EE 16B Final, December 13, 2016

Name: ________________

SID #: ________________

Important Instructions:

- **Show your work.** An answer without explanation is not acceptable and does not guarantee any credit.
- **Only the front pages will be scanned and graded.** If you need more space, please ask for extra paper instead of using the back pages.
- **Do not remove pages**, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

Problem	**Points**	**Score**
1	15	
2	15	
3	15	
4	15	
5	10	
6	10	
7	10	
8	10	
Total	100	

1. (15 points) Consider the discrete-time system

$$\vec{x}(t+1) = A\vec{x}(t) + Bu(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

a) (3 points) Show that the system is controllable.

b) (5 points) We wish to move the state vector from $\vec{x}(0) = 0$ to

$$\vec{x}(T) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

Find the smallest possible time T and an input sequence $u(0), \ldots, u(T-1)$ to accomplish this task.

c) (7 points) Now suppose that the control input is subject to the constraint $|u(t)| \leq 1$ for all t; that is, we can't apply inputs with magnitude over 1. Find the smallest possible T under this constraint and an input sequence $u(0), \ldots, u(T-1)$ such that $\vec{x}(T)$ is as specified in part (b).

2. (15 points) In parts (a) and (b) below find a singular value decomposition,

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T.$$

a) (5 points)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\sigma_1 =$	$\vec{u}_1 =$	$\vec{v}_1^T =$
$\sigma_2 =$	$\vec{u}_2 =$	$\vec{v}_2^T =$

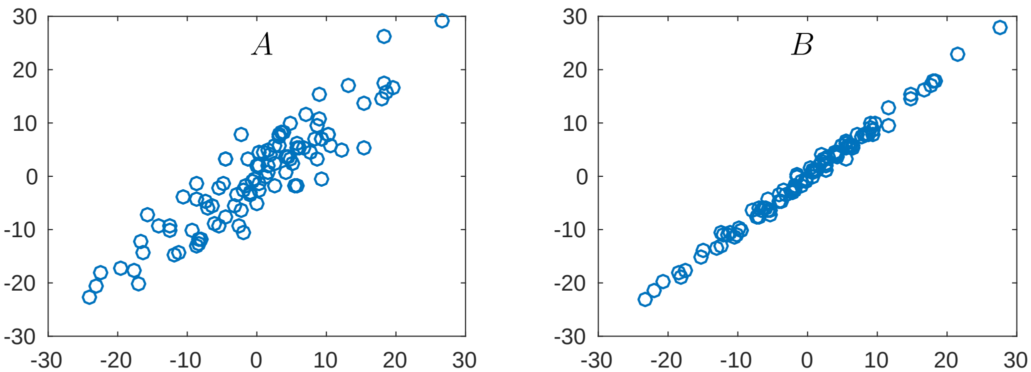
b) (6 points)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\sigma_1 =$ $\vec{u}_1 =$ $\vec{v}_1^T =$

$\sigma_2 =$ $\vec{u}_2 =$ $\vec{v}_2^T =$

c) (4 points) The plots below correspond to two 2×100 matrices, A and B. Each of the 100 points on the left plot represents a column vector of A and the right plot is constructed similarly for B. Answer the questions below based on the qualitative features of the plots rather than precise numerical values.



i) (2 points) Which matrix has the **biggest** ratio σ_1/σ_2 of the largest singular value σ_1 to the second singular value σ_2?

Answer: _____

ii) (2 points) Go to the plot A or B corresponding to your answer in part (i), and draw one line that shows the direction of the vector $\vec{u}_1$ and another line that shows the direction of the vector $\vec{u}_2$ in the singular value decomposition

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T.$$

Clearly label each line to indicate which one is for $\vec{u}_1$ and which one for $\vec{u}_2$.

3. (15 points) The continuous-time functions in parts (a)-(e) below are sampled with period $T = 1$. For each one determine whether the function can be recovered from its samples by sinc interpolation. If your answer is no, indicate what other continuous function would result from sinc interpolation.

a) $\cos\left(\frac{3\pi}{4}t\right)$

b) $\cos\left(\frac{5\pi}{4}t\right)$

For parts (c)-(e) below sketching the samples of the function with period $T = 1$ may help in determining whether the function can be recovered from these samples and, if not, what function results from sinc interpolation.

c) $\operatorname{sinc}(t) \triangleq \begin{cases} \frac{\sin(\pi t)}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases}$

d) $f(t) = \begin{cases} 1 - |t| & |t| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$

e) $\cos\left(\pi t + \frac{\pi}{3}\right)$

4. (15 points)

a) (6 points) Find a **<u>real-valued</u>** length-4 sequence $x(t)$, $t = 0, 1, 2, 3$, such that the DFT coefficients for $k = 1$ and $k = 2$ are

$$X(1) = 1 - j \quad X(2) = 0.$$

Is the answer unique? If not, give an example of another real-valued $x(t)$ with the same $X(1)$ and $X(2)$.

b) (5 points) Suppose a length-N sequence, where N is even, satisfies

$$x\left(t+\frac{N}{2}\right) = -x(t), \quad t = 0, 1, \ldots, \frac{N}{2} - 1$$

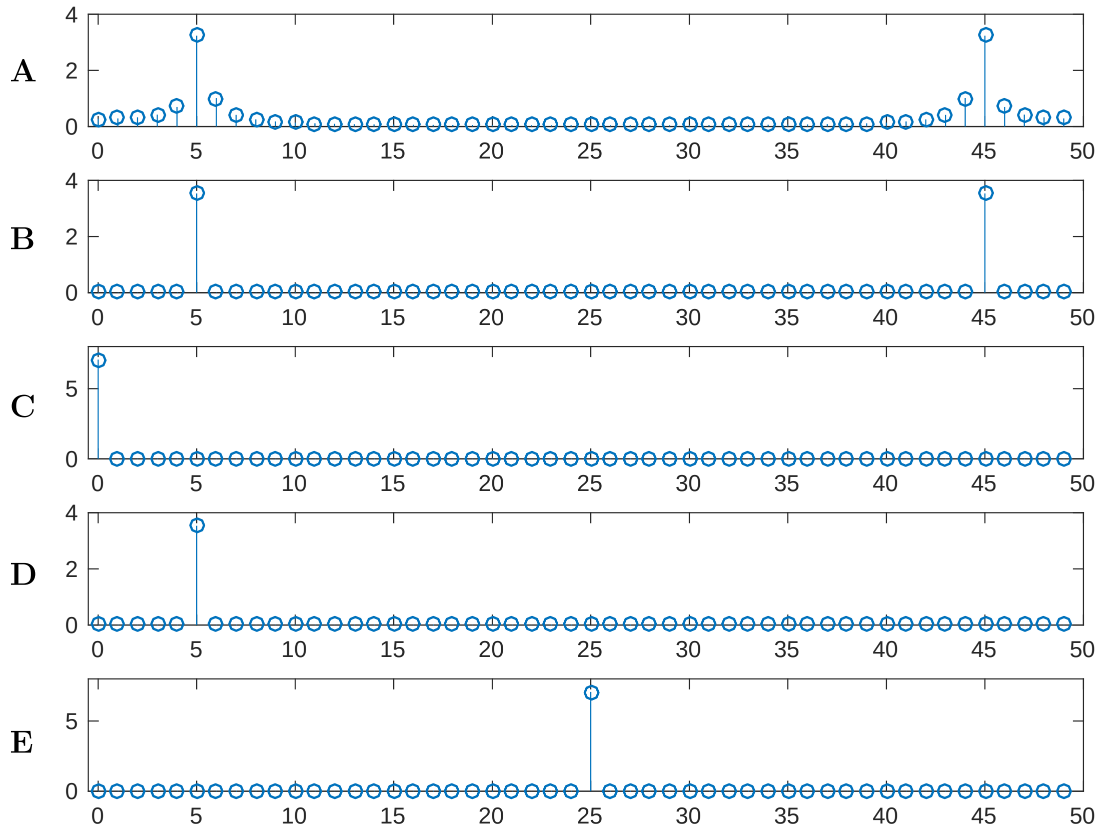
that is, the second half of the sequence is the negative of the first half. Show that

$$X(k) = 0 \quad \text{when } k \text{ is even.}$$

Hint: First find a relation between W_k^t and $W_k^{t+\frac{N}{2}}$ where $W_k = e^{jk\frac{2\pi}{N}}$.

c) (4 points) Plots A-E show the magnitude of the DFT for the length-50 sequences below. Match each sequence to a DFT plot:

$$\begin{array}{lcl} 1 \text{ for all } t & \rightarrow & \text{Plot } \underline{\hspace{2em}} \\ (-1)^t & \rightarrow & \text{Plot } \underline{\hspace{2em}} \\ \frac{1}{2}e^{j0.2\pi t} & \rightarrow & \text{Plot } \underline{\hspace{2em}} \\ \cos(0.2\pi t) & \rightarrow & \text{Plot } \underline{\hspace{2em}} \\ \cos(0.21\pi t) & \rightarrow & \text{Plot } \underline{\hspace{2em}} \end{array}$$



5. (10 points) Consider a system described by the input/output relation:

$$y(t) = a_0 u(t) + a_1 u(t-1) + \cdots + a_M u(t-M)$$

where M is a positive integer and $a_0, a_1, \ldots, a_M$ are constants.

a) (2 points) Determine if this system is linear and time-invariant. Explain your reasoning.

b) (2 points) Suppose the system above has impulse response

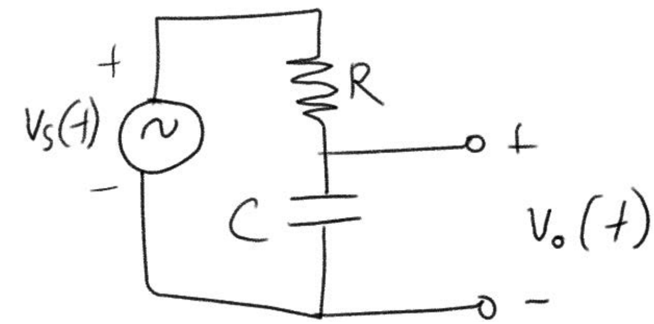
$$h(t) = \begin{cases} 0.5 & \text{when } t = 0 \\ -0.5 & \text{when } t = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the integer M and the coefficients $a_0, a_1, \ldots, a_M$.

c) (4 points) Find the DFT of $h(t)$, $t = 0, 1, \ldots, N-1$, where the length $N \geq 2$ is arbitrary. Your answer should provide a formula for $H(k)$, $k = 0, 1, \ldots, N-1$, that depends on k and the length N.

d) (2 points) Continuing part (c) now specify $H(k)$ when $k = 0$ and $k = N/2$ (assuming N is even). Which range of input frequencies (high or low) does this system suppress?

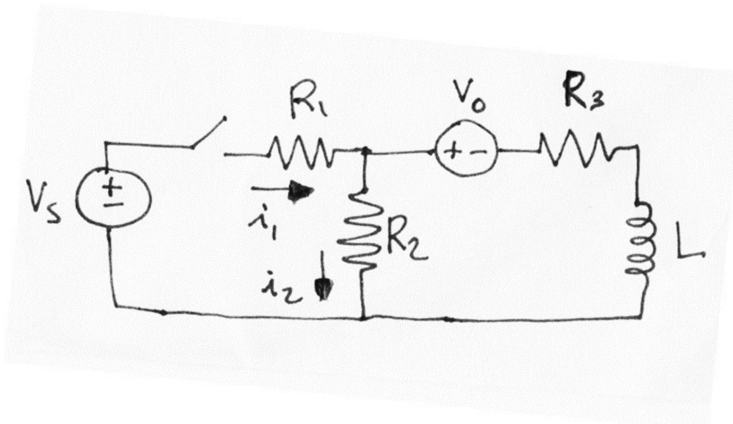
6. (10 points) Consider the circuit below where $v_s(t)$ is a sinusoidal voltage at a single frequency, ω.



a) (5 points) Provide a symbolic expression for $v_0(t)$ as a function of $v_s(t)$.

b) (5 points) If $C = 1\mu F$ and $\omega = 1$ rad/s find R such that the output is phase shifted by $\pi/4$ from the input.

7. (10 points) Consider the circuit below. The switch is closed until $t = 0$ s, then opened. V_s, V_0, R_1, R_2, R_3, and L are given.

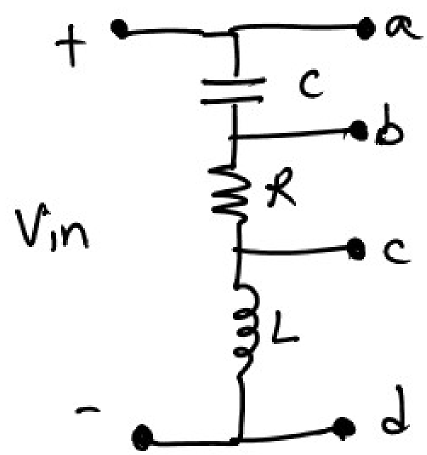


a) (2.5 points) Provide a symbolic expression for i_1 at $t < 0$ s.

b) (2.5 points) Provide a symbolic expression for i_2 at $t < 0$ s.

c) (5 points) Provide a symbolic expression for i_2 at $t \geq 0$ s.

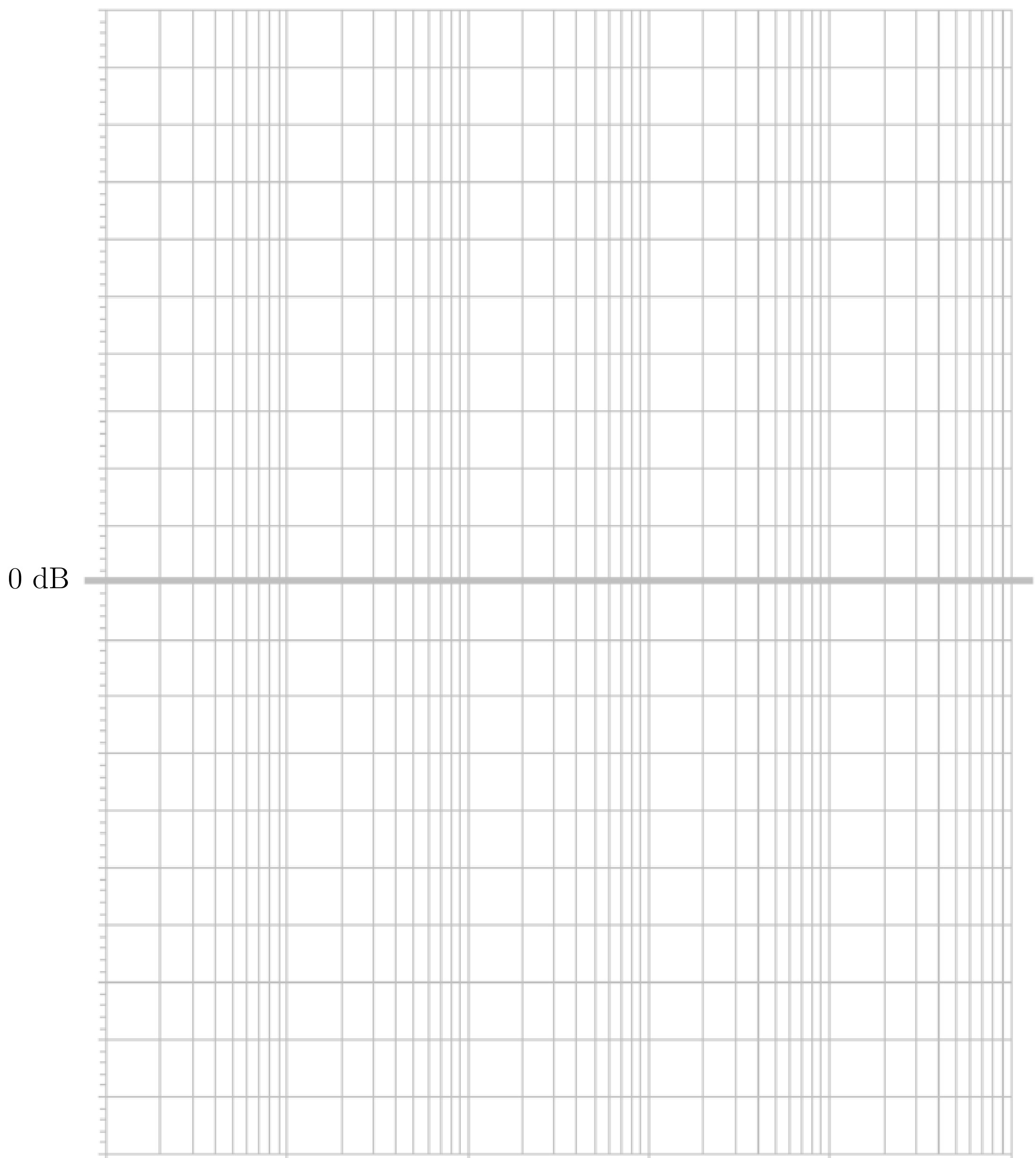
8. (10 points) Consider the circuit below.



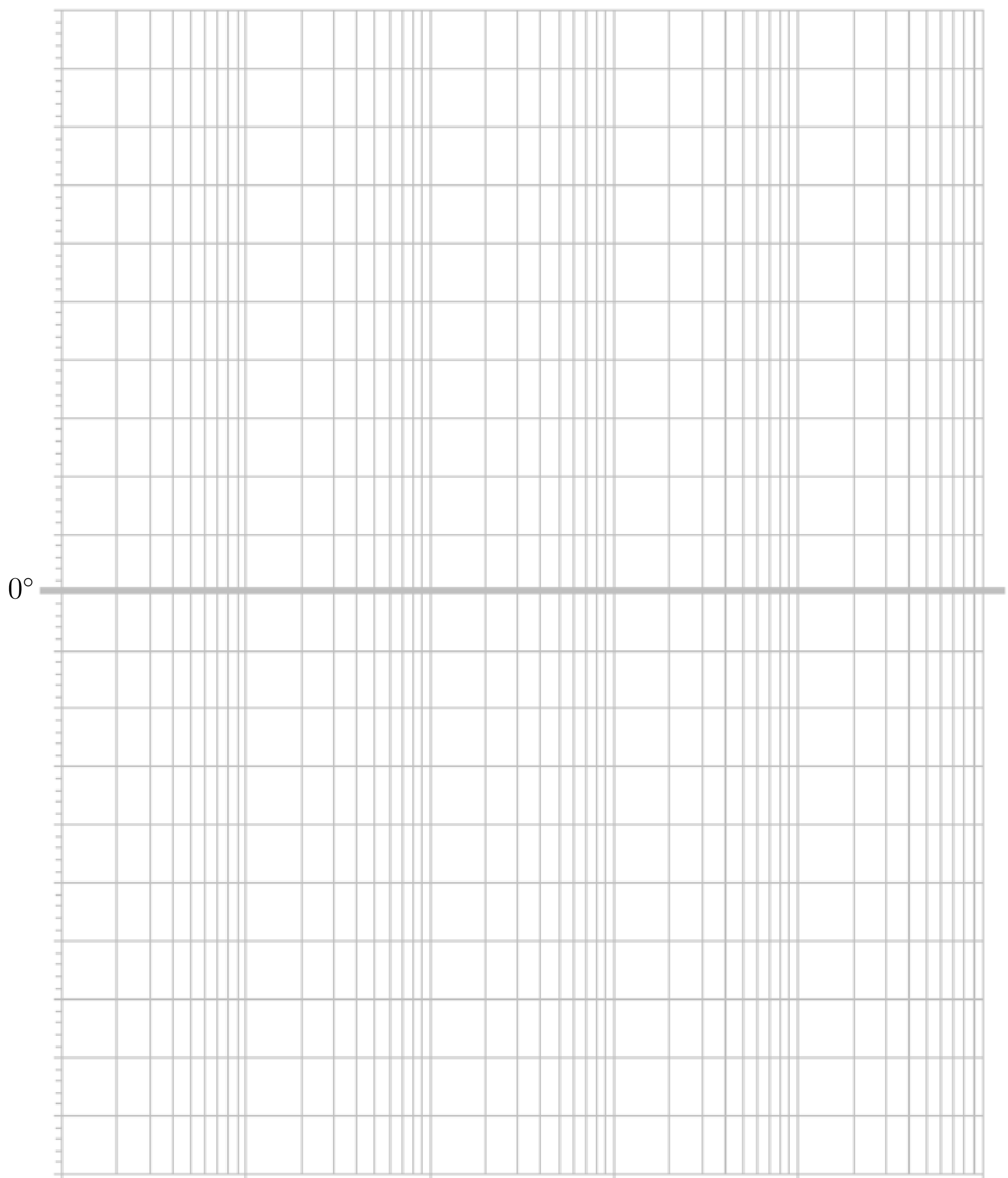
a) (2.5 points) Across which two output terminals would the voltage transfer function look like a low pass filter with respect to V_{in}?

b) (2.5 points) Write the transfer function for your answer in (a).

c) (2.5 points) Plot the magnitude Bode plot of the transfer function. (You can use the table on page 32.)

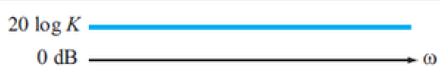
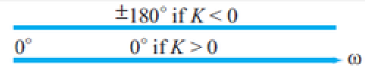
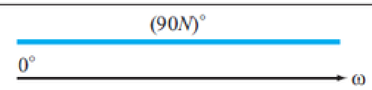
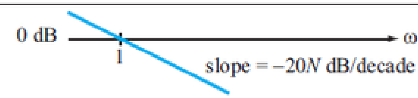
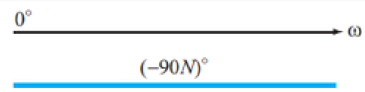
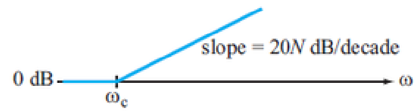
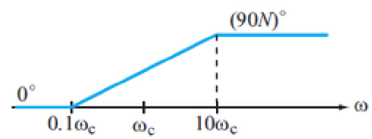
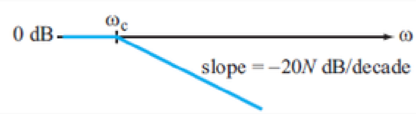
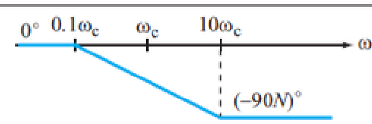
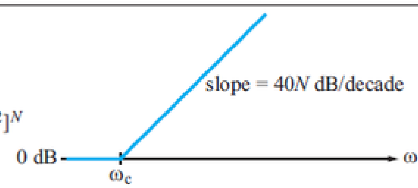
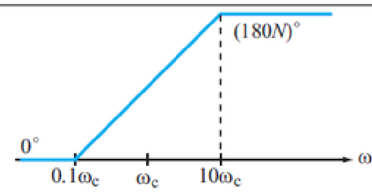


d) (2.5 points) Plot the phase Bode plot of the transfer function. (You can use the table on page 32.)



Additional workspace for Problems 8c and 8d.

Table 9-2: Bode straight-line approximations for magnitude and phase.

Factor	Bode Magnitude	Bode Phase
Constant K		
Zero @ Origin $(j\omega)^N$		
Pole @ Origin $(j\omega)^{-N}$		
Simple Zero $(1 + j\omega/\omega_c)^N$		
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$		
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$		
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$