EECS 16B Designing Information Devices and Systems II
Summer 2022 UC Berkeley Lab Note 5

Lab 5: Sensing Part 2

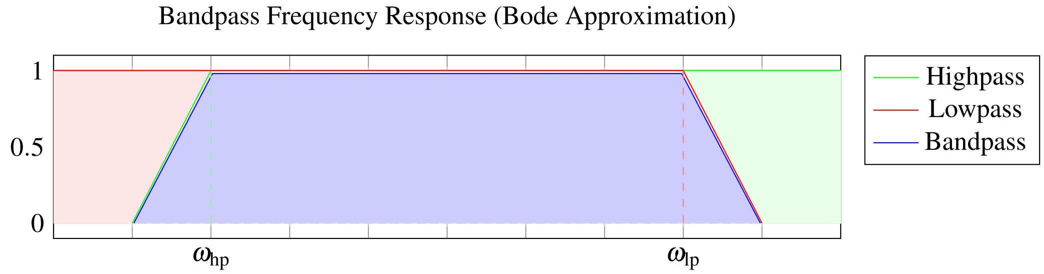
In this part of lab, we will explore cascade design, one of the most popular methods for designing filters.

Part 1: Caught in the Midrange

First of all, we will be building a band-pass filter to isolate the mid-range frequencies. This filter can be made simply by chaining/cascading a low-pass filter and a high-pass filter together. You will need to choose two new cutoff frequencies ω_{hp} and ω_{lp}. Important questions to ask yourself while designing this filter include:

- Which of the two cutoff frequencies should be higher?
- What's a good amount of space to leave between your bandpass cutoffs and your bass and treble filter cutoffs?

The frequency response of the bandpass will look something like this:



Sanity check question: *Does the order of the filters matter? I.e. does it matter whether we chain low-pass into high-pass or vice versa?*

In order to accomplish this, we want the transfer functions of our two filters to multiply. However, we need to be careful: we cannot simply plug the output of one of the filters into the input of the other directly; doing so would "load" the first filter and affect its cutoff frequency because the second filter ends up drawing current from the first. If we want to make sure the transfer functions to multiply and give us the desired band-pass frequency response without affecting each other, we need to somehow isolate the first filter from the second while still passing the output of the first to the input of the second. Here is where buffers come into play.

Buffers

You can think of a buffer as providing an impedance transformation between two *cascaded* circuits. When you observe an undesired loading effect between two circuits, placing a buffer between them changes the load impedance of the first circuit to a very high value and the source impedance of the second to a very low value in accordance with (approximately) ideal op-amp characteristics. As the op-amp does not allow any current to flow into its input terminals, this prevents the second filter from drawing current from the first filter and affecting the frequency response. Instead, the second filter draws its current from the output of the op-amp, which is a replica of the first filter's output due to this being a buffer circuit. This allows you to build very modular circuits easily, without having to do lots of ugly algebra.

By placing a buffer in between our two filters that make up the band-pass filter, *cascading them does not change the transfer functions of the individual circuits* and the overall transfer function of the cascade is simply the product of the transfer functions of the individual circuits. This is why buffers are so useful in filter design. However, op-amps (and therefore these buffer circuits) do have their limits; they are not quite as perfect as the golden rules describe them to be. More details about this can be found in the appendix.

Go to the ipython notebook and complete Part 1.

Part 2: Notch filter

So far, we have built low-pass, high-pass, and band-pass filters with resistors and capacitors to isolate signals at different frequencies. Are they enough for all applications? Of course not. There are more interesting filters that can be designed with the help of another electric device, the inductor. Let's consider the following RLC circuit:

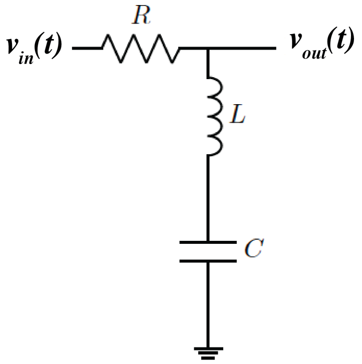


Figure 1: Notch filter.

Its transfer function can be written as:

$$H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \frac{Z_L + Z_C}{Z_L + Z_C + Z_R} = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

It's obvious that $|H(j\omega)|$ will be equal to 0 if $\omega L = \frac{1}{\omega C}$, which means that, if we know the exact frequency of the interference signal, we can use this type of RLC filter (notch filter), to create a notch at that frequency and then completely filter the interference signal out, which is impossible to be achieved by an RC filter. The frequency of the notch location, also known as the natural frequency of the LC tank, is $\omega_0 = \frac{1}{\sqrt{LC}}$.

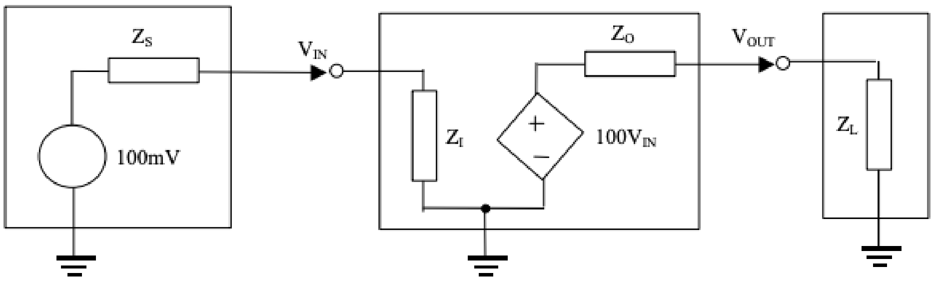
Sanity check question: *Does the resistor value in the notch filter matter? I.e. how does it impact the response of the notch filter?*

Now go to the ipython notebook and complete Part 2.

Appendix A: Amplifier Loading

Recall the impedance characteristics of the ideal op-amp: its input impedance is infinite (no current flows into its inputs) and its output impedance is 0. This allows it to act like an ideal voltmeter at the input and supply infinite current at its output. But what happens if we don't have those characteristics?

Let's assume we have a noninverting amplifier with a gain of 100, with input impedance Z_I and output impedance Z_o, as in the schematic below. The middle box represents the amplifier.



We now see that V_{in} depends on the source output impedance Z_S and the amplifier input impedance Z_I (because of the voltage divider formed by the two), and V_{out} depends on the amplifier output impedance Z_O and the load impedance Z_L. Recalling the voltage divider equation,

$$V_{out} = (100V_{IN})\frac{Z_L}{Z_L + Z_O}$$

But $100V_{IN}$ is our desired V_O! To keep that approximately correct and avoid "loading" the output and reducing the voltage noticeably from what we expect, we need Z_L to be considerably larger than Z_O to keep $\frac{Z_L}{Z_L+Z_O}$ as close to 1 as possible.

For our circuits, including the ones in this lab, you may assume that the load impedance is always sufficiently large enough and the op-amps' output impedance is small enough so that the loading effect is negligible. Thus, the outputs of our op-amp circuits, like the buffers we're using in this lab, remain unaffected by whatever we connect it to, and we can safely use a buffer to build our band-pass filter.

References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.
Sedra, A. and Smith, K. (2015). *Microelectronic Circuits*. 7th ed. New York: Oxford University Press, ch 17.

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