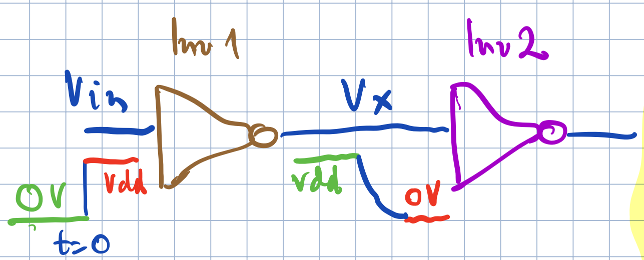
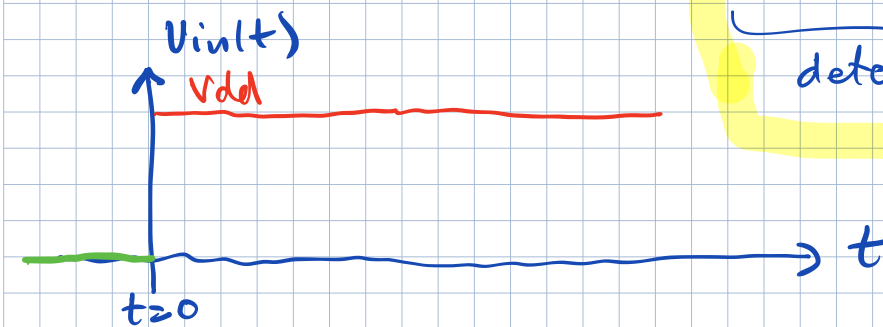
Lecture 3

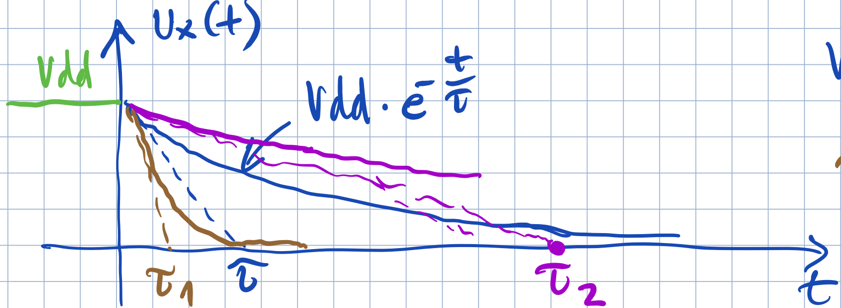
* Computing : Transistors & Logic
  * RC transients finish
  * Non-homogeneous diff. equns.
    * constant input
    * piece-wise constant input
    * continuous input
* Sensing



$$V_x(t) = V_{dd}\, e^{-\frac{t}{\tau}}, \quad t \geq 0$$

$$\underbrace{\tau = R_{on,n_1} \cdot (C_{Gn_2} + C_{Gp_2})}_{\text{determines the speed of transition}}$$





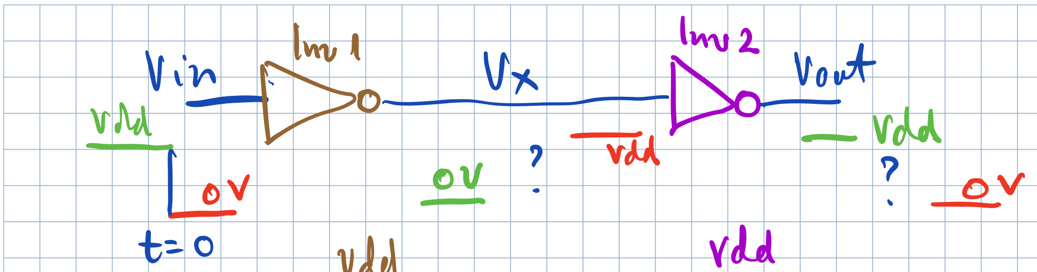
What happens for

$\tau_1 < \tau < \tau_2$

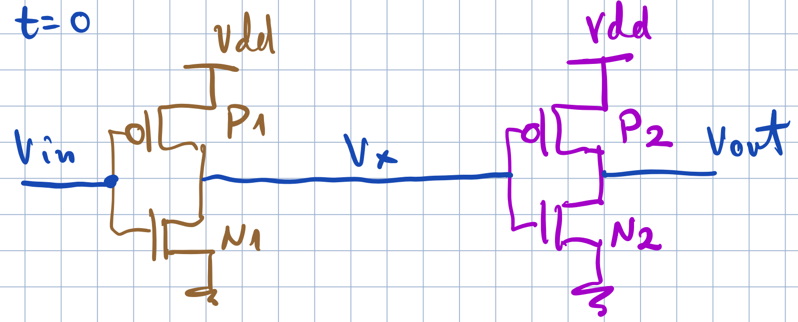
If you make transistor smaller $\Rightarrow \tau \downarrow$

Dennard scaling $\Rightarrow$ Moore's law
(economics law)

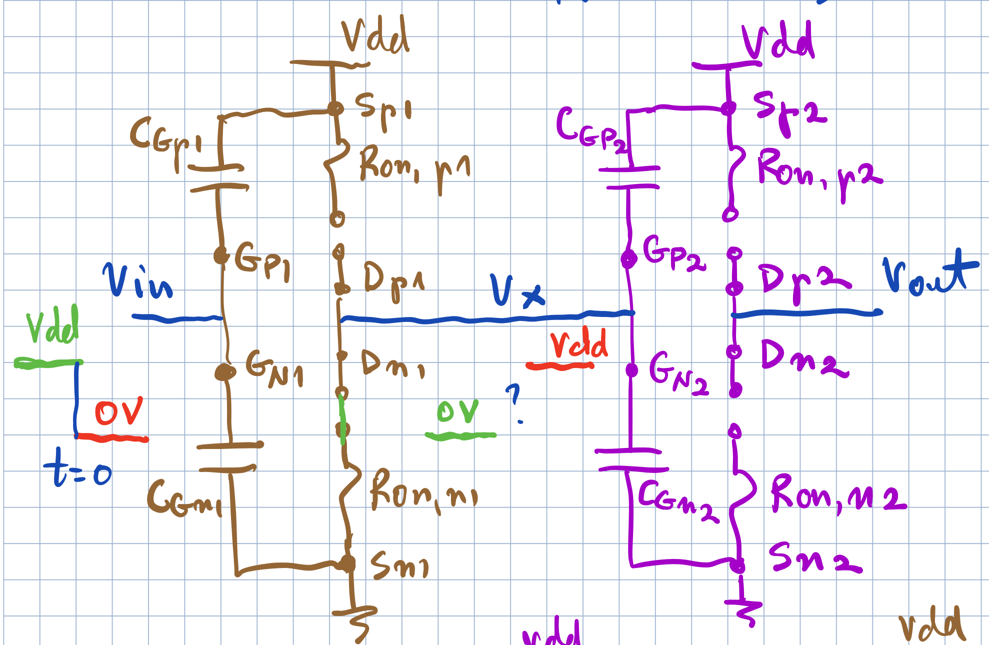
Logic view

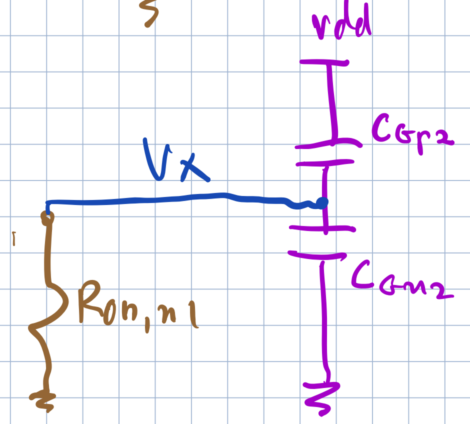


Schematic view (CMOS Logic)



⇓ RC models



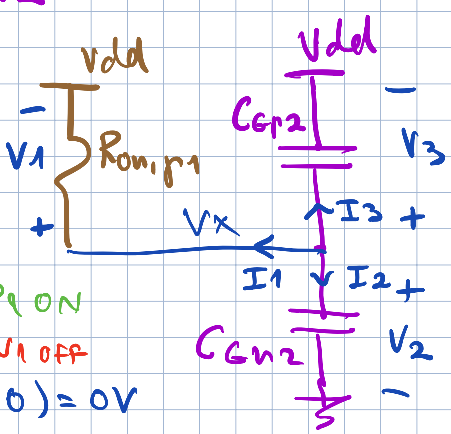


t=0 ⇒

P1 ON

N1 OFF

$V_x(0) = 0V$



$t < 0$, $V_{in} = V_{dd}$, N1 ON, P1 OFF

$V_x = 0V$

$t \geq 0$

KCL: $I_1 + I_2 + I_3 = 0$

Voltages:

$V_1 = V_x - V_{dd}$

$V_2 = V_x$

$V_3 = V_x - V_{dd}$

Elements:

$V_1 = I_1 \cdot R_{on,p1}$

$I_2 = C_{Gn2} \cdot \frac{dV_2}{dt}$

$I_3 = C_{Gp2} \cdot \frac{dV_3}{dt}$

From KCL & Elements:

$$\underbrace{\frac{V_1}{R_{on,p1}}}_{I_1} + \underbrace{C_{Gn2}\frac{dV_2}{dt}}_{I_2} + \underbrace{C_{Gp2}\frac{dV_3}{dt}}_{I_3} = 0$$

From voltages:

$$\frac{V_x - V_{dd}}{R_{on,p1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{d}{dt}(V_x - V_{dd}) = 0$$

$$\left[\frac{d}{dt} V_{dd} = 0\right]$$

$$\frac{V_x - V_{dd}}{R_{on,p1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{dV_x}{dt} = 0$$

(1) $$\frac{V_x - V_{dd}}{R_{on,p1}} + (C_{Gn2} + C_{Gp2})\frac{dV_x}{dt} = 0$$

$$\frac{dV_x}{dt} = \underbrace{-\frac{V_x}{R_{on,p1}(C_{Gn2} + C_{Gp2})}}_{\text{homogeneous term}} + \underbrace{\frac{V_{dd}}{R_{on,p1}(C_{Gn2} + C_{Gp2})}}_{\text{non-homogeneous term}}$$

Form: $\frac{d}{dt} x(t) = \lambda x(t)$ (homogeneous)

$\frac{d}{dt} x(t) = \lambda x(t) + a$

(non-homogeneous)

Go back to (1)

$$\frac{V_x - V_{dd}}{R_{on1,p1}} + (C_{on2} + C_{op2}) \cdot \frac{d}{dt}(V_x - V_{dd}) = 0$$

Try change of variables to transform the problem into one we know how to solve.

$$\boxed{\tilde{V}_x = V_x - V_{dd}}$$

$$\frac{\tilde{V}_x}{R_{on1,p1}} + (C_{on2} + C_{op2}) \frac{d}{dt}\tilde{V}_x = 0$$

$$\boxed{\frac{d}{dt}\tilde{V}_x = -\frac{\tilde{V}_x}{R_{on1,p1} \cdot (C_{on2} + C_{op2})}}$$

We already know how to solve this!

☺

homogeneous diff. eqn.

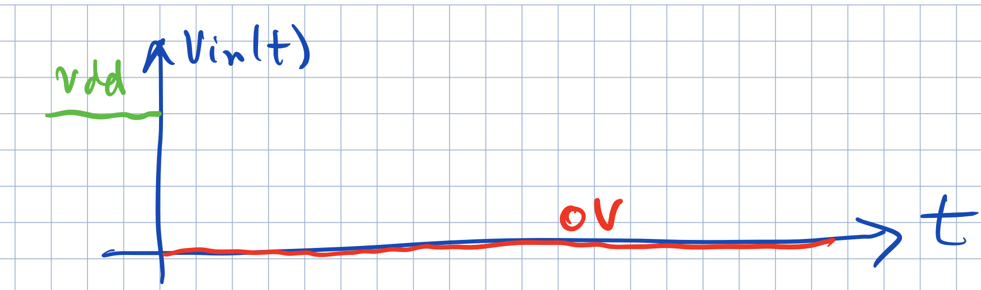
$$\tau = R_{on1,p1}(C_{on2} + C_{op2})$$

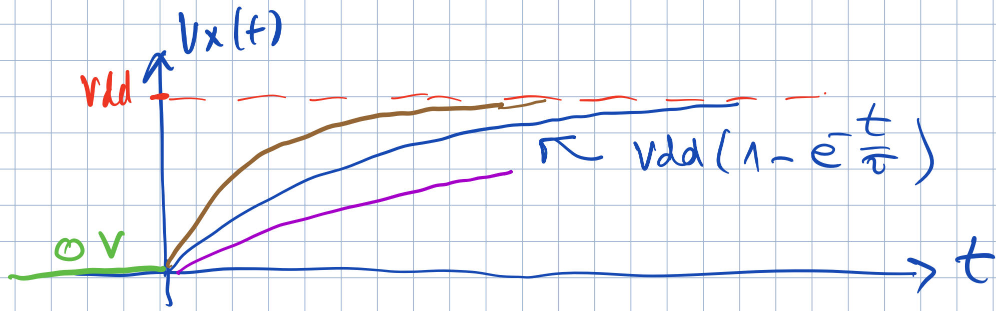
$$\tilde{V}_x(t) = \tilde{V}_x(0) \cdot e^{-\frac{t}{\tau}} \quad , t \geq 0$$

$V_x(0) = 0V$

$$V_x(t) - V_{dd} = (\cancel{V_x(0)}^{0} - V_{dd}) \cdot e^{-\frac{t}{\tau}}$$

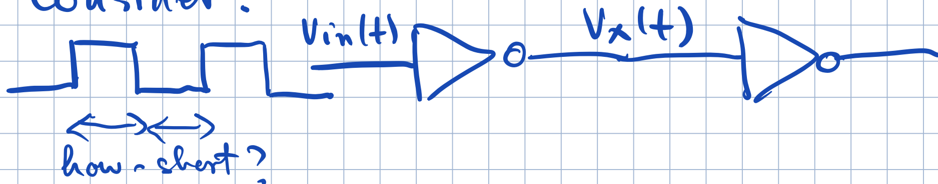
$$\boxed{V_x(t) = V_{dd}\left(1 - e^{-\frac{t}{\tau}}\right) \quad , t \geq 0}$$

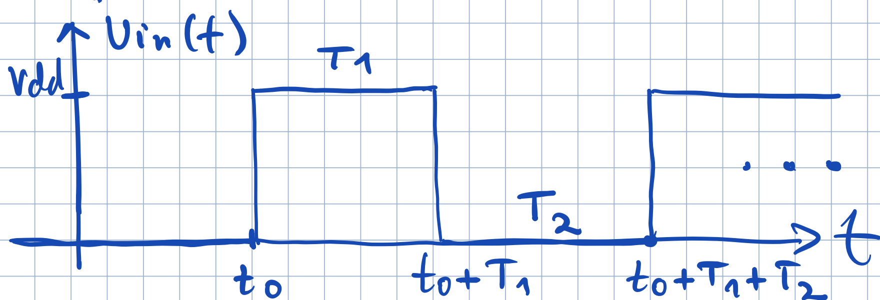




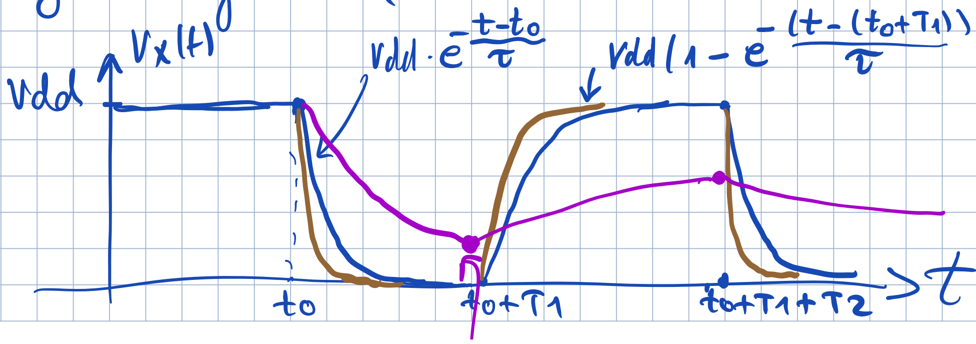
$\tau_1 < \tau < \tau_2$

Consider :





Will Vx(t) be able to follow these changes as a "logic" signal (i.e. to reach 0V or Vdd) ?



$\tau_1 < \tau < \tau_2$

init. condition is no longer ov.
it is actually $V_x(t_0+T_1) =$
$= Vdd \cdot e^{-\frac{t_0+T_1-t_0}{\tau_2}} =$
$= Vdd \cdot e^{-\frac{T_1}{\tau_2}}$

Use previous $V_x(t)$ solution as an initial condition for the next interval.

Solutions for piece-wise constant input:

Form: $\frac{d}{dt}x(t) = \lambda x(t) - \lambda u(t) \leftarrow$ constant or piece-wise constant

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What we also want to solve is $u(t) = u_c(t)$ ← continuous time.

$u_c(t)$

$u(t) \neq u_c(t)$

$u(t) \xrightarrow[\Delta \to 0]{} u_c(t)$

$u(t) = \text{const}, \; t \in [t_0, t_0+\Delta)$

iterate & use the previous solution as initial condition

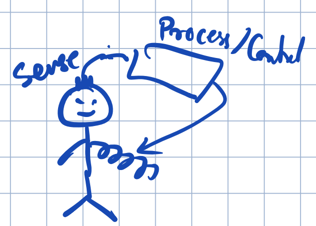
Then let $\Delta \to 0$ for $u(t)$ to approach $u_c(t)$ in the limit (disc. + hw)

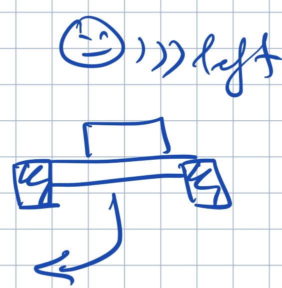
Why do we want the know the response to continuous time input?

EECS 16AB Pipeline

Analog world → Sense → Processing → Actuate → Analog world

analog ckts ← digital ckts





Sensing: brain signals or voice signals

$\underbrace{\text{signal of interest}}$ + ~~interference~~

(AC power, wifi/cellular, others talking in the lab, ...)

Design sensor ckts to pick this up

unwanted

The goal:

Filter/select signals of interest and reject interference.

How can a circuit become a filter - process the input continuous-time signal?