\) is connected to \(V_A\); the gate of \(P_B\) is connected to \(V_B\).

(b) If \(V_A = V_{DD}\) and \(V_B = V_{DD}\), which transistors act like open switches? Which transistors act like closed switches? What is \(V_{out}\)?

**Solution:** \(P_A\) and \(P_B\) are off (open switches). \(N_B\) and \(N_A\) are on (closed switches). \(V_{out} = 0V\) because it is connected to ground through a closed circuit consisting of \(N_A\) and \(N_B\) (and detached...
from $V_{DD}$).

(c) If $V_A = 0V$ and $V_B = V_{DD}$, what is $V_{out}$?

Solution: $P_B$ and $N_A$ are off (open switches). $P_A$ and $N_B$ are on (closed switches). $V_{out} = V_{DD}$ because it is connected to $V_{DD}$ through a closed circuit consisting of $P_A$ (and detached from ground, since both $N_A$ and $N_B$ must be closed for $V_{out}$ to be connected to ground).

(d) If $V_A = V_{DD}$ and $V_B = 0V$, what is $V_{out}$?

Solution: $P_A$ and $N_B$ are off (open switches), $P_B$ is on (closed switch). So, $V_{out} = V_{DD}$ because it is connected to $V_{DD}$ through a closed switch.

(e) If $V_A = 0V$ and $V_B = 0V$, what is $V_{out}$?

Solution: $N_B$ is off, creating an open circuit. $P_A$ and $P_B$ are on, creating a closed circuit. $V_{out} = V_{DD}$ because it is connected by closed circuit to $V_{DD}$.

(f) Write out the truth table for this circuit.

<table>
<thead>
<tr>
<th>$V_A$</th>
<th>$V_B$</th>
<th>$V_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V_{DD}$</td>
</tr>
<tr>
<td>0</td>
<td>$V_{DD}$</td>
<td>0</td>
</tr>
<tr>
<td>$V_{DD}$</td>
<td>0</td>
<td>$V_{DD}$</td>
</tr>
<tr>
<td>$V_{DD}$</td>
<td>$V_{DD}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution:
2. RC Circuits - Part I

In this problem, we will find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining four functions over time: \( I(t) \) is the current at time \( t \), \( V(t) \) is the voltage across the circuit at time \( t \), \( V_R(t) \) is the voltage across the resistor at time \( t \), and \( V_C(t) \) is the voltage across the capacitor at time \( t \).

Recall from 16A that the voltage across a resistor is defined as \( V_R = RI_R \) where \( I_R \) is the current across the resistor, and the voltage across a capacitor is defined as \( V_C = \frac{Q}{C} \) where \( Q \) is the charge across the capacitor.

(a) Starting from the given charge-voltage relation for a capacitor, find an equation that relates the current across the capacitor \( I(t) \) with the voltage across the capacitor \( V_C(t) \).

**Solution:** As noted in the problem statement, we start from the \( Q-V \) relationship of the capacitor:

\[
Q(t) = CV_C(t)
\]  

(1)

Differentiating \( V_C(t) = \frac{Q(t)}{C} \) in terms of \( t \), we get

\[
\frac{dV_C(t)}{dt} = \frac{dQ(t)}{dt} \frac{1}{C}.
\]  

(2)

By definition, the change in charge is the current across the capacitor, so

\[
\frac{dV_C(t)}{dt} = I(t) \frac{1}{C}
\]  

(3)

(b) Analyzing the circuit, write an equation that relates the functions \( I(t), V_C(t), \) and \( V(t) \).

**Solution:** From KCL, we have

\[
\frac{V(t) - V_C(t)}{R} - I(t) = 0
\]  

(4)

\[
RI(t) + V_C(t) = V(t)
\]  

(5)

(c) So far, we have an equation that involves both \( I(t) \) and \( V_C(t) \). To solve this equation, we can remove \( I(t) \) (one of the unknowns) using what we found in part 2.a. **Rewrite the previous equation in part 2.b in the form of a differential equation.** You will pick up with this in the next discussion.
Solution: From part (a), we have

\[ I(t) = \frac{dV_C(t)}{dt} C \quad (6) \]

Substituting this into eq. (5) gives us

\[ RC \frac{dV_C(t)}{dt} + V_C(t) = V(t) \quad (7) \]