Discussion 2D

The following notes are useful for this discussion: Note 3 and Note 4.

1. Introduction to Inductors

An inductor is a circuit element analogous to a capacitor; its voltage is proportional to the derivative of the current across it. That is:

$$V_L(t) = L\frac{\mathrm{d}I_L(t)}{\mathrm{d}t} \tag{1}$$

When first studying capacitors, we analyzed a circuit where a current source was directly attached to a capacitor. In Figure 1, we form the counterpart circuit for an inductor:

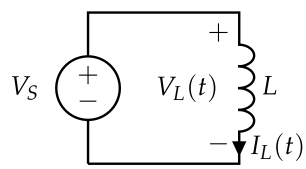


Figure 1: Inductor in series with a voltage source.

(a) **What is the current through an inductor as a function of time? If the inductance is $L = 3\,\mathrm{H}$, what is the current at $t = 6\,\mathrm{s}$?** Assume that the voltage source turns from $0\,\mathrm{V}$ to $5\,\mathrm{V}$ at time $t = 0\,\mathrm{s}$, and there's no current flowing in the circuit before the voltage source turns on, i.e $I_L(0) = 0\,\mathrm{A}$.

(b) Now, we add some resistance in series with the inductor, as in Figure 2.

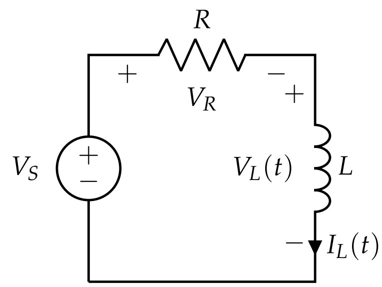
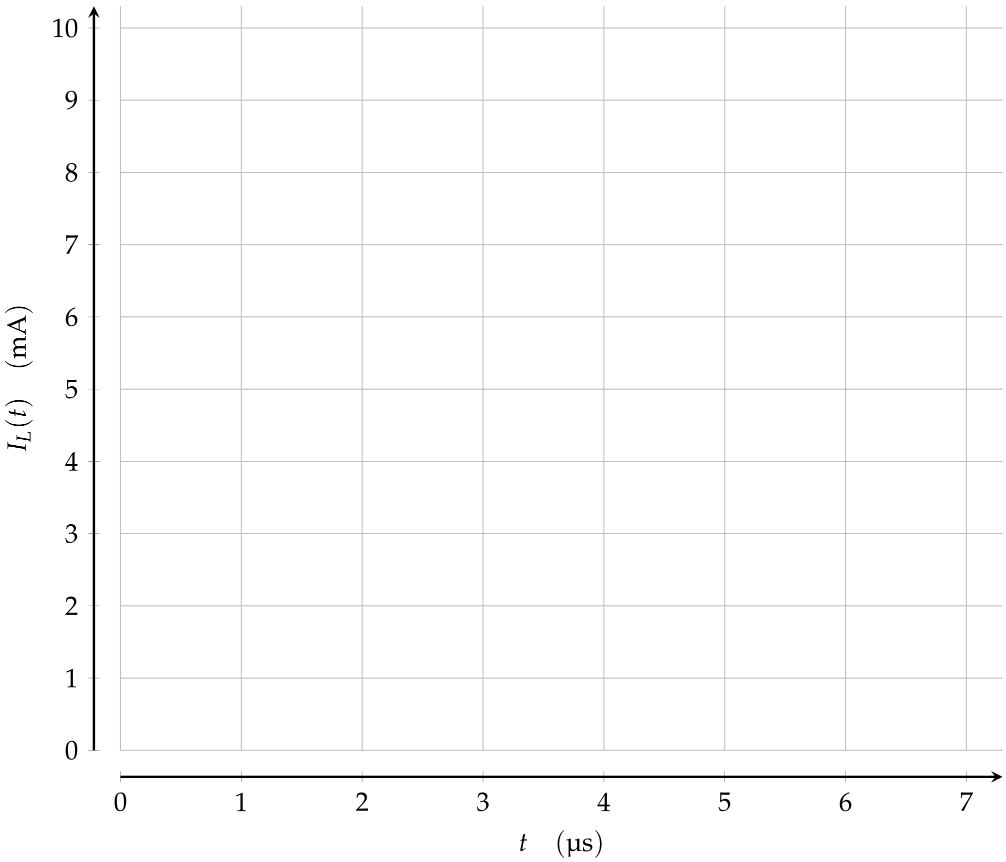
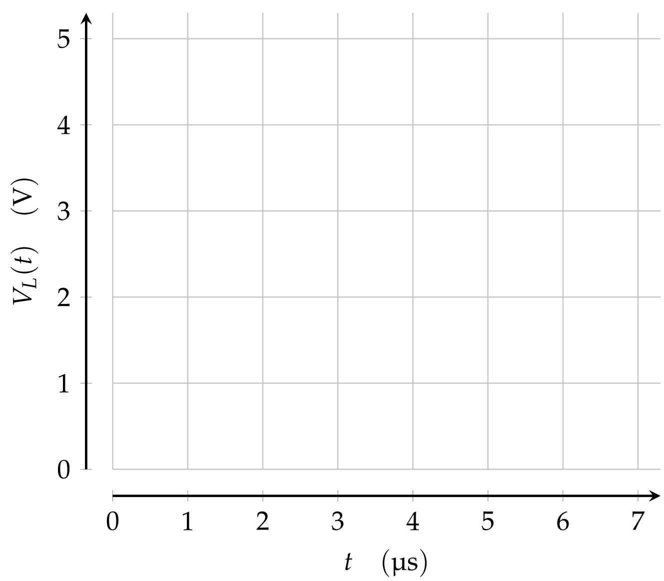


Figure 2: Inductor in series with a voltage source.

Solve for the current $I_L(t)$ and voltage $V_L(t)$ in the circuit over time, in terms of R, L, V_S, t. Note that $I_L(0) = 0\,\text{A}$.

(c) **Suppose $R = 500\,\Omega, L = 1\,\text{mH}, V_S = 5\,\text{V}$. Plot the current through and voltage across the inductor ($I_L(t), V_L(t)$), as these quantities evolve over time.**


10
9
8
7
6
5
4
3
2
1
0
$I_L(t)$ (mA)
0 1 2 3 4 5 6 7
t (µs)


5
4
3
2
1
0
$V_L(t)$ (V)
0 1 2 3 4 5 6 7
t (µs)

2. Differential Equations with Complex Numbers

Recall the steps we take to solve a non-diagonal (coupled) system of differential equations. So far, we have dealt with matrices that have purely real eigenvalues. In this problem, we will apply the same principles to solve a system with complex eigenvalues.

(a) Consider the following system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{z}(t) = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \vec{z}(t) \tag{2}$$

with the initial condition $\vec{z}(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. **Solve the differential equation to find $\vec{z}(t)$.** We will have to perform a change of variables, since the system is not diagonal. It may help to recall the change of variables strategy in fig. 3. You may use the fact that $\begin{bmatrix} \mathrm{j} \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -\mathrm{j} \\ 1 \end{bmatrix}$ are eigenvectors of $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$.

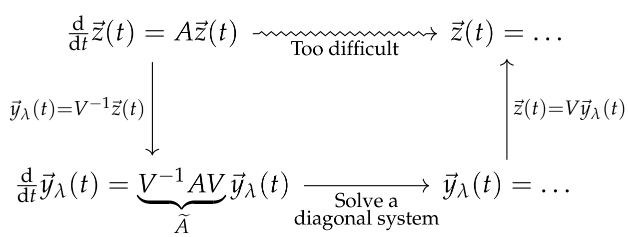


Figure 3: Change of Variables Strategy

(b) Notice that the solution in part 2.a is purely real, even though we had complex eigenvalues in our system. Now, we will investigate why this is the case. Indeed, we can define

$$\vec{z}(t) = \begin{bmatrix} a & \overline{a} \\ b & \overline{b} \end{bmatrix} \vec{y}(t) \tag{3}$$

where $\vec{y}(t) = \begin{bmatrix} c_0 e^{\lambda t} \\ \overline{c_0} e^{\overline{\lambda} t} \end{bmatrix}$ and $a, b, c_0, \lambda \in \mathbb{C}$ are arbitrary (nonzero) constants. **Show $\vec{z}(t)$ will be purely real.** *HINT: First show $x + \overline{x}$ is real for $x \in \mathbb{C}$. Also, recall $\overline{(x \cdot y)} = \overline{x} \cdot \overline{y}$ for $x, y \in \mathbb{C}$.* **Use this result to conclude that the solution in part 2.a is real.**

Contributors:

- Anish Muthali.
- Neelesh Ramachandran.
- Kumar Krishna Agrawal.
- Anant Sahai.
- Regina Eckert.
- Kuan-Yun Lee.

