EE 16B Midterm 1, February 15, 2017

Name:____

SID #:____

Discussion Section and TA:______________________
Discussion Section and TA:______________________
Lab Section and TA:____________________________

Name of left neighbor:__________________________
Name of right neighbor:_________________________

Important Instructions:

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

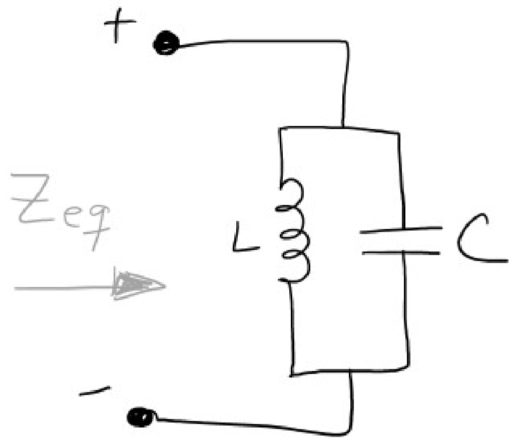
Do not remove pages, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

PROBLEM	MAX
1	20
2	20
3	15
4	30
5	15

"Well, Diotallevi and I are planning a reform in higher education. A School of Comparative Irrelevance, where useless or impossible courses are given. The school's aim is to turn out scholars capable of endlessly increasing the number of unnecessary subjects."
— Umberto Eco, *Foucault's Pendulum*

Problem 1 *Warm up* (20 points)

a) Consider the following circuit. Z_{eq} is the impedance looking into the circuit from the left, as shown. Provide an expression for Z_{eq}.



$$Z_{eq} = \boxed{\frac{j\omega L}{1-\omega^2 LC}}$$

$$Z_{eq} = Z_L \,||\, Z_C = j\omega L \,||\, \frac{1}{j\omega C}$$

$$= \frac{j\omega L\left(\frac{1}{j\omega C}\right)}{j\omega L + \frac{1}{j\omega C}} = \frac{\frac{j\omega L}{j\omega C}}{\frac{j^2\omega^2 LC + 1}{j\omega C}}$$

$$= \frac{j\omega L}{1 + j^2\omega^2 LC} = \frac{j\omega L}{1-\omega^2 LC}$$

b) If this impedance is driven by a sinusoidal source at frequency, ω [rad/s], for what ω is $Z_{eq} = \infty$?

$$\omega = \frac{1}{\sqrt{LC}}$$

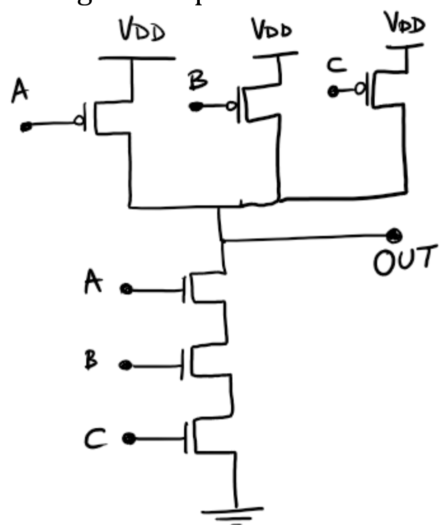
$$\frac{j\omega L}{1-\omega^2 LC} \rightarrow \infty$$ when the denominator $\rightarrow 0$

$$1-\omega^2 LC = 0$$

$$\omega^2 LC = 1$$

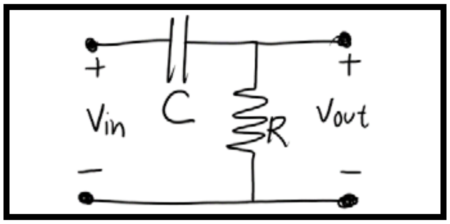
$$\omega^2 = \frac{1}{LC} \qquad \omega = \frac{1}{\sqrt{LC}}$$

c) What logic function does the following circuit perform?



$$OUT = \overline{(A \text{ and } B \text{ and } C)}$$

d) Consider the following four circuits. For each, we define the voltage transfer function, $\mathbf{H}_V(\omega) = \mathbf{V}_{out}/\mathbf{V}_{in}$. With respect to $\mathbf{H}_V(\omega)$, circle what class of frequency response each circuit performs.



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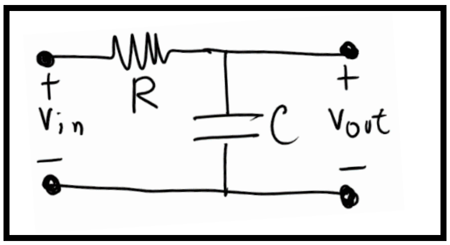
$Z_C \to \infty$ as $\omega \to 0$

$Z_C \to 0$ as $\omega \to \infty$



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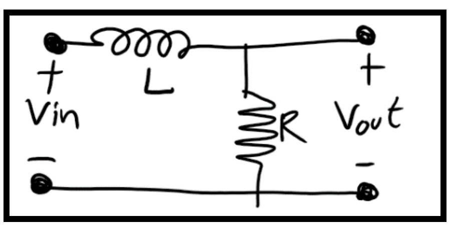

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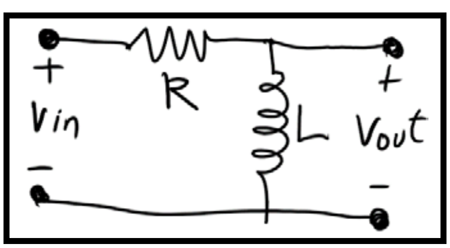
$Z_L \to \infty$ as $\omega \to \infty$

$Z_L \to 0$ as $\omega \to 0$



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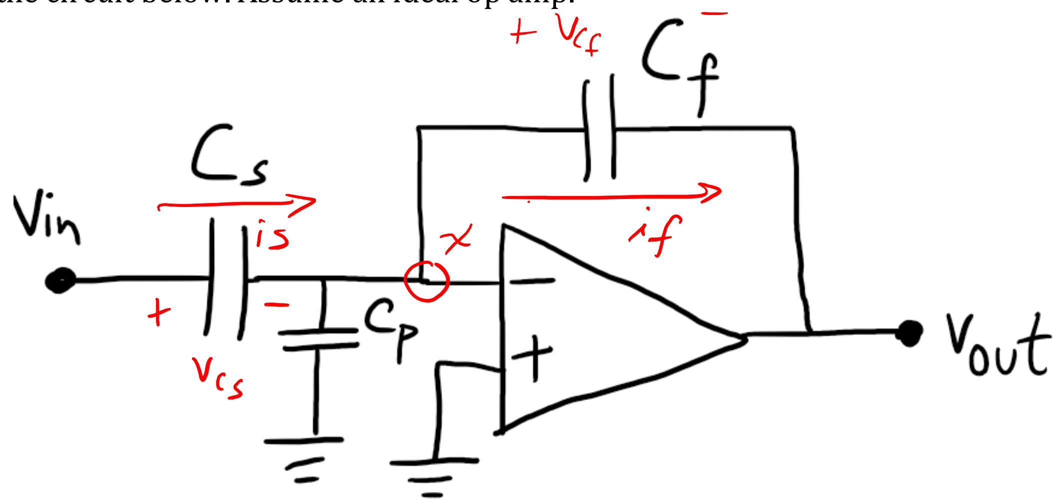
"You can tell you've found a really interesting question when nobody wants you to answer it."
— James S.A. Corey, *Nemesis Games*

Problem 2 (20 points)

Consider the circuit below. Assume an ideal op amp.



a) Find an expression that relates the derivative of v_{out} (dv_{out}/dt) to the input voltage (v_{in}) and/or its derivative (dv_{in}/dt).

$$\boxed{\frac{dv_{out}}{dt} = -\frac{C_s}{C_f}\frac{dv_{in}}{dt} = -\frac{1}{5}\cdot 5 = -1}$$

b) Now given that $C_s = 1$ nF, $C_f = 5$ nF, $C_p = 1$ nF, $v_{Cf}(t<0) = v_{Cs}(t<0) = 0$ and $v_{in}(t\geq 0) = 5*t$ [volts], provide an expression for $V_{out}(t)$ for $t\geq 0$.

$V_x = 0$

Cp does nothing as both terminals are at the same potential.

$i_s = i_f$

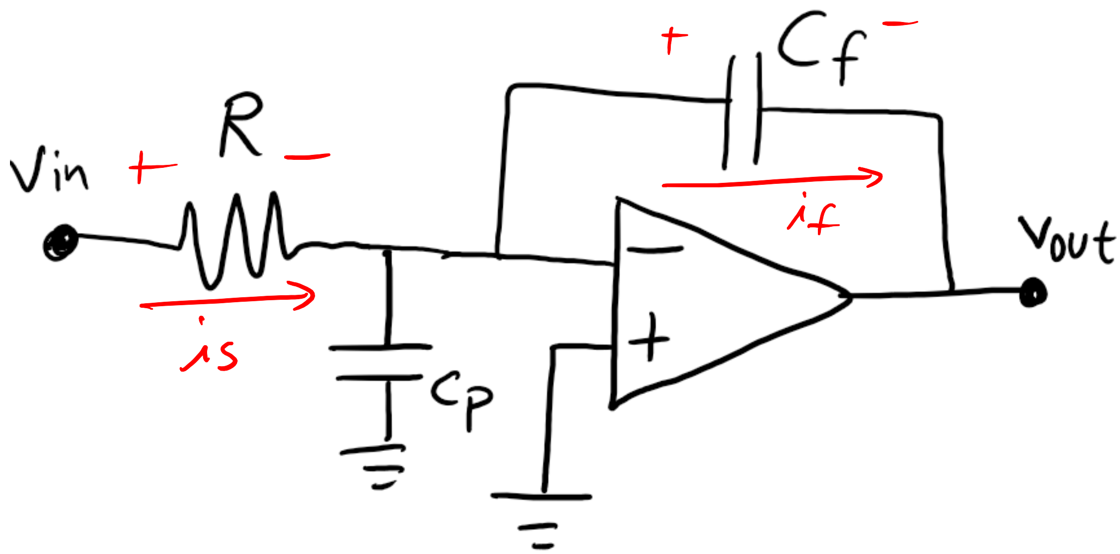
$$i_s = C_s\frac{dv_s}{dt} = C_s\frac{dv_{in}}{dt}$$

$$i_f = -C_f\frac{dv_{out}}{dt}$$

$$C_s\frac{dv_{in}}{dt} = -C_f\frac{dv_{out}}{dt}$$

$$\frac{dv_{out}}{dt} = -\frac{C_s}{C_f}\frac{dv_{in}}{dt}$$

Consider now the different circuit below. Assume an ideal op amp.



c) Provide a symbolic expression for $v_{out}(t)$ for $t \geq 0$.

$v_{out}(t) =$ $-\dfrac{\int V_{in}\,dt}{RC}$

d) Assume $v_{in}(t \geq 0) = 5{*}t$ [volts] and $v_{Cf}(t<0) = 0$. What is the value of $v_{out}(t)$ at t=1 s?

$v_{out}(t) =$ $-\dfrac{5}{2} \cdot \dfrac{1}{RC}$

$i_s = i_f$

$\dfrac{V_{in}}{R} = -C\dfrac{dv_{out}}{dt}$

$\dfrac{dV_{out}}{dt} = \left(-\dfrac{1}{RC}\right)V_{in}$

$V_{out} = \dfrac{-\int V_{in}\,dt}{RC}$

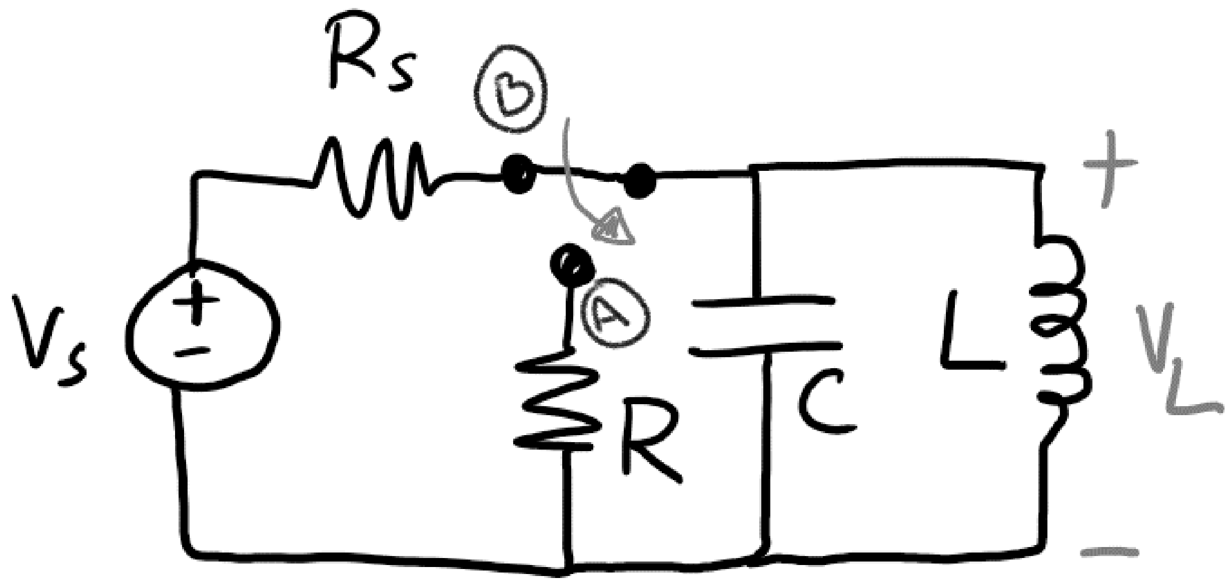
(extra space)

$$\int_0^1 5t \, dt = \frac{5t^2}{2}\Big]_0^1 = \frac{5}{2}$$

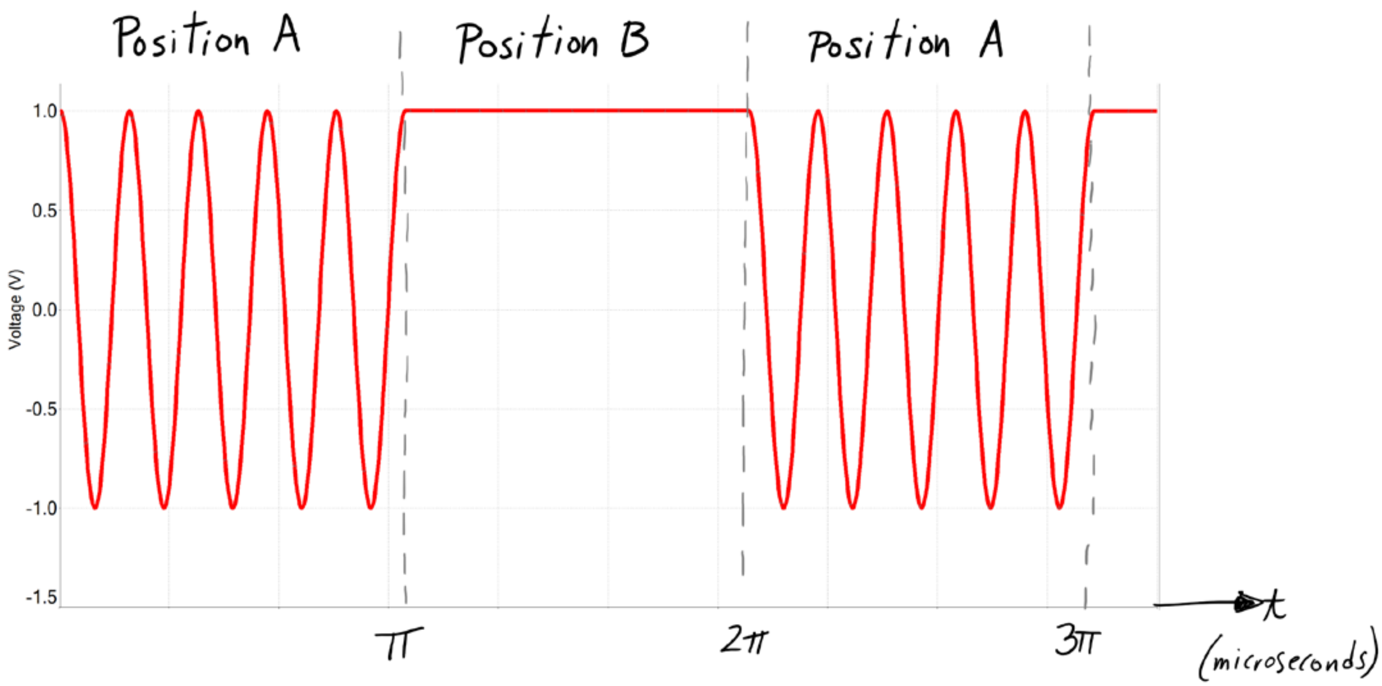
"One should never mistake pattern for meaning."
— Iain Banks, *The Hydrogen Sonata*

Problem 3 (15 points)

The following circuit is part of a near field communication system. A realistic voltage source (Vs, Rs) is connected through a switch onto a three component circuit. The inductor represents an antenna; the voltage across it modulates how much energy is radiated away from the system. The switch alternates continuously between position A and position B; it has been doing this since t = -∞. It spends π microseconds at each position.



We want the voltage on the inductor, V_L, to follow the curve plotted below. Specifically, we want to fulfill the following condition.
Condition: *The inductor voltage should oscillate 5 times during period when the switch is in position A.*



Plot of V_L as a function of time with switch positions labeled. **Note the units of time (10^{-6} seconds)!**

a) If R → ∞ and L is non-zero and known, provide an expression for C such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

C = $\frac{1}{10^{14} L}$ [F]

If $R \to \infty$, LC circuit will oscillat at

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \frac{1}{\sqrt{LC}} = 10^7$; $\frac{1}{LC} = 10^{14}$

$C = \frac{1}{10^{14} L}$

$\omega_0 = 2\pi f_0 = \frac{(2\pi) 5}{\pi 10^{-6}} = 10^7$ rad/s

b) Unfortunately, a colleague tells you that R ≠ ∞; if L and C are known, provide an expression for R such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

R = $\left(\frac{L}{4C(1 - 10^{14} LC)} \right)^{1/2}$

If $R \neq \infty$, then the circuit will oscillat

at $\omega_D = \sqrt{\omega_0^2 - \alpha^2}$ where $\omega_0 = \frac{1}{\sqrt{LC}}$

and $\alpha = \frac{1}{2RC}$

$10^7 = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$

$10^{14} = \frac{1}{LC} - \frac{1}{2R^2C^2}$

(extra space)

$$10^{14} = \frac{4R^2C^2 - LC}{(4R^2C^2)(LC)}$$

$$10^{14} = \frac{(4R^2C - L)\,C}{(4R^2C^2L)\,C}$$

$$= \frac{4R^2C - L}{4R^2C^2L}$$

$$4\cdot 10^{14} R^2C^2L = 4R^2C - L$$

$$(4\cdot 10^{14} C^2L - 4C)R^2 = -L$$

$$(4C - 4\cdot 10^{14} C^2L)R^2 = L$$

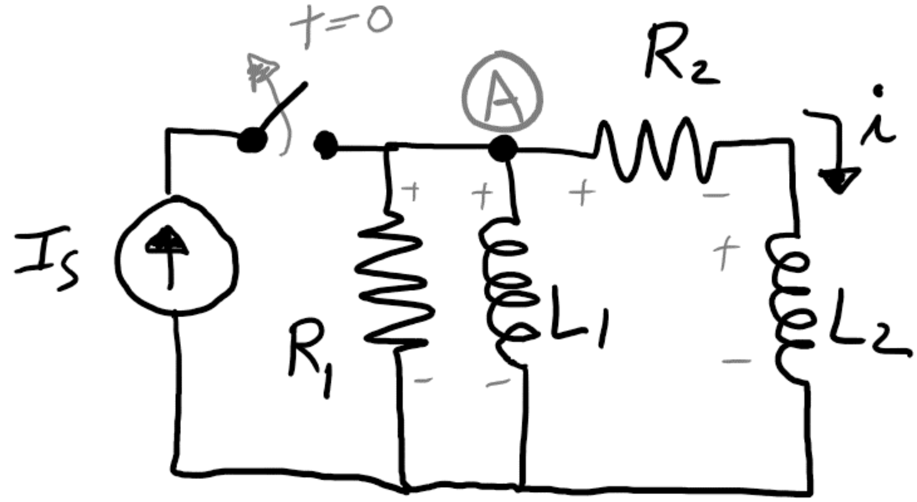
$$R^2 = \frac{L}{4(C - 10^{14} C^2L)}$$

$$R = \left(\frac{L}{4C(1 - 10^{14}LC)}\right)^{1/2}$$

“Chang Tzu tells us of a persevering man who after three laborious years mastered the art of dragon-slaying. For the rest of his days, he had not a single opportunity to test his skills.”
— Jorge Luis Borges, *The Book of Imaginary Beings*

Problem 4 (30 points)

Consider the circuit below.



a) What is i(0)?
Hint. What is the current flowing through L1 before the switch opens? Consequently, what is the current flowing through L2?

Solution. Because inductors behave like shorts to direct currect, the current flowing through L_1 is I_s. Since all the current is flowing through L_1, it must mean that the current flowing through L_2 is 0 from KCL.

b) What is di/dt (0)?

$i_{L1}(0^-) = I_s$

$i_{L2}(0^-) = 0$

$V_{R1} = -I_s R_1 = V_{L1} = V_{L2}$

$V = L\frac{di}{dt}$ ∴ $\frac{di_{L1}}{dt} = \frac{-I_s R_1}{L_1}$

$\frac{di_{L2}}{dt} = \frac{-I_s R_1}{L_2}$

c) What is the relationship between the voltages across L1 and R1?

Solution. They are the same.

d) Use KCL on Node A and the relationship derived above to arrive at a differential equation of the form,

$$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2}(t) + a_1 \frac{\mathrm{d}i}{\mathrm{d}t}(t) + a_0 i(t) = 0$$

where i(t) is the current going through L2.

Solution. Let the current going through R_1 be i_0 and the current going through L_1 be i_1. Then,

$$i_0 + i_1 + i = 0 \implies \frac{\mathrm{d}i_0}{\mathrm{d}t} + \frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

This means that,

$$\frac{\mathrm{d}i_0}{\mathrm{d}t} = -\frac{\mathrm{d}i}{\mathrm{d}t} - \frac{V_{L_1}}{L_1}$$

or,

$$\frac{\mathrm{d}i_0}{\mathrm{d}t} = -\frac{\mathrm{d}i}{\mathrm{d}t} - i\frac{R_2}{L_1} + \frac{L_2}{L_1}\frac{\mathrm{d}i}{\mathrm{d}t}$$

Now, taking the derivative with respect to time of,

$$V_{L_1} = V_{R_2} + V_{L_2}$$

we get,

$$R_1\frac{\mathrm{d}i_0}{\mathrm{d}t} = R_2\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}^2 t}{\mathrm{d}t^2}$$

Plugging in the expression for $\frac{\mathrm{d}i_0}{\mathrm{d}t}$, we get,

$$a_1 = \frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1} \text{ and } a_0 = \frac{R_1 R_2}{L_1 L_2}$$

e) Let R1 = R2 = R and L1 = L2 = L. Recall that the above differential equation can be reshaped into the follow linear algebra problem:

$$\begin{bmatrix} \frac{\mathrm{d}i}{\mathrm{d}t} \\ \frac{\mathrm{d}^2 i}{\mathrm{d}t^2} \end{bmatrix} = A \begin{bmatrix} i \\ \frac{\mathrm{d}i}{\mathrm{d}t} \end{bmatrix}$$

What is the A matrix and what are its eigenvalues?

Solution. We have,

$$\begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}$$

with,

$$a_1 = \frac{3R}{L} \text{ and } a_0 = \frac{R^2}{L^2}$$

This tells us that the eigenvalues are,

$$\lambda = \frac{R}{2L}(-3 \pm \sqrt{5})$$

f) Will this circuit exhibit any oscillations?

Solution. The eigenvalues we calculated does not have any imaginary terms, so no.

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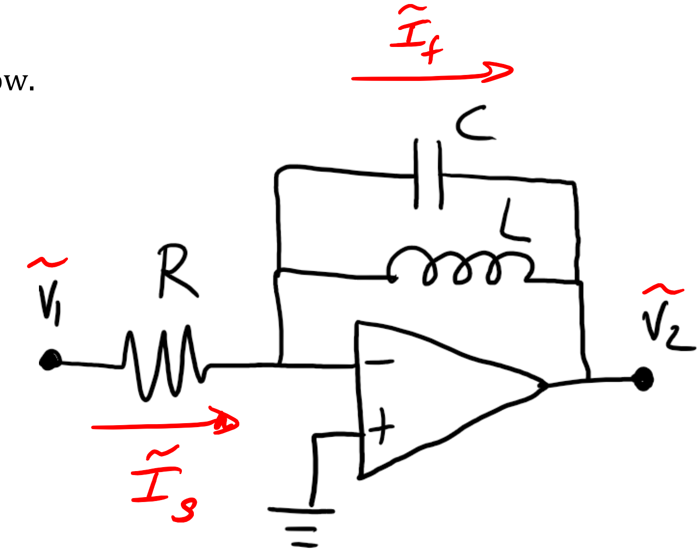
"I am Groot."
- Groot, *Guardians of the Galaxy*

Problem 5 (15 points)

Consider the circuit below.



a) Given an input voltage, $v_1(t)$, which is a sinusoid at frequency ω, and phasors corresponding to the input and output voltages, $\mathbf{V}_1$ and $\mathbf{V}_2$, find an expression for $\mathbf{V}_2/\mathbf{V}_1$.

$$\frac{\boldsymbol{V}_2}{\boldsymbol{V}_1} = \frac{-j\omega L}{R - \omega^2 RLC}$$

$$\tilde{I}_s = \tilde{I}_f \qquad Z_f = \frac{j\omega L}{1-\omega^2 LC}$$

$$\frac{\tilde{V}_1}{R} = \frac{-\tilde{V}_2}{Z_f}$$

$$\frac{\tilde{V}_2}{\tilde{V}_1} = -\frac{Z_f}{R} = \boxed{\frac{-j\omega L}{R - \omega^2 RLC}}$$

b) If $v_1(t) = \cos(\omega t)$ where $\omega = 10^6$ rad/s and L = 1 μH, R = 1 Ω, and C = 0.5 μF, solve for $v_2(t)$.

$v_2(t) =$ $-2\cos\left(\omega t + \frac{\pi}{2}\right)$

$\tilde{V}_1 = 1$

$$\tilde{V}_2 = \left(\frac{-j\omega L}{R - \omega^2 RLC}\right)\tilde{V}_1$$

$$= -\left(\frac{j\,10^6\,10^{-6}}{1 - 10^{12}(1)(10^{-6})(0.5\times10^{-6})}\right)$$

$$= \frac{-j}{1-(0.5)} = -2j$$

$$\tilde{V}_2 = -2j = -2e^{j\pi/2}$$

$$v_2(t) = -2\cos(\omega t + \pi/2)$$