EECS 16B Designing Information Devices and Systems II
Fall 2021 Midterm Exam Midterm 1

Exam location: <examloc>

PRINT your student ID: ____

PRINT AND SIGN your name: ________________, ________________ ____________________
(last) (first) (signature)

PRINT your discussion sections and (u)GSIs (the ones you attend): ______________________________

Row Number (front row is 1): __________ Seat Number (left most is 1): __________

Name and SID of the person to your left: ____

Name and SID of the person to your right: ____

Name and SID of the person in front of you: ____

Name and SID of the person behind you: ____

Section 0: Pre-exam questions (4 points)

1. **Honor Code: Please copy the following statement in the space provided below and sign your name.**

   As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will follow the rules and do this exam on my own.

2. **What's your favorite thing about this semester? (2 pts)**

3. **Who is a movie, TV, or fiction character that inspires you? (2 pts)**

Do not turn this page until the proctor tells you to do so. You can work on Section 0 above before time starts.

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4. Complex Numbers (2 points)

(a) (1 pt) Express the complex number j in polar form $j = Ae^{j\theta}$.
What is A? What is θ? Your θ should be between $-\pi\mathrm{rad}$ and $+\pi\mathrm{rad}$.

(b) (1 pt) Express the complex number $e^{j\frac{\pi}{3}}$ in rectangular form $e^{j\frac{\pi}{3}} = a + jb$.
What is a? What is b?

The following table may be useful:

Angle θ (rad)	$\sin(\theta)$	$\cos(\theta)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

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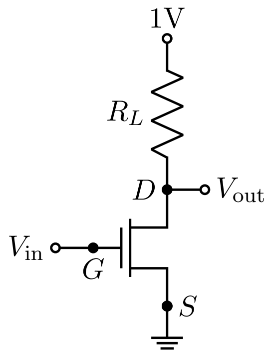
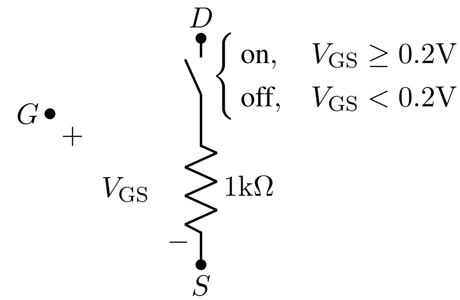
5. NMOS Transistor Inverter (4 points)

Consider the following schematic and NMOS model.



(a) An NMOS Transistor circuit



(b) Resistor and switch model for NMOS transistor.

Figure 1: NMOS figures.

What value of R_L is required to produce $V_{\text{out}} = 0.1\text{V}$ when $V_{\text{in}} = 1\text{V}$? Show your work.

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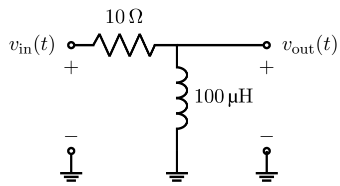
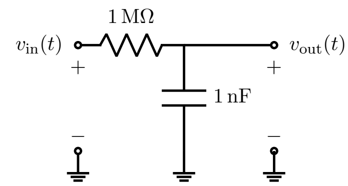
6. Transfer Function Matching (8 points)

Below, you have filter circuits A,B,C,D, each with specific component values. **Fill in the bubbles to match each filter to its corresponding magnitude Transfer Function Plot out of choices I, II, III, IV.**

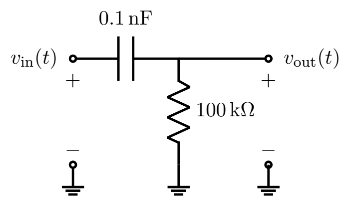
Note that each plot may be assigned to filters once, more than once, or not at all. Each filter has exactly one corresponding plot. *A table of SI Prefixes and some info. about common filters is on the next page (scratch).*



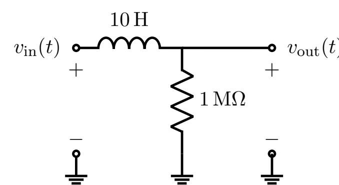
(A) Filter A.



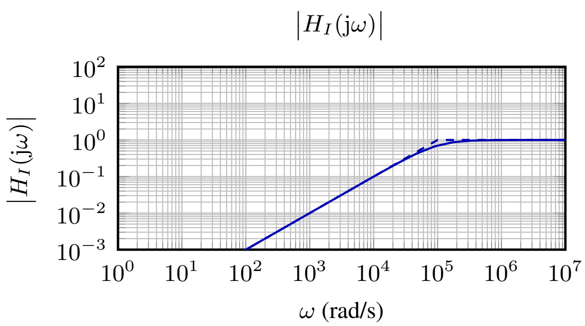
(B) Filter B.



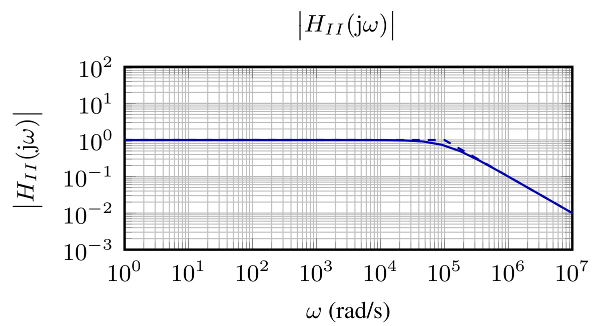
(C) Filter C.



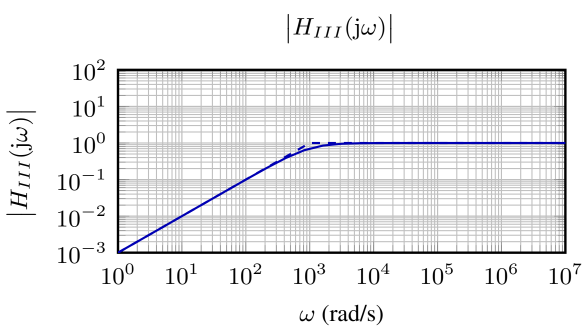
(D) Filter D.



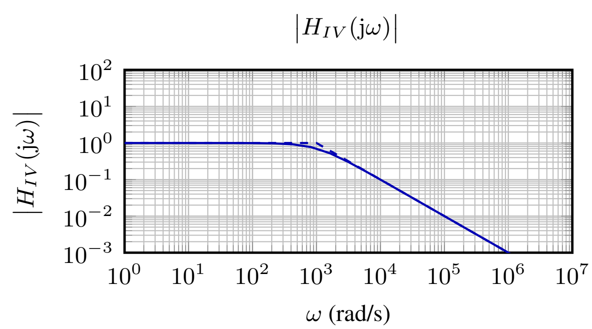
(I) Plot I.



(II) Plot II.



(III) Plot III.



(IV) Plot IV.

Filter Letter	Plot I	Plot II	Plot III	Plot IV
A	○	○	○	○
B	○	○	○	○
C	○	○	○	○
D	○	○	○	○

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[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

SI Prefix	Exponent Definition
nano (n)	10^{-9}
micro (μ)	10^{-6}
milli (m)	10^{-3}
kilo (k)	10^{3}
mega (M)	10^{6}
giga (G)	10^{9}

- RC low-pass filter: $H(\mathrm{j}\omega) = \frac{1}{1+\mathrm{j}\frac{\omega}{\omega_c}}$ $\omega_c = \frac{1}{RC}$
- RC high-pass filter: $H(\mathrm{j}\omega) = \frac{\mathrm{j}\frac{\omega}{\omega_c}}{1+\mathrm{j}\frac{\omega}{\omega_c}}$ $\omega_c = \frac{1}{RC}$
- RL low-pass filter: $H(\mathrm{j}\omega) = \frac{1}{1+\mathrm{j}\frac{\omega}{\omega_c}}$ $\omega_c = \frac{R}{L}$
- RL high-pass filter: $H(\mathrm{j}\omega) = \frac{\mathrm{j}\frac{\omega}{\omega_c}}{1+\mathrm{j}\frac{\omega}{\omega_c}}$ $\omega_c = \frac{R}{L}$

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7. Magnitude, Phase, and Cascades (6 points)

Suppose you have the transfer function $H(\mathrm{j}\omega)$ for a system as given in eq. (1) below:

$$H(\mathrm{j}\omega) = \frac{10}{1 + \mathrm{j}\frac{\omega}{10^6}} \tag{1}$$

Answer the following questions.

(a) (2 pts) **What is the transfer function's approximate magnitude $\left|H(\mathrm{j}\omega)\right|$ at $\omega = 1 \times 10^4\ \frac{\text{rad}}{\text{s}}$? Select the closest option from the list below by filling in a bubble.**

- ◯ 100
- ◯ $10\sqrt{2}$
- ◯ 10
- ◯ $\frac{10}{\sqrt{2}}$
- ◯ 2
- ◯ $\sqrt{2}$
- ◯ 1
- ◯ 0.1
- ◯ 0.01
- ◯ 0.001

(b) (2 pts) **What is the transfer function's approximate phase $\measuredangle H(\mathrm{j}\omega)$ at $\omega = 1 \times 10^9\ \frac{\text{rad}}{\text{s}}$? Select the closest option from the list below by filling in a bubble.**

- ◯ $90°, \frac{\pi}{2}\,\text{rad}$
- ◯ $84°$
- ◯ $45°, \frac{\pi}{4}\,\text{rad}$
- ◯ $6°$
- ◯ $0°, 0\,\text{rad}$
- ◯ $-6°$
- ◯ $-45°, -\frac{\pi}{4}\,\text{rad}$
- ◯ $-84°$
- ◯ $-90°, -\frac{\pi}{2}\,\text{rad}$

(c) (2 pts) You cascade three of the systems as defined by the transfer function in eq. (1). You place one after the other, with unity-gain buffers in between. **Write the overall transfer function $H_{\text{ov}}(\mathrm{j}\omega)$ in terms of the given transfer function $H(\mathrm{j}\omega)$.** *You do not have to simplify your answer.*

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8. Recurrence Stability (6 points)

Just as you have seen second-order vector differential equations, we can define similar second-order recurrence relations in discrete-time. Consider:

$$x[i+1] = x[i] - 3x[i-1] + w[i] \tag{2}$$

where $x[0] = 0, x[1] = 0$ and $\left|w[i]\right| \leq \varepsilon$.

This can be written in vector form for $i \geq 1$ as:

$$\begin{bmatrix} x[i] \\ x[i+1] \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -3 & 1 \end{bmatrix}}_{A_d} \begin{bmatrix} x[i-1] \\ x[i] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w[i], \tag{3}$$

with initial condition $\begin{bmatrix} x[0] \\ x[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Is the discrete-time system above stable or unstable? Fill in the appropriate bubble and then explain below.

◯ Stable

◯ Unstable

The following information about the matrix A_d may be useful. You may use as much or as little of the following information as you find helpful. It is here to simplify calculations you may want to perform.

Eigenvalues of A_d :	$\lambda_1 = \frac{1}{2}\left(1 + \mathrm{j}\sqrt{11}\right)$	$\lambda_2 = \frac{1}{2}\left(1 - \mathrm{j}\sqrt{11}\right)$
Eigenvalue magnitudes:	$\lvert\lambda_1\rvert \approx 1.73$	$\lvert\lambda_2\rvert \approx 1.73$
Eigenvectors for associated λ_i :	$\vec{v}_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}$	$\vec{v}_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}$
Matrix inverse:	$A_d^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$	

If stable, explain why. If unstable, just give a sequence of bounded scalar disturbances $w[i]$ that would cause the states $x[i]$ to grow unboundedly. You don't need to justify your choice of $w[i]$ sequence here.

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9. Nonhomogeneous Solution (4 points)

Solve the following differential equation for $t \geq 0$:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -3x(t) + 6 \tag{4}$$

with initial condition $x(0) = -2$. **What is** $x(t)$**?**

You do not have to show any work for this problem.

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10. Get Real! (8 points)

(a) (2 pts) Suppose you have the input signal in eq. (5):

$$v_{\text{in}}(t) = 4\cos\left(10^4 t + \frac{\pi}{4}\right) \tag{5}$$

For the sinusoidal input voltage signal given in eq. (5), what is the angular frequency ω?

(b) (4 pts) You now have a circuit given in fig. 4. We want to add an element Z_E in series to the R and C such that the equivalent impedance of the series combination is purely real. Notice the same input signal eq. (5) is also labeled in the diagram fig. 4.

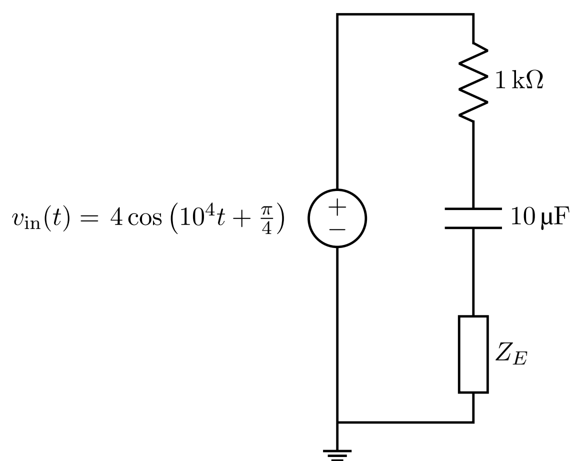

Figure 4: A given circuit for which we want to make the equivalent series impedance purely real.

In order to make the equivalent impedance of the series interconnection of R, C and Z_E purely real at the input angular frequency ω you identified above, **what should the impedance $Z_E(\mathrm{j}\omega)$ be? Show your work.** Give a specific numerical value, and do not worry about either units or the dependence on ω.

(c) (2 pts) **What single circuit element could you use to get the above Z_E? Select one of the options below by filling in a bubble.**

◯ Resistor
◯ Capacitor
◯ Inductor

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11. System ID (8 points)

Consider a scalar system we want to model by:

$$x[i+1] = a_1 x[i] + a_2 x[i-1] + bu[i] + w[i], \tag{6}$$

where $w[i]$ is a hopefully small disturbance.

We collect a trace of values for $x[i]$ and $u[i]$ from time $i = 0, 1, \ldots, \ell$. Setup a least squares problem for the unknown parameters a_1, a_2, b, in the form of solving an approximate system of equations $D\vec{p} \approx \vec{s}$. **What are the matrix D, the vector of unknowns $\vec{p}$, and the vector $\vec{s}$?**

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12. s-Impedance Derivation (8 points)

(a) (4 pts) You are given a component with the I-V relationship $V(t) = p_0 \frac{\mathrm{d}^2}{\mathrm{d}t^2} I(t) - p_1 \frac{\mathrm{d}}{\mathrm{d}t} I(t)$. Suppose you apply $I(t) = e^{st}$. **What is $V(t)$?**

(b) (4 pts) In (a) you should have noticed the answer is proportional to e^{st}. The s-impedance is defined as the ratio $\frac{V(t)}{I(t)}$ when $I(t) = e^{st}$. **What is the s-impedance for this component?**

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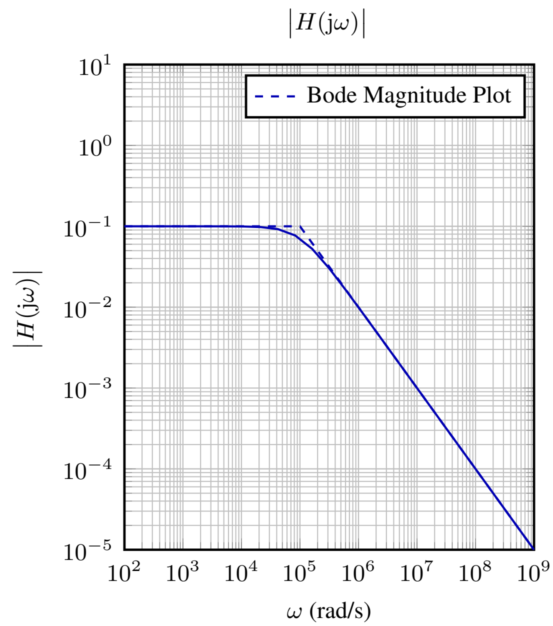
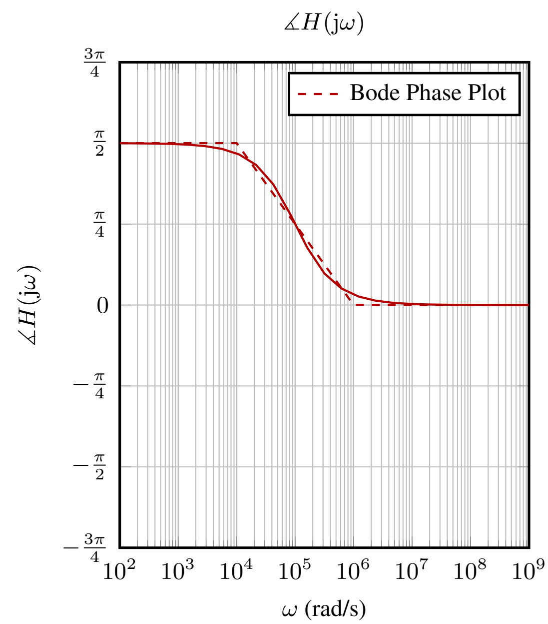
13. Conceptual Transfer Functions and Bode Plot Analysis (10 points)

In fig. 5 are magnitude and phase Bode plots for some filter with transfer function $H(j\omega)$.



(i) Part a) Magnitude Plot



(ii) Part a) Phase Plot

Figure 5: Part a) Magnitude and Phase Bode Plots for a transfer function $H(j\omega)$.

(a) (4 pts) Suppose $v_{\text{in},1}(t)$ below is an input voltage signal to the above filter with transfer function $H(j\omega)$.

$$v_{\text{in},1}(t) = 3\sin\left(10^3 t + \frac{\pi}{3}\right) \tag{7}$$

Label the input angular frequency of $v_{\text{in},1}(t)$ on both plots above, in fig. 5, using a vertical line. Next, compute the output voltage $v_{\text{out},1}(t)$ after this input signal passes through the filter defined by $H(j\omega)$.

Your answer for the output voltage should have the form $A_1 \sin(\omega_1 t + \phi_1)$. What are A_1, ω_1, and ϕ_1?

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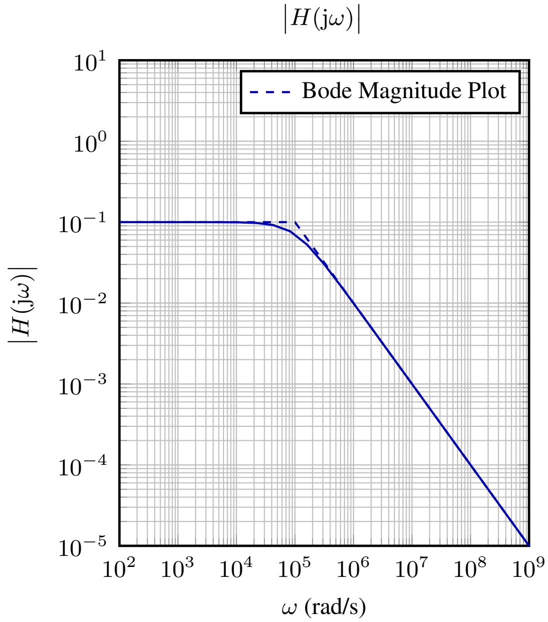
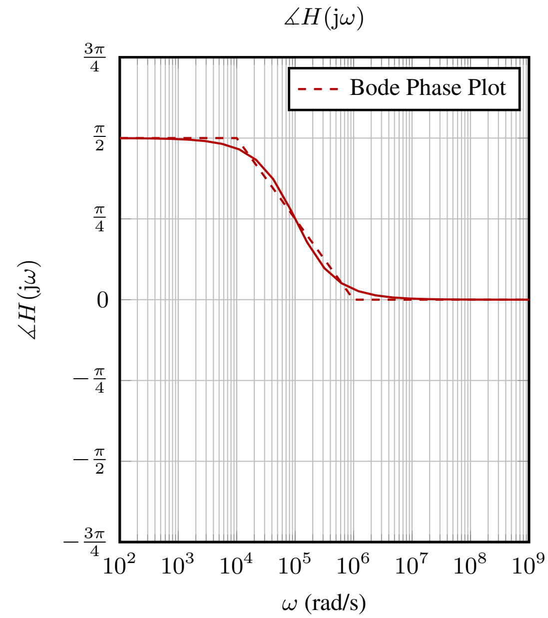
(b) (4 pts) For convenience, we have copied the same plots from part (a) below.



(i) Part b) Magnitude Plot



(ii) Part b): Phase Plot

Figure 6: Part b): Magnitude and Phase Bode Plots for a transfer function $H(j\omega)$.

Suppose $v_{\text{in},2}(t)$ below is a second input voltage signal to the filter with transfer function $H(j\omega)$ on the previous page.

$$v_{\text{in},2}(t) = 6\cos\left(10^{8}t - \frac{\pi}{6}\right) \tag{8}$$

Label the input angular frequency of $v_{\text{in},2}(t)$ on both plots in fig. 6, using a vertical line.

Next, compute the output voltage $v_{\text{out},2}(t)$ after this input signal passes through the filter defined by $H(j\omega)$.

Your answer for the output voltage should have the form $A_2\cos(\omega_2 t + \phi_2)$. What are A_2, ω_2, and ϕ_2?

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(c) (2 pts) **Based on your answers in parts (a) and (b), if $v_{\text{in}}(t) = v_{\text{in},1}(t) + v_{\text{in},2}(t)$, what is the corresponding $v_{\text{out}}(t)$?**

You can leave your answer in terms of the A_i, ω_i, ϕ_i from the previous two parts. You don't have to substitute in values.

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14. Being able to control with feedback (22 points)

Consider the discrete-time dynamic system:

$$\vec{x}[i+1] = A\vec{x}[i] + \vec{b}u[i] + \vec{w}[i], \tag{9}$$

where the purely real matrix $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) (3 pts) **Show that if $\beta = 0$, then $\vec{b}$ is an eigenvector of A.**
(HINT: What does it mean to be an eigenvector?)

(b) (5 pts) Let η_1 and η_2 be a pair of complex numbers that are complex conjugates. (i.e. $\eta_1 = \overline{\eta_2}$.)
Show that the polynomial $(\lambda - \eta_1)(\lambda - \eta_2) = \lambda^2 + c_1\lambda + c_0$ has purely real coefficients c_1 and c_0.

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(c) (14 pts) Suppose we knew that $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is **not** an eigenvector of the real matrix $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$.

Choose real feedback gains f_1, f_2 **so that** $u[i] = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \vec{x}[i]$ **places the eigenvalues of the closed-loop matrix** $A_{cl} = (A + \vec{b} \begin{bmatrix} f_1 & f_2 \end{bmatrix})$ **at the complex conjugate pair** η_1 **and** η_2 **from the previous part. Show your work.**

It is fine if you leave your answer in terms of c_1 and c_0 from the previous part.

(HINT: The final gains f_1 *and* f_2 *are going to depend on the* $\alpha, \beta, \gamma, \delta$ *that define* A *as well as* η_1 *and* η_2 *through* c_1 *and* c_0 *from part (b).*

At some point in the derivation, your life might become easier if you solve for f_2 *first before trying to solve for* f_1*.)*

(Potentially heavier algebra warning for this part. Don't get bogged down for too long.)

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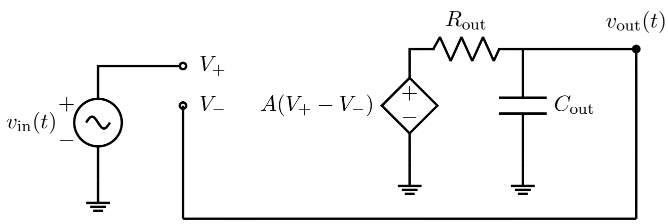
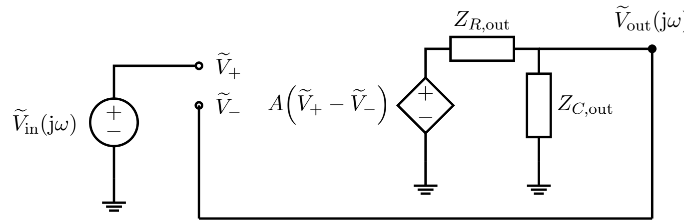
15. Op-Amp Model Analysis (16 points)

(a) (10 pts) You are given the op-amp circuit model in fig. 7i. This circuit has a phasor-domain representation as given in fig. 7ii. **Find the transfer function from v_{in} to v_{out}. i.e. Find $H(j\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_{\text{in}}}$ corresponding to fig. 7ii. Show your work.**

You don't have to fully simplify your answer but it has to be a valid transfer function.



(i) Op-amp Model with feedback (in "Time-Domain").



(ii) Op-amp Model with feedback to compute the Transfer Function (in "Phasor-Domain").

Figure 7: Op-amp Models in "Time-Domain" and "Phasor-Domain".

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(b) (6 pts) Now suppose that your A is large so that you can approximate your transfer function $H(\mathrm{j}\omega)$ as $\widehat{H}(\mathrm{j}\omega)$ as in eq. (10).

$$\widehat{H}(\mathrm{j}\omega) = \frac{1}{1 + \frac{\mathrm{j}\omega}{\omega_0 A}} \tag{10}$$

For the transfer function $\widehat{H}(\mathrm{j}\omega)$, sketch a Bode Plot (straight-line approximations are fine) for the magnitude on fig. 8 with $w_0 = 10^2 \frac{\text{rad}}{\text{s}}$ and $A = 10^5$.

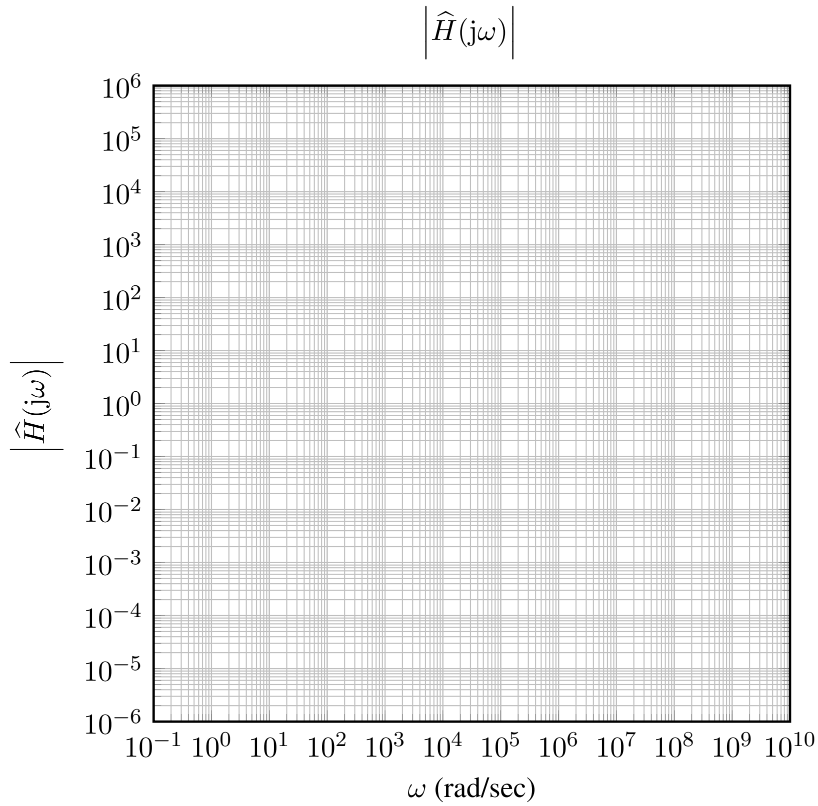

Figure 8: Magnitude Bode Plots: Template for part (b).

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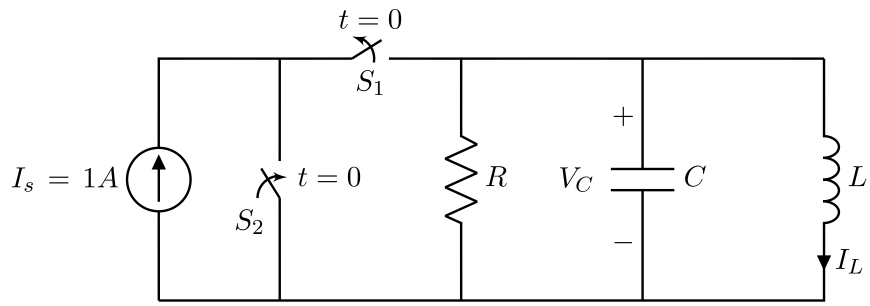
16. Parallel RLC with Current Source (28 points)

Consider the following circuit:



(a) (10 pts) At $t = 0$, switch S_1 became open and switch S_2 became closed. We first need to construct our state space system for $t \geq 0$. Our natural state variables are the current through the inductor $x_1(t) = I_L(t)$ and the voltage across the capacitor $x_2(t) = V_C(t)$ since these are the quantities whose derivatives show up in the system of equations governing our circuit.

Find the system of differential equations in terms of our state variables that describes this circuit for $t \geq 0$. Leave the system symbolic in terms of $I_s, R, L,$ and C. Write the system of differential equations in vector/matrix form with the vector state variable:

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} \tag{11}$$

This should be in the form $\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t)$ with a 2×2 matrix A.
Show your work.

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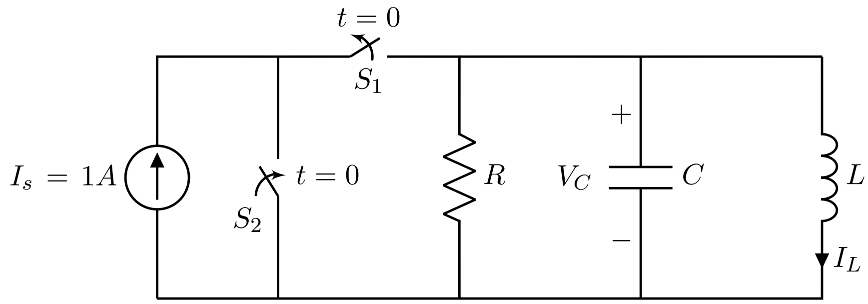

(b) (3 pts) For a long time in the past $t < 0$, assume that the switch S_1 had remained closed and S_2 had remained open. **What is $I_L(0)$ and $V_C(0)$? Give a brief justification as well.**

(c) (3 pts) For the rest of this problem, assume $R = 1\,\mathrm{M}\Omega, L = 25\,\mu\mathrm{H}, C = 10\,\mathrm{nF}$. With these values, we get the following eigenvalues and eigenvectors for A in the differential equation $\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t)$.

$$\lambda_1 = -Z_0 + \mathrm{j}\omega_0 \qquad \lambda_2 = -Z_0 - \mathrm{j}\omega_0, \tag{12}$$

$$\text{where } Z_0 = \sqrt{\frac{L}{C}} = 50 \text{ and } \omega_0 = \frac{1}{\sqrt{LC}} = 2 \times 10^6$$

$$V = \begin{bmatrix} \vec{v}_{\lambda_1} & \vec{v}_{\lambda_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ a & \overline{a} \end{bmatrix} \tag{13}$$

$$V^{-1} = \begin{bmatrix} b & -\mathrm{j}0.01 \\ \overline{b} & \mathrm{j}0.01 \end{bmatrix} \tag{14}$$

Consider a nice coordinate system for which we can write $\vec{x}(t) = V\vec{\widetilde{x}}(t)$.

What is the $\widetilde{A}$ so that $\frac{\mathrm{d}}{\mathrm{d}t}\vec{\widetilde{x}}(t) = \widetilde{A}\vec{\widetilde{x}}(t)$? You can leave the answer symbolic in terms of λ_1, λ_2. Note that you **do not** need to find/evaluate a, b. You can still answer all the questions below without knowing these values.

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(d) (12 pts) Now, suppose that our initial conditions for $I_L(0)$ and $V_C(0)$ have changed to the following:

$$\begin{bmatrix} I_L(0) \\ V_C(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{15}$$

Using the information from (c), **find** $I_L(t)$ **for** $t \geq 0$ **and write the answer in a form involving real exponentials and sinusoids. Does** $I_L(t)$ **converge as** $t \to \infty$**? If so, what does it converge to? Show your work.**

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[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.

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