Homework 6

This homework is due on Sunday, July 31 at 11:59 pm PT.

1. Spectral Theorem for Real Symmetric Matrices

We want to show that every real symmetric matrix can be diagonalized by a matrix of its orthonormal eigenvectors. In other words, a symmetric matrix $S \in \mathbb{R}^{n\times n}$, i.e., a matrix S such that $S = S^\top$, can be written as $S = V\Lambda V^\top$, where $V \in \mathbb{R}^{n\times n}$ is an orthonormal matrix of eigenvectors of S and $\Lambda \in \mathbb{R}^{n\times n}$ is a diagonal matrix of corresponding real eigenvalues of S. This is called the Spectral Theorem for real symmetric matrices.

To do this, we will use a proof which is similar to the proof of existence of the Schur decomposition. Along the way, we will practice block matrix manipulation and the induction proof technique.

(a) One part of the spectral theorem can be proved without any further delay. **Prove that the eigenvalues λ of a real, symmetric matrix S are real.**

(HINT: Let λ be an eigenvalue of S with corresponding nonzero eigenvector $\vec{v}$. Evaluate $\overline{\vec{v}}^\top S\vec{v}$ in two different ways: $\overline{\vec{v}}^\top (S\vec{v})$ and $(\overline{\vec{v}}^\top S)\vec{v}$. What does this show about λ?)

(b) For the main proof that every real symmetric matrix is diagonalized by a matrix of its orthonormal real eigenvectors, we will proceed by *induction*.

Recall that an inductive proof trying to prove a statement that depends on n, say P_n[1], is true for all positive integers n, has two steps:

- A base case – prove that P_1 is true.
- An inductive step – for every $n \geq 2$, given that P_{n-1} is true, prove that P_n is true.[2]

By doing these two steps, we show P_n is true for all n.

In our case, the statement P_n is "every $n \times n$ symmetric matrix S can be diagonalized as $S = V\Lambda V^\top$, where V is the real orthonormal matrix of eigenvectors of S, and Λ is the real diagonal matrix of corresponding eigenvalues of S."

Show the base case: every 1×1 symmetric matrix S can be written as $S = V\Lambda V^\top$, where V is a real and orthonormal matrix of eigenvectors of S, and Λ is a real and diagonal matrix of corresponding eigenvalues of S.

(HINT: Every 1×1 matrix is symmetric, and also diagonal, by definition; the only real orthonormal 1×1 matrices are $\begin{bmatrix}1\end{bmatrix}$ and $\begin{bmatrix}-1\end{bmatrix}$.)

[1] Lecture used S_n, but S is already being used for symmetric matrix here.

[2] This is the so-called *weak induction* paradigm; it contrasts with *strong induction*, which you can learn in future classes like CS70.

(c) With the base case done, we are now in the inductive step. Let S be an arbitrary $n \times n$ symmetric matrix; ultimately, we want to show that $S = V\Lambda V^\top$, where V is a real and orthonormal matrix of eigenvectors of S, and Λ is a real and diagonal matrix of corresponding eigenvalues of S.

To start, let λ be an eigenvalue of S, and let $\vec{q}$ be any normalized eigenvector of S corresponding to eigenvalue λ. Let $\widetilde{Q} \in \mathbb{R}^{n\times(n-1)}$ be a set of orthonormal vectors chosen so that $Q := \begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix} \in \mathbb{R}^{n\times n}$ is an orthonormal matrix.[3] **Show the following equality:**

$$Q^\top S Q = \begin{bmatrix} \lambda & \vec{0}_{n-1}^\top \\ \vec{0}_{n-1} & S_0 \end{bmatrix} \qquad \textbf{where} \qquad S_0 := \widetilde{Q}^\top S \widetilde{Q}. \tag{1}$$

(HINT: Expand Q as a block matrix $\begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix}$ *and multiply* $Q^\top S Q = \begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix}^\top S \begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix}$*.)*
(HINT: Since Q is orthonormal, we have $Q^\top Q = I_n$*. What does this mean for the values of* $\vec{q}^\top \vec{q}$ *and* $\widetilde{Q}^\top \vec{q}$*? Use block matrix multiplication on* $Q^\top Q = \begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix}^\top \begin{bmatrix} \vec{q} & \widetilde{Q} \end{bmatrix}$ *again.)*

(d) **Show that the matrix S_0 is a real symmetric matrix.**

(e) Since S_0 is a real symmetric $(n-1) \times (n-1)$ matrix, by our inductive assumption, S_0 can be orthonormally diagonalized as $S_0 = V_0 \Lambda_0 V_0^\top$, where Λ_0 is a real diagonal matrix of eigenvalues of S_0 and $V_0 \in \mathbb{R}^{(n-1)\times(n-1)}$ is a real orthonormal matrix of corresponding eigenvectors of S_0.

Define

$$V := Q \begin{bmatrix} 1 & \vec{0}_{n-1}^\top \\ \vec{0}_{n-1} & V_0 \end{bmatrix} \qquad \textbf{and} \qquad \Lambda := V^\top S V. \tag{2}$$

i. **Show that V is orthonormal.**

ii. **Show that Λ is diagonal.**

iii. **Show that $S = V\Lambda V^\top$.**

(HINT: Use block matrix multiplication again.)

Thus, we have found a real orthonormal V and real diagonal Λ such that $S = V\Lambda V^\top = V\Lambda V^{-1}$. We have seen in a previous homework that if $A = V\Lambda V^{-1}$, then Λ are the eigenvalues of A, and V are the corresponding eigenvectors. Thus, given P_{n-1} – the fact that we can orthonormally diagonalize $(n-1) \times (n-1)$ real symmetric matrices – we have proven P_n – the fact that we can orthonormally diagonalize $n \times n$ real symmetric matrices. Thus, we've proved the Spectral Theorem for real symmetric matrices by induction!

[3] This matrix $\widetilde{Q}$ can be generated via Gram-Schmidt, for example.

2. SVD

(a) Consider the matrix

$$A = \begin{bmatrix} -1 & 1 & 5 \\ 3 & 1 & -1 \\ 2 & -1 & 4 \end{bmatrix}.$$

Observe that the columns of matrix A are mutually orthogonal with norms $\sqrt{14}, \sqrt{3}, \sqrt{42}$.

Verify numerically that columns $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ **and** $\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}$ **are orthogonal to each other.**

(b) **Write $A = BD$, where B is an orthonormal matrix and D is a diagonal matrix.** What is B? What is D?

(c) **Write out a singular value decomposition of $A = U\Sigma V^\top$ using the previous part.** Note the ordering of the singular values in Σ should be from the largest to smallest. *(HINT: There is no need to compute the eigenvalues of anything. Use Theorem 14, Note 16.)*

(d) Given the matrix

$$A = \frac{1}{\sqrt{50}} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} + \frac{3}{\sqrt{50}} \begin{bmatrix} -4 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}, \tag{3}$$

write out a singular value decomposition of matrix A in the form $U\Sigma V^\top$. Note the ordering of the singular values in Σ should be from the largest to smallest. *(HINT: You don't have to compute any eigenvalues for this. Some useful observations are that*

$$\begin{bmatrix} 3, 4 \end{bmatrix} \begin{bmatrix} -4 \\ 3 \end{bmatrix} = 0, \quad \begin{bmatrix} 1, -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0, \quad \| \begin{bmatrix} 3 \\ 4 \end{bmatrix} \| = \| \begin{bmatrix} -4 \\ 3 \end{bmatrix} \| = 5, \quad \| \begin{bmatrix} 1 \\ -1 \end{bmatrix} \| = \| \begin{bmatrix} 1 \\ 1 \end{bmatrix} \| = \sqrt{2}.$$

)

(e) Define the matrix

$$A = \begin{bmatrix} -1 & 4 \\ 1 & 4 \end{bmatrix}.$$

Find the SVD of A by following the standard algorithm introduced in Note 16, i.e. by computing the eigendecomposition of $A^\top A$. Also find the eigenvectors and eigenvalues of A. Is there a relationship between the eigenvalues or eigenvectors of A with the SVD of A?

3. The Moore-Penrose pseudoinverse

Say we have a set of linear equations given by $A\vec{x} = \vec{y}$. If A is invertible, then the unique solution for $\vec{x}$ is $\vec{x} = A^{-1}\vec{y}$. However, what if A is not a square matrix, and we still wanted to find an $\vec{x}$ that satisfied $A\vec{x} = \vec{y}$? We know that we could use a linear least-squares approach for "tall" matrices A where it isn't possible to find a solution that exactly matches all the measurements. The linear least-squares solution gives us a reasonable answer that asks for the "best" match in terms of reducing the norm of the error vector.

How about when the matrix A is "wide", i.e. A has more columns than rows? In this case, there are generally going to be lots of possible solutions — so which should we choose? To address this, we introduce the *Moore-Penrose pseudoinverse*, which generalizes the idea of the matrix inverse and can be calculated using the singular value decomposition.

Since the SVD of a matrix always exists, the Moore-Penrose pseudoinverse does as well. Another useful property of the Moore-Penrose pseudoinverse $A^\dagger$ is that the solution it gives, $\hat{\vec{x}} = A^\dagger\vec{y}$, satisfies a minimality property: $\|\hat{\vec{x}}\| \leq \|\vec{z}\|$ for all $\vec{z}$ such that $A\vec{z} = \vec{y}$.

(a) Say we have the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

To find the Moore-Penrose pseudoinverse we start by calculating the SVD of A. That is to say, we find orthonormal matrices U and V, and diagonal matrix Σ, such that $A = U\Sigma V^\top$.

Here we give you the decomposition of A as:

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{4}$$

where:

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{5}$$

$$\Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \tag{6}$$

$$V^\top = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{7}$$

It is a good idea to be able to calculate the SVD yourself as you may be asked to solve similar questions on your own in the exam. **You do not have to do any work for this part.**

(b) Suppose we have non-zero singular values $\sigma_1, \ldots, \sigma_r$, and that we have written the SVD matrices so that Σ is in the form

$$\Sigma = \underbrace{\begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_r & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}}_{\text{Dimension: } m \times n}. \tag{8}$$

Consider the action of Σ on $\vec{v} \in \mathbb{R}^n$, i.e. $\Sigma\vec{v}$. **What is the effect of Σ on each element of $\vec{v}$?**

Let us define the following matrix:

$$\widetilde{\Sigma} = \underbrace{\begin{bmatrix} \frac{1}{\sigma_1} & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_2} & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sigma_r} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}}_{\text{Dimension: } n \times m}. \tag{9}$$

What is $\widetilde{\Sigma}\Sigma$? What is the effect of $\widetilde{\Sigma}\Sigma$ on $\vec{v} \in \mathbb{R}^n$?

(c) Consider when $A = U\Sigma V^\top$ acts on $\vec{x}$ to give the result $\vec{y}$, i.e.

$$A\vec{x} = U\Sigma V^\top \vec{x} = \vec{y}. \tag{10}$$

Observe that $V^\top \vec{x}$ rotates $\vec{x}$ without changing its length, and U rotates $\Sigma V^\top \vec{x}$ again. The *Moore-Penrose pseudoinverse* $A^\dagger$ is given as

$$A^\dagger = V\widetilde{\Sigma}U^\top, \tag{11}$$

where $\widetilde{\Sigma}$ is given in (c). Consider if we apply the Moore-Penrose pseudoinverse to find a candidate solution $A^\dagger y$:

$$\vec{y} = U\Sigma V^\top \vec{x} \tag{12}$$

$$A^\dagger y = (V\widetilde{\Sigma}U^\top)(U\Sigma V^\top)\vec{x}. \tag{13}$$

Qualitatively, what are the effects of the matrices V, $\widetilde{\Sigma}$, and $U^\top$ in the Moore-Penrose pseudoinverse when finding a solution?

(d) **What does the Moore-Penrose pseudoinverse give as a solution $\vec{x}$ in the following system of equations?**

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

Confirm that your solution indeed satisfies the system of equations.

4. Rank 1 Decomposition

In this problem, we will decompose a few images into linear combinations of rank 1 matrices. Remember that outer product of two vectors $\vec{s}\vec{g}^\top$ gives a rank 1 matrix. It has rank 1 because the column span is one-dimensional — multiples of $\vec{s}$ only — and the row span is also one dimensional — multiples of $\vec{g}^\top$ only.

These decompositions are useful for understanding the outer product form of the SVD.

(a) Consider a standard 8×8 chessboard shown in Figure 1. Assume that black colors represent -1 and that white colors represent 1.

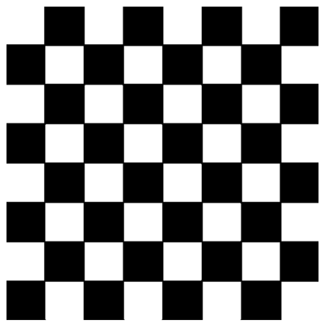


Figure 1: 8×8 chessboard.

In particular, the chessboard is given by the following 8×8 matrix C_1:

$$C_1 = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix} \tag{14}$$

Express C_1 as a linear combination of outer products.

(HINT: In order to determine how many rank 1 matrices you need to combine to represent the matrix, find the rank of the matrix you are trying to represent.)

(b) For the same chessboard shown in Figure 1, now assume that black colors represent 0 and that white colors represent 1.

In particular, the chessboard is given by the following 8×8 matrix C_2:

$$C_2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{15}$$

Express C_2 as a linear combination of outer products.

(HINT: If you find yourself getting stuck, one way to proceed is to note that $\text{rank}(C_2) = 2$*, so you will need to sum two outer products. Try to have one outer product that fills in the odd columns and is* 0 *elsewhere, and one outer product that fills in the even columns and is* 0 *elsewhere.)*

(c) Now consider the Swiss flag shown in Figure 2. Assume that red colors represent 0 and that white colors represent 1.

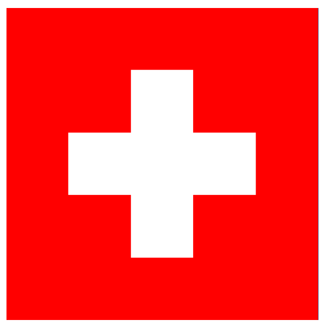


Figure 2: Swiss flag.

Assume that the Swiss flag is given by the following 5×5 matrix S:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

Furthermore, we know that the Swiss flag can be viewed as a superposition of the following pairs of images:

Figure 3: Pairs of images - Option 1

Figure 4: Pairs of images - Option 2

Express the S in two different ways:

i. **as a linear combination of outer products inspired by the Option 1 images.**

ii. **as a linear combination of outer products inspired by the Option 2 images.**

5. Frobenius Norm

In this problem we will investigate the basic properties of the Frobenius norm.

Similar to how the norm of vector $\vec{x} \in \mathbb{R}^n$ is defined as $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$, the Frobenius norm of a matrix $A \in \mathbb{R}^{m\times n}$ is defined as

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |A_{ij}|^2}. \tag{17}$$

A_{ij} is the entry in the i^{th} row and the j^{th} column. This is basically the norm that comes from treating a matrix like a big vector filled with numbers.

(a) With the above definitions, **show that for a** 2×2 **matrix** A**:**

$$\|A\|_F = \sqrt{\operatorname{tr}\left(A^\top A\right)}. \tag{18}$$

Note: The trace of a matrix is the sum of its diagonal entries. For example, let $A \in \mathbb{R}^{m\times n}$, then,

$$\operatorname{tr}(A) = \sum_{i=1}^{\min(n,m)} A_{ii} \tag{19}$$

Think about how/whether this expression eq. (18) generalizes to general $m \times n$ matrices.

(b) **Show that if** U **and** V **are square orthonormal matrices, then**

$$\|UA\|_F = \|AV\|_F = \|A\|_F. \tag{20}$$

(HINT: Use the trace interpretation from part (a).)

(c) **Use the SVD decomposition to show that** $\|A\|_F = \sqrt{\sum_{i=1}^n \sigma_i^2}$**, where** $\sigma_1, \ldots, \sigma_n$ **are the singular values of** A**.**

(HINT: The previous part might be quite useful.)

6. Movie Ratings and PCA

Recall from the lecture on PCA that we can think of movie ratings as a structured set of data. For every person i and movie j, we have that person's rating $R_{i,j}$ (thought of as a real number).

Suppose that there are m movies and n people. Let's think about arranging this data into a big $n \times m$ matrix R with people corresponding to rows and movies corresponding to columns. So the entry in the i-th row and j-th column should be $R_{i,j}$ above. Note that this is organized differently from how it was in lecture. Each row corresponds to a unique person and each column to a unique movie.

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix} \tag{21}$$

(a) Suppose we believe that there is actually an underlying pattern to this rating data and that a separate study has revealed that every movie is characterized by a set of characteristics: say action and comedy. This means that every movie j has a pair of numbers a_j (for action) and c_j (for comedy) that define it. At the same time, every person i has a pair of sensitivities f_i and g_i that defines that person's preferences for action vs. comedy movies respectively. A person i will rate the movie j as $R_{i,j} = f_i a_j + g_i c_j$.

If we arrange the sensitivities into a pair of n-dimensional vectors $\vec{f}, \vec{g}$ for our group of n people, and the movie characteristics into a pair of m-dimensional vectors $\vec{a}, \vec{c}$ for our group of m movies, **use outer products to express the rating matrix R in terms of these vectors $\vec{f}, \vec{g}, \vec{a}, \vec{c}$.**

(b) Now suppose that we want to discover the underlying nature of movies from the data R itself.

Suppose for this part, that you have four observed rating data vectors (corresponding to four different movies being rated by six individuals).

All of the movie data vectors just happened to be multiples of the following 6-dimensional vector $\vec{w} = \begin{bmatrix} 2 \\ -2 \\ 3 \\ -4 \\ 4 \\ 0 \end{bmatrix}$. (For your convenience, note that $\|\vec{w}\| = 7$.)

You arrange the movie data vectors as the columns of a matrix R given by:

$$R = \begin{bmatrix} | & | & | & | \\ -\vec{w} & -2\vec{w} & 2\vec{w} & 4\vec{w} \\ | & | & | & | \end{bmatrix} \tag{22}$$

You want to perform PCA (for movies) using the SVD of the matrix R to better understand the pattern in your data.

The first "principal component vector" is a unit vector that tells which direction we would want to project the columns of R onto to get the best rank-1 approximation for R.

Find this first principal component vector of the columns of R to explain the nature of your movie data points.

(HINT: You don't need to actually compute any SVDs in this simple case. Also, be sure to think about what size vector you want as the answer. Don't forget that you want a unit vector!)

(c) Suppose that now, we have two more data points (corresponding to two more movies being rated by the same set of six people, i.e. we added two columns to our matrix) that are multiples of a different vector $\vec{p}$ where:

$\vec{p} = \begin{bmatrix} 6 \\ 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. (For your convenience, note that $\|\vec{p}\| = 7$ and that $\vec{p}^\top \vec{w} = 0$.)

We augment our ratings data matrix with these two new data points to get:

$$R = \begin{bmatrix} | & | & | & | & | & | \\ -\vec{w} & -2\vec{w} & 2\vec{w} & 4\vec{w} & -3\vec{p} & 3\vec{p} \\ | & | & | & | & | & | \end{bmatrix} \tag{23}$$

Find the first two principal components corresponding to the nonzero singular values of R. This is what we would use to best project the movie data points onto a two-dimensional subspace.

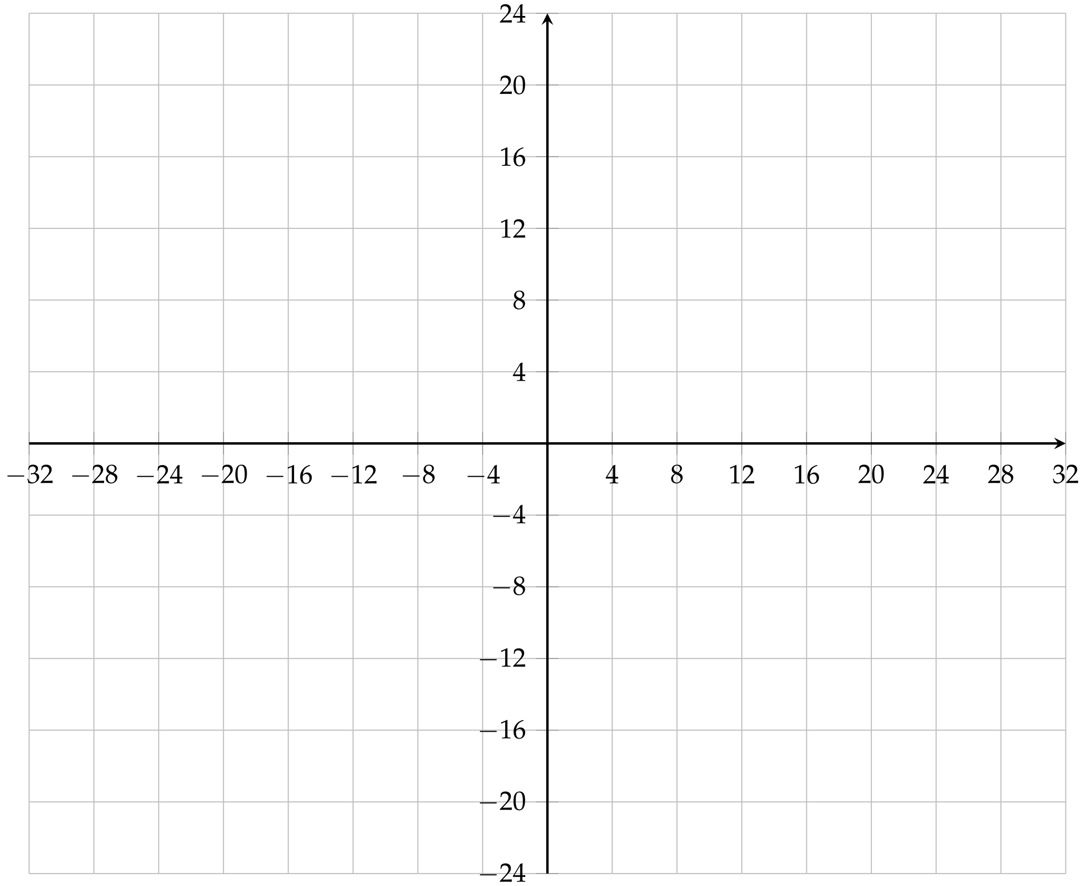
What is the first principal component vector? What is the second principal component vector? Justify your answer. *(HINT: Think about the inner product of $\vec{w}$ and $\vec{p}$ and what that implies for being able to appropriately decompose R. Again, very little computation is required here.)*

(d) In the previous part, you had

$$R = \begin{bmatrix} | & | & | & | & | & | \\ -\vec{w} & -2\vec{w} & 2\vec{w} & 4\vec{w} & -3\vec{p} & 3\vec{p} \\ | & | & | & | & | & | \end{bmatrix}$$

with $\|\vec{w}\| = 7$ and $\|\vec{p}\| = 7$, satisfying $\vec{p}^\top \vec{w} = 0$.

If we use $\vec{r}_i$ to denote the i-th column of R, **plot the movie data points $\vec{r}_i$ (for all i) projected onto the first and second principal component vectors along the columns of R.** The coordinate along the first principal component should be represented by horizontal axis and the coordinate along the second principal component should be the vertical axis. **Label each point, and the axes. Remember that principal component vectors are normalized**.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7. Homework Process, Study Group, and Course Weekly Survey

Citing sources and collaborators are an important part of life, including being a student!

We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

At the same time, we want to check-in weekly regarding Discussions, Lectures, Lab, and Office Hours and see how effective they have all been for you as students.

Please fill out this survey link. For your submission, please attach a screenshot indicating that you have completed the survey this week.

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