## EE 16B Midterm 1, February 15, 2017

Name: $\qquad$
SID \#: $\qquad$
Discussion Section and TA: $\qquad$
Discussion Section and TA: $\qquad$
Lab Section and TA:

Name of left neighbor: $\qquad$
Name of right neighbor: $\qquad$

## Important Instructions:

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

Do not remove pages, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

| PROBLEM | MAX |
| :---: | :---: |
| $\mathbf{1}$ | 20 |
| 2 | 20 |
| 3 | 15 |
| 4 | 30 |
| 5 | 15 |

"Well, Diotallevi and I are planning a reform in higher education. A School of Comparative Irrelevance, where useless or impossible courses are given. The school's aim is to turn out scholars capable of endlessly increasing the number of unnecessary subjects."

- Umberto Eco, Foucault's Pendulum

Problem 1 Warm up (20 points)
a) Consider the following circuit. $\mathrm{Z}_{\text {eq }}$ is the impedance looking into the circuit from the left, as shown. Provide an expression for $\mathrm{Z}_{\mathrm{eq}}$.

b) If this impedance is driven by a sinusoidal source at frequency, $\omega[\mathrm{rad} / \mathrm{s}]$, for what $\omega$ is $\mathrm{Z}_{\mathrm{eq}}=\infty$ ?
$\omega=$
c) What logic function does the following circuit perform?

d) Consider the following four circuits. For each, we define the voltage transfer function, $\mathbf{H}_{\mathrm{v}}(\omega)=\mathbf{V}_{\text {out }} / \mathbf{V}_{\text {in }}$. With respect to $\mathbf{H}_{v}(\omega)$, circle what class of frequency response each circuit performs.


Lowpass filter
Highpass filter
Bandpass
filter

Bandstop
filter


Lowpass
filter
Highpass
filter
Bandpass
filter
Bandstop filter


Lowpass filter
Highpass
filter
Bandpass
filter
Bandstop
filter


Lowpass
filter
Highpass
filter
Bandpass
filter
Bandstop
filter
"You can tell you've found a really interesting question when nobody wants you to answer it." - James S.A. Corey, Nemesis Games

Problem 2 (20 points)
Consider the circuit below. Assume an ideal op amp.

a) Find an expression that relates the derivative of $v_{\text {out }}\left(d_{\text {vout }} / d t\right)$ to the input voltage ( $\mathrm{vin}_{\mathrm{in}}$ ) and/or its derivative ( $\mathrm{d}_{\mathrm{vin}} / \mathrm{dt}$ ).
$\frac{d v_{\text {out }}}{d t}=$
b) Now given that $\mathrm{C}_{\mathrm{s}}=1 \mathrm{nF}, \mathrm{C}_{\mathrm{f}}=5 \mathrm{nF}, \mathrm{C}_{\mathrm{p}}=1 \mathrm{nF}, \operatorname{VCf}(\mathrm{t}<0)=\operatorname{VCs}(\mathrm{t}<0)=0$ and $\operatorname{Vin}(\mathrm{t} \geq 0)=$ $5^{*} t$ [volts], provide an expression for $V_{\text {out }}(t)$ for $t \geq 0$.

Consider now the different circuit below. Assume an ideal op amp.

c) Provide a symbolic expression for $V_{\text {out }}(t)$ for $t \geq 0$.
$\mathrm{V}_{\text {out }}(\mathrm{t})=$
d) Assume $V_{\text {in }}(t \geq 0)=5^{*} t[$ Volts $]$ and $\operatorname{VCf}(\mathrm{t}<0)=0$. What is the value of $V_{\text {out }}(\mathrm{t})$ at $\mathrm{t}=1 \mathrm{~s}$ ?
$\mathrm{v}_{\text {out }}(\mathrm{t})=$
"One should never mistake pattern for meaning."

- Iain Banks, The Hydrogen Sonata


## Problem 3 (15 points)

The following circuit is part of a near field communication system. A realistic voltage source (Vs, Rs) is connected through a switch onto a three component circuit. The inductor represents an antenna; the voltage across it modulates how much energy is radiated away from the system. The switch alternates continuously between position A and position B; it has been doing this since $t=-\infty$. It spends $\pi$ microseconds at each position.


We want the voltage on the inductor, $\mathrm{V}_{\mathrm{L}}$, to follow the curve plotted below. Specifically, we want to fulfill the following condition.
Condition: The inductor voltage should oscillate 5 times during period when the switch is in position $A$.


Plot of $V_{\mathrm{L}}$ as a function of time with switch positions labeled. Note the units of time ( $\mathbf{1 0}^{-6}$ seconds)!
a) If $R \rightarrow \infty$ and $L$ is non-zero and known, provide an expression for $C$ such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)
$\mathrm{C}=$
b) Unfortunately, a colleague tells you that $\mathrm{R} \neq \infty$; if L and C are known, provide an expression for R such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)
$\mathbf{R}=$
(extra space)
"Chang Tzu tells us of a persevering man who after three laborious years mastered the art of dragonslaying. For the rest of his days, he had not a single opportunity to test his skills."

- Jorge Luis Borges, The Book of Imaginary Beings


## Problem 4 (30 points)

Consider the circuit below.

a) What is $i(0)$ ?

Hint. What is the current flowing through L1 before the switch opens? Consequently, what is the current flowing through L2?
b) What is di/dt (0)?
c) What is the relationship between the voltages across L1 and R1?
d) Use KCL on Node A and the relationship derived above to arrive at a differential equation of the form,

$$
\frac{\mathrm{d}^{2} i}{\mathrm{~d} t^{2}}(t)+a_{1} \frac{\mathrm{~d} i}{\mathrm{~d} t}(t)+a_{0} i(t)=0
$$

where $\mathrm{i}(\mathrm{t})$ is the current going through L 2 .
e) Let $\mathrm{R} 1=\mathrm{R} 2=\mathrm{R}$ and $\mathrm{L} 1=\mathrm{L} 2=\mathrm{L}$. Recall that the above differential equation can be reshaped into the follow linear algebra problem:

$$
\left[\begin{array}{c}
\frac{\mathrm{d} i}{\mathrm{~d} t} \\
\frac{\mathrm{~d}^{2} i}{\mathrm{~d} t^{2}}
\end{array}\right]=A\left[\begin{array}{c}
i \\
\frac{\mathrm{~d} i}{\mathrm{~d} t}
\end{array}\right]
$$

What is the A matrix and what are its eigenvalues?
f) Will this circuit exhibit any oscillations?
"I am Groot."

- Groot, Guardians of the Galaxy

Problem 5 (15 points)
Consider the circuit below.

a) Given an input voltage, $v_{1}(t)$, which is a sinusoid at frequency $\omega$, and phasors corresponding to the input and output voltages, $\mathbf{V}_{1}$ and $\mathbf{V}_{2}$, find an expression for $\mathbf{V}_{2} / \mathbf{V}_{1}$.
$\frac{\boldsymbol{V}_{2}}{\boldsymbol{V}_{1}}=$
b) If $v_{1}(t)=\cos (\omega t)$ where $\omega=10^{6} \mathrm{rad} / \mathrm{s}$ and $\mathrm{L}=1 \mu \mathrm{H}, \mathrm{R}=1 \Omega$, and $\mathrm{C}=0.5 \mu \mathrm{~F}$, solve for $\mathrm{v}_{2}(\mathrm{t})$.
$\mathrm{V}_{2}(\mathrm{t})=$

