EECS 16B Summer 2020 Midterm 1 (Form 1)

Instructions

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Justify your answers. A correct result without justification will not receive full credit.

Honor Code

COPY, SIGN, and DATE the UC Berkeley Honor Code by hand. (*If you fail to do so, your exam may not be accepted.* Email the instructors if you are unable.)

> *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

. . . .

COPY, SIGN, and DATE the following addendum by hand. (*If you fail to do so, your exam may not be accepted.* Email the instructors if you are unable.)

> *Should I commit academic misconduct during this exam, let me receive a failing grade in EECS 16B or dismissal from the University.*

. . . .

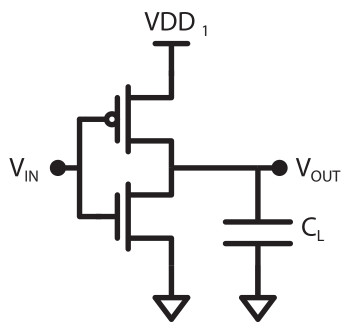
1 Digital logic

Designers of modern computer processors must navigate the trade-off between higher speed and lower power. Increasing the power supply voltage enables higher speed operation, but costs more power. Let's examine where the increased power draw and increased speed come from and where they do not. (Answers within 1-2 significant figures are correct if work is shown.)

Solution

Values for this problem vary, solutions are given in symbolic form. Plug in the values from your version to get the final answer.

a) Let's examine an ideal inverter driving a signal line with capacitance C_L. Solve the first-order differential equation for V_{out}, given $V_{\text{in}}(t < 0) = 1$ and $V_{\text{in}}(t \geq 0) = 0$, mosfet switch resistance $R_n = R_p = 2\,\Omega$, gate capacitance $C_n = C_p = 5 \times 10^{-12}$ F, voltage threshold $V_{\text{TH}} = 0.5\text{VDD}_1$ with $\text{VDD}_1 = 1$ V, and signal line capacitance $C_L = 2 \times 10^{-9}$ F.



Solution

The output node is composed of R_p from VDD_1 to the out node, and C_L from the out node to GND. C_p, C_n, and R_n have no effect on the output node for the given circuit and input.

Writing the voltage current relation for R_p and C_L gives us

$$\frac{\mathrm{d}}{\mathrm{d}t} V_{\text{out}}(t) C_L = I_C(t)$$

$$\text{VDD}_1 - V_{\text{out}}(t) = I_R(t) R_p$$

From KCL on the node "out," we know that $I_R(t) = I_C(t)$

$$\frac{\mathrm{d}}{\mathrm{d}t} V_{\text{out}}(t) C_L = \frac{\text{VDD}_1 - V_{\text{out}}(t)}{R_p}$$

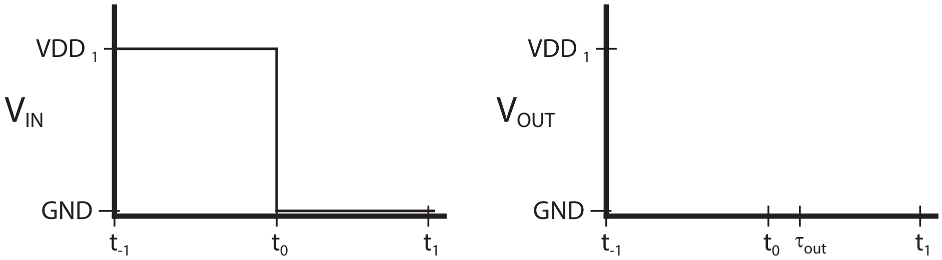
$$\frac{\mathrm{d}}{\mathrm{d}t} V_{\text{out}}(t) = \frac{\text{VDD}_1}{R_p C_L} - \frac{V_{\text{out}}(t)}{R_p C_L}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} V_{\text{out}}(t) + \frac{V_{\text{out}}(t)}{R_p C_L} = \frac{\text{VDD}_1}{R_p C_L}$$

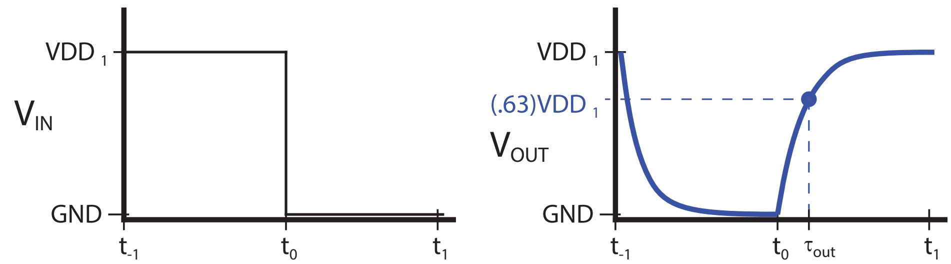
We recognize this form as a first order differential equation with a constant input similar to the general first order differential equation with constant input $\frac{\mathrm{d}}{\mathrm{d}t}x(t) + ax(t) = b$, with solution $x(t) = \frac{b}{a} + (x_0 - \frac{b}{a})e^{-at}$. Which gives us the final answer.

$$V_{\mathrm{out}}(t) = \mathrm{VDD}_1(1 - e^{-\frac{t}{R_p C_L}})$$

b) Draw V_{out} for V_{in} as a periodic square wave with one period shown below. Indicate the value of $V_{\text{out}}(\tau_{\text{out}})$ on the y-axis, where τ_{out} is an arbitrary RC time constant of the output node.



Solution



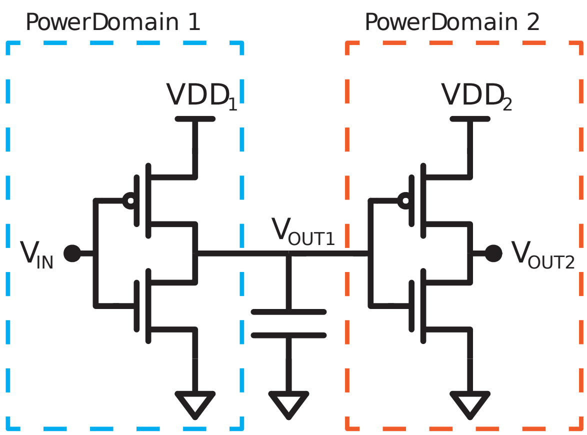
Must mark the y-axis value at one time constant τ_{out} for full credit.

c) How much energy does C_L store just before time t_1, for $\tau_{\text{out}} \ll (t_1 - t_0)$?

Solution

$$E = \frac{1}{2} C V_C^2$$
$$E = \frac{1}{2} C_L \text{VDD}_1^2$$
$$E = \frac{1}{2} C_L$$

d) Say V_{out1} is the input to a second power domain, where the inverters are connected to different VDDs. If each inverter's threshold is half of its respective VDD ($V_{\text{THX}} = 0.5\text{VDD}_{\text{X}}$), how long does it take for V_{out1} to reach V_{TH2} and flip the second inverter if $\text{VDD}_2 = 2\text{VDD}_1$?



Solution

V_{out1} asymptotically approaches $\text{VDD}_1 = \frac{1}{2}\text{VDD}_2 = V_{\text{TH2}}$. Inverter 2 *never* flips. ($t_{flip} = \text{inf}$)

e) Let α be the ratio VDD_2/VDD_1. In the previous question, $\alpha = 2$ wasn't a great value. Calculate a new α so that the second inverter flips at $t_{\text{flip}} = 3\tau_{\text{out1}}$, where τ_{out1} is the RC time constant related to node OUT1.

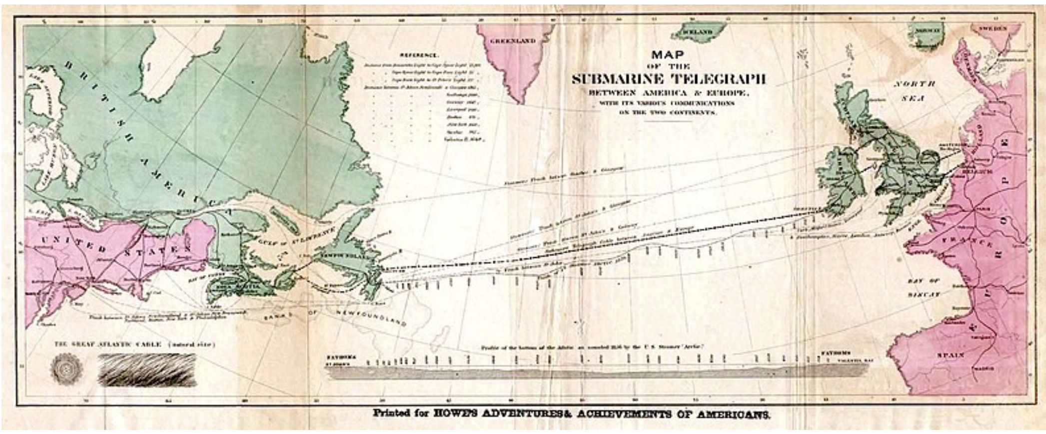
Solution

Inverter 2 flips at $V_{\text{out}_1}(t_{flip}) = V_{\text{TH2}} = \frac{1}{2}VDD_2$. From the first part we know that $V_{\text{out}_1}(t_{flip}) = \text{VDD}_1(1 - e^{-\frac{t_{flip}}{\tau_{out_1}}})$. Plug in $\alpha VDD_1 = VDD_2$ and $t_{flip} = X\tau_{out_1}$.

$$
\begin{aligned}
\text{VDD}_1(1 - e^{-\frac{t_{flip}}{\tau_{out_1}}}) &= V_{\text{TH2}} \\
\text{VDD}_1(1 - e^{-\frac{t_{flip}}{\tau_{out_1}}}) &= \frac{1}{2}VDD_2 \\
\text{VDD}_1(1 - e^{-\frac{t_{flip}}{\tau_{out_1}}}) &= \alpha\frac{1}{2}VDD_1 \\
\text{VDD}_1(1 - e^{-\frac{X\tau_{out_1}}{\tau_{out_1}}}) &= \alpha\frac{1}{2}VDD_1 \\
(1 - e^{-X}) &= \alpha\frac{1}{2} \\
\alpha &= 2(1 - e^{-X})
\end{aligned}
$$

2 Problem 2

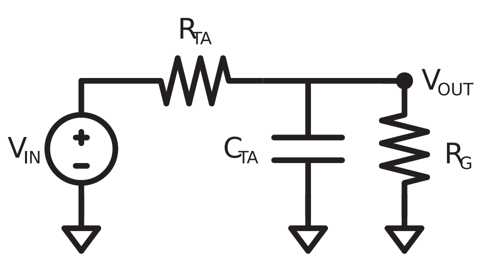
In 1858, the Atlantic Telegraph Company completed the first transatlantic telegraph cable. It took 16 hours to send Queen Victoria's inaugural 98 words, over a distance usually covered in 10 days. This would have been a success if the project hadn't been plagued with errors and the cable broken just days later. In this problem, you will use all of your modern engineering to understand all of things that went wrong. (Note, all of these assumptions are drastic over simplifications, but they are still a lot more than designers at the time knew!) (Answers within 1-2 significant figures are correct if work is shown.)



Solution

Fun history note[1]

a) While the concept of capacitance was understood at the time (Maxwell himself was consulting on the transatlantic cable) modeling or predicting capacitance was much more difficult. The designers had very little idea how slow they would have to send symbols until the cable was in place.



i) Write the fully factored transfer function, $H(j\omega)$, in terms of the above variables for our model of the transatlantic cable.

Solution

Recognize that the schematic is a voltage divider between impedances R_{TA} and $Z_{C_{\text{TA}}}||R_G$.

$$Z_{C_{\text{TA}}}||R_G = \frac{\frac{1}{j\omega C_{\text{TA}}} R_G}{\frac{1}{j\omega C_{\text{TA}}} + R_G}$$

[1]`https://en.wikipedia.org/wiki/Transatlantic_telegraph_cable`

$$
\begin{aligned}
H(j\omega) = \frac{V_{out}}{V_{in}} &= \frac{Z_{C_{\text{TA}}}||R_G}{(Z_{C_{\text{TA}}}||R_G) + R_{\text{TA}}} \\
&= \frac{\frac{\frac{1}{j\omega C_{\text{TA}}}R_G}{\frac{1}{j\omega C_{\text{TA}}}+R_G}}{\left(\frac{\frac{1}{j\omega C_{\text{TA}}}R_G}{\frac{1}{j\omega C_{\text{TA}}}+R_G}\right) + R_{\text{TA}}} \\
&= \frac{\frac{R_G}{j\omega C_{\text{TA}}}}{\frac{R_{\text{TA}}}{j\omega C_{\text{TA}}} + R_{\text{TA}}R_G + \frac{R_G}{j\omega C_{\text{TA}}}} \\
&= \frac{R_G}{R_{\text{TA}} + j\omega C_{\text{TA}}R_{\text{TA}}R_G + R_G} \\
&= \frac{R_G}{(R_{\text{TA}} + R_G) + j\omega C_{\text{TA}}R_{\text{TA}}R_G} \\
H(j\omega) &= \frac{\frac{R_G}{R_{\text{TA}}+R_G}}{1 + j\omega\frac{C_{\text{TA}}R_{\text{TA}}R_G}{R_{\text{TA}}+R_G}}
\end{aligned}
$$

ii) The transatlantic cable could send Morse code at 6 words/hr and quality began to degrade at any faster frequency. Knowing this cutoff frequency and having our model of the cable allows us to estimate the capacitance of the cable. Solve for R_{TA} and C_{TA} if $R_G = 1\,\Omega$, $\omega_c = 0.05\,\text{rad/s}$, and $H(j0) = 1 \times 10^{-6}$.

Solution

From the previous part

$$
H(j\omega) = \frac{\frac{R_G}{R_{\text{TA}}+R_G}}{1 + j\omega\frac{C_{\text{TA}}R_{\text{TA}}R_G}{R_{\text{TA}}+R_G}}
$$

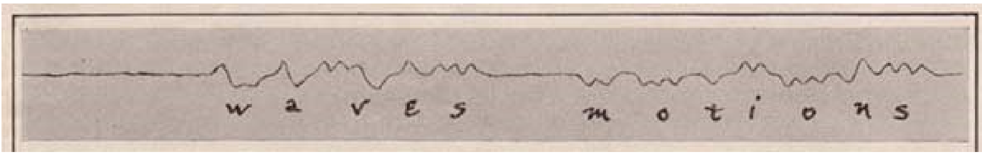
Solving first for R_{TA} using $H(j0) = 1 \times 10^{-6}$.

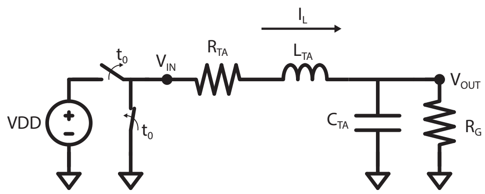
$$
\begin{aligned}
H(j0) = 1 \times 10^{-6} &= \frac{R_G}{R_G + R_{\text{TA}}} \\
\frac{1}{10^6} &= \frac{1\Omega}{1\Omega + R_{\text{TA}}} \approxeq \frac{1}{R_{\text{TA}}} \\
R_{\text{TA}} &= 10^6\Omega
\end{aligned}
$$

Solving for C_{TA} using $\omega_c = 0.05\,\text{rad/s}$

$$
\begin{aligned}
\omega_c = 0.05 &= \frac{R_{\text{TA}} + R_G}{C_{\text{TA}}R_{\text{TA}}R_G} \\
0.05 &= \frac{10^6\Omega + 1\Omega}{C_{\text{TA}}(10^6\Omega)(1\Omega)} \\
C_{\text{TA}} &\approxeq \frac{10^6}{0.05(10^6)} \\
C_{\text{TA}} &= 20\,\text{F}
\end{aligned}
$$

b) A telegraph operator would send morse code symbols through a telegraph cable by connecting a switch between VDD or GND, so the input signal would look much like a square wave. The output signal, however, had a ripple (ringing) by the time it reached the other end of the cable. The effect of inductance on a telegraph line was rarely considered, until the transatlantic cable failed and a bunch of clever scientists wanted to know why.





i) Find the symbolic values of matrix A in terms of L for the vector differential equation which describes our improved cable model with new values $R_{\mathrm{TA}} = 10,000\,\Omega$, $C_{\mathrm{TA}} = 1 \times 10^{-6}$, and $R_G = 1\,\Omega$, with $V_{\mathrm{OUT}}(t_0) = 1$ and $V_{\mathrm{IN}}(t) = 0$ for all t.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} V_{\mathrm{OUT}} \\ I_{\mathrm{L}} \end{bmatrix} = A \begin{bmatrix} V_{\mathrm{OUT}} \\ I_{\mathrm{L}} \end{bmatrix}$$

Solution

From KVL and KCL we can relate currents and voltages in our schematic.

$$\begin{aligned} I_L &= I_C + I_{R_G} \\ I_C &= I_L - I_{R_G} \\ V_{OUT}(t) &= -V_L - V_{R_{\mathrm{TA}}} \\ V_L &= -V_{OUT}(t) - V_{R_{\mathrm{TA}}} \end{aligned}$$

We can also write our voltage and current relationship for R_G, R_{TA}, L_{TA}, and C_{TA}.

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} V_{OUT}(t) C_{\mathrm{TA}} &= I_C \\ \frac{\mathrm{d}}{\mathrm{d}t} I_L(t) L_{\mathrm{TA}} &= V_L \\ V_{R_{\mathrm{TA}}} &= I_{R_{\mathrm{TA}}} R_{\mathrm{TA}} \\ V_{R_G} &= I_{R_G} R_G \end{aligned}$$

Combining all of the above we get

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} V_{OUT}(t) C_{\mathrm{TA}} &= I_L - \frac{V_{OUT}(t)}{R_G} \\ \frac{\mathrm{d}}{\mathrm{d}t} I_L(t) L_{\mathrm{TA}} &= -V_{OUT}(t) - R_{\mathrm{TA}} I_L \end{aligned}$$

Which we rearrange to write in a form that matches our vector differential equation.

$$\frac{\mathrm{d}}{\mathrm{d}t}V_{OUT}(t) = -\frac{V_{OUT}(t)}{R_G C_{\mathrm{TA}}} + \frac{I_L}{C_{\mathrm{TA}}}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}I_L(t) = -\frac{V_{OUT}(t)}{L_{\mathrm{TA}}} - \frac{R_{\mathrm{TA}} I_L}{L_{\mathrm{TA}}}$$
$$A = \begin{bmatrix} -\frac{1}{R_G C_{\mathrm{TA}}} & \frac{1}{C_{\mathrm{TA}}} \\ -\frac{1}{L_{\mathrm{TA}}} & -\frac{R_{\mathrm{TA}}}{L_{\mathrm{TA}}} \end{bmatrix}$$
$$A = \begin{bmatrix} -\frac{1}{1\times10^{-6}} & \frac{1}{1\times10^{-6}} \\ -\frac{1}{L_{\mathrm{TA}}} & -\frac{10{,}000}{L_{\mathrm{TA}}} \end{bmatrix}$$

ii) Choose any value of L that will cause ringing at the output of this cable for a square wave input. Justify your choice by solving for the eigenvalues of A with your chosen L.

Solution

Solving for the determinant or guessing with ipython were both acceptable methods.

$$\begin{aligned}
\det\begin{pmatrix} -10^6-\lambda & 10^6 \\ -\frac{1}{L_{\text{TA}}} & -\frac{10^3}{L_{\text{TA}}}-\lambda \end{pmatrix} &= (-10^6-\lambda)(-\frac{10^3}{L_{\text{TA}}}-\lambda)+\frac{10^6}{L_{\text{TA}}} \\
&= -\frac{10^9}{L_{\text{TA}}}+10^6\lambda+\frac{10^3\lambda}{L_{\text{TA}}}+\lambda^2+\frac{10^6}{L_{\text{TA}}} \\
&= \lambda^2+(10^6-\frac{10^3}{L_{\text{TA}}})\lambda-\frac{(10^9-10^6)}{L_{\text{TA}}} \\
&\approxeq \lambda^2+10^6\lambda-\frac{(10^9)}{L_{\text{TA}}} \\
\lambda &= \frac{-10^6\pm\sqrt{(10^6)^2-4\frac{10^9}{L_{\text{TA}}}}}{2}
\end{aligned}$$

Choose L_{TA} s.t.

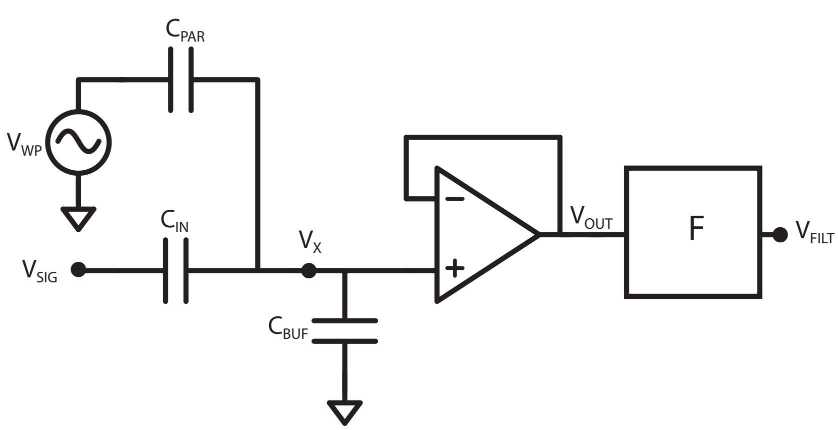
$$\begin{aligned}
10^{12}-4\frac{10^9}{L_{\text{TA}}} &< 0 \\
10^{12} &< 4\frac{10^9}{L_{\text{TA}}} \\
L_{\text{TA}} &< 4\frac{10^9}{10^12} \\
L_{\text{TA}} &< 4\times 10^{-3}\,\text{H}
\end{aligned}$$

3 Problem 3

Any circuit designer who has spent enough time debugging in a lab will come across a mysterious 60 Hz signal. Why 60 Hz? Because that's the frequency of AC wall power. If you are not very careful, that frequency might sneak into your signal lines and create enormous interference. Let's examine how this happens. (Answers within 1-2 significant figures are correct if work is shown.)

a) You are given an approximate model of a signal line node with input voltage V_{sig} which goes through a unit buffer. There is some parasitic capacitance C_{par} that comes from sloppy wiring and a cheap power source that capacitively couples wall power to your signal node. So even when your signal is off, you're seeing a 60Hz sinusoid on your output node.

The wall power voltage is $V_{\text{wp}}(t) = 120\cos\left(\omega_{\text{wp}}t\right)$. If the desired signal is $V_{\text{sig}}(t) = 50\cos\left(\omega_{\text{sig}}t\right)$ where $\omega_{\text{sig}} = 2\pi 1000$ (rad/sec) design a passive filter for box F that will attenuate a 60 Hz tone by $\frac{1}{10}$ or more, without significantly attenuating V_{sig}. You need only write the factored form transfer function, $H_F(j\omega)$, and justify why it fulfills the design specifications.



Solution

Need a high pass filter to cancel low frequency noise and keep higher frequency signal. Let's use a first order high pass filter from the general factored form. (Can also solve a CR high pass filter to get the transfer function.)

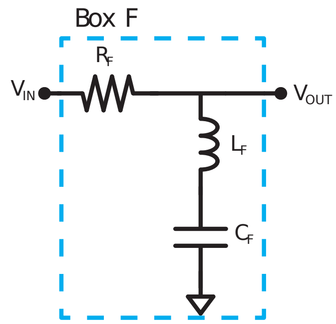
$$H_F(j\omega) = \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}}$$

Because the filter is first order, we estimate that signals are attenuated proportionally with $\frac{\omega}{\omega_c}$ until the corner frequency. So $\omega_c = 600 - 1000$Hz will attenuate 60Hz by $\frac{1}{10}$ or more, without significantly attenuating 1000Hz.

You could also solve this plugging in 60Hz and various ω_c using guess and check in python, or by setting $|H(j\omega)| = \frac{1}{10}$ and solving for ω_c.

b) Say, now, that $V_{\text{sig}}(t) = 0.5\cos\left(\omega_{\text{sig}1}t\right) + 0.5\cos\left(\omega_{\text{sig}2}t\right)$ where $\omega_{\text{sig}1} = 2\pi 1000$ (rad/sec) and $\omega_{\text{sig}2} = 2\pi 5$ (rad/sec). We would a need a filter which is the opposite of a bandpass filter, one which lets through low and high signals, but removes middle frequencies. Such a filter is called a notch filter.

i) Explain what is happening to the impedance of the following notch filter at $\omega_{\text{wp}} = 2\pi 60$ if L_F and C_F are in resonance.



Solution

The imaginary inductor and capacitor impedance will cancel out at resonance. This will cause the voltage divider between R_F and $Z_{L_F} + Z_{C_F}$ to scale any input at resonance to zero.

ii) Calculate C_F for the notch filter shown below if the only available inductor and resistor are $L_F = 1 \times 10^{-6}\,\text{H}$ and $R_F = 1000\,\Omega$ and we wish to remove ω_{wp}.

Solution

$$\begin{aligned} \omega_R^2 &= \frac{1}{L_F C_F} \\ (2\pi 60)^2 &= \frac{1}{10^{-6} C_F} \\ C_F &= \frac{1}{10^{-6}(2\pi 60)^2} \end{aligned}$$

4 Diagonalization I

In this problem all matrices are over the complex numbers. You may use the fact that if x is a complex number, there is a complex number $\sqrt{x}$ such that $\left(\sqrt{x}\right)^2 = x$.

a) State a basis in which A has a diagonal representation, and give coordinates for A relative to this basis. If you are not able to do so, explain why.

$$A = \begin{pmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Solution

The characteristic polynomial of A is

$$\begin{aligned} \chi_A(s) &= \det \begin{pmatrix} s-\alpha & \beta & 0 \\ -\beta & s-\alpha & 0 \\ 0 & 0 & s \end{pmatrix} \\ &= s\left((s-\alpha)^2 + \beta^2\right) = s\left(s - (\alpha + j\beta)\right)\left(s - (\alpha + j\beta)\right) \end{aligned}$$

Therefore the eigenvalues of A are $\lambda_1 = 0$, $\lambda_2 = \alpha + j\beta$, and $\lambda_3 = \alpha - j\beta$. $v_1 \propto (0, 0, 1)$, which is in the null space of A. As for v_2:

$$A - \lambda_2 I = \begin{pmatrix} -j\beta & -\beta & 0 \\ \beta & -j\beta & 0 \\ 0 & 0 & -\alpha - j\beta \end{pmatrix},$$

so $v_2 \propto (j, 1, 0)$. Likewise, $v_1 \propto (1, j, 0)$.

b) State the eigenvalues of A in the simplest form possible. If you are not able to do so, explain why.

$$A = \begin{pmatrix} a^2 & \sqrt{2}ab + b^2 j \\ \sqrt{2}ab - b^2 j & -a^2 \end{pmatrix}$$

Solution

The characteristic polynomial of A is

$$\begin{aligned} \chi_A(s) &= \det \begin{pmatrix} s - a^2 & -\sqrt{2}ab - b^2 j \\ -\sqrt{2}ab + b^2 j & s + a^2 \end{pmatrix} \\ &= s^2 - \left(a^4 + 2a^2b^2 + b^4\right) \end{aligned}$$

Setting this equal to zero,

$$\begin{aligned} s^2 &= a^4 + 2a^2b^2 + b^4 = \left(a^2 + b^2\right)^2 \\ s &= \pm\left(a^2 + b^2\right) \end{aligned}$$

c) State the eigenvalues of A in polar form. If you are not able to do so, explain why.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & k \\ k & 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 \\ 0 & 0 & k & 0 & 0 \\ 0 & 0 & 0 & k & 0 \end{pmatrix}$$

Solution

The three forms had different matrices, but the process is the same. Solving directly for λ in $Av = \lambda v$ results in $\lambda^5 = k^5$. The eigenvalues are the 5th roots of 1, multiplied by k: $ke^{0\frac{2\pi}{5}j}$, $ke^{1\frac{2\pi}{5}j}$, $ke^{2\frac{2\pi}{5}j}$, $ke^{3\frac{2\pi}{5}j}$, and $ke^{4\frac{2\pi}{5}j}$. Different angles (equivalent modulo 2π) are accepted.

5 Diagonalization II

a) If $T \in \mathbb{C}^{m\times n}$ is a matrix, then T^*T has nonnegative real eigenvalues. How does the construction of the SVD of T rely on this fact? (Length: 1 sentence)

Solution

Singular values are the principal square roots of this matrix's eigenvalues.

b) Does there exist a matrix $A \in \mathbb{C}^{2\times 2}$ that is *not* diagonalizable? If so, give and explain an example. If no, explain why not. (Length: 1–2 sentences)

Solution

Yes. The following matrix has an eigenspace of dimension 1 for its only eigenvalue.

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

c) Does there exist a matrix $A \in \mathbb{C}^{n\times n}$ that does not have an eigenvector? If so, give and explain an example. If not, explain why not. (Length: 1–2 sentences)

Solution

No. The existence of an eigenvector is guaranteed by the Fundamental Theorem of Algebra. (This is used in the proof of the Spectral Theorem.)

d) We proved that a matrix $A \in \mathbb{C}^{n\times n}$ satisfying $A = A^*$ can be factored $A = PDP^*$, where P is unitary and D is diagonal. How did we use mathematical induction in the process? (Length: 1 sentence for base case, 1 sentence for inductive step)

Solution

Base case: matrices are diagonal when $n = 1$.

Inductive step: if the Spectral Theorem is true for $(n-1)\times(n-1)$ matrices, then it holds for $n \times n$ matrices.

e) Answer True/False: a matrix of noisy instrument measurements is more likely to have high rank than low rank.

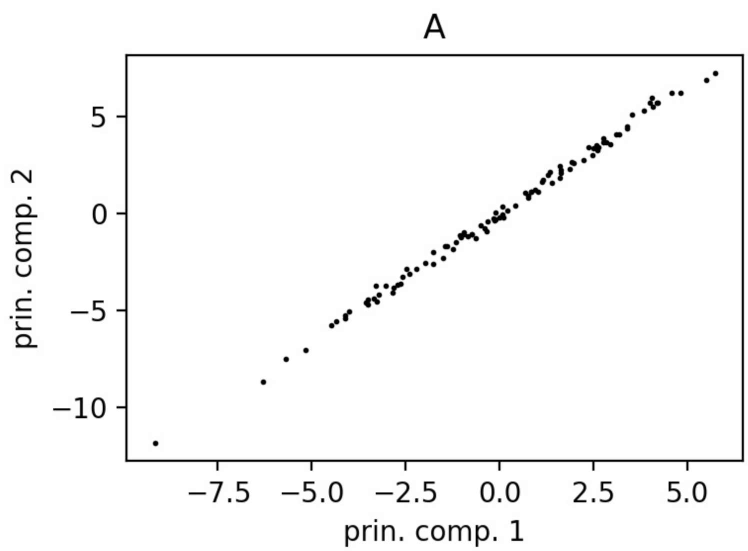
Solution

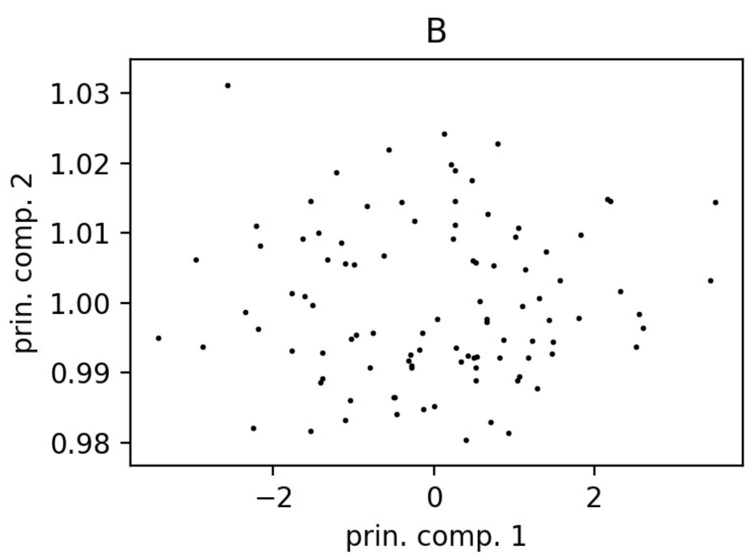
True

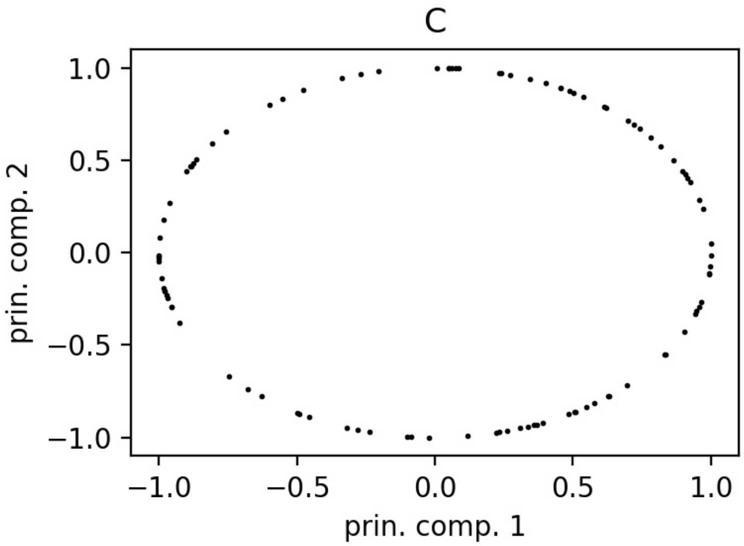
6 PCA

Answer whether each scatter plot resembles the scatter plot of the projections of a dataset onto its first two principal components, in which the first principal component is shown on the horizontal axis and the second is shown on the vertical axis. (True/False: *True* if it does, *False* if it doesn't)

If not possible, state that there is not enough information to do so. Justify each answer with 1–2 sentences.



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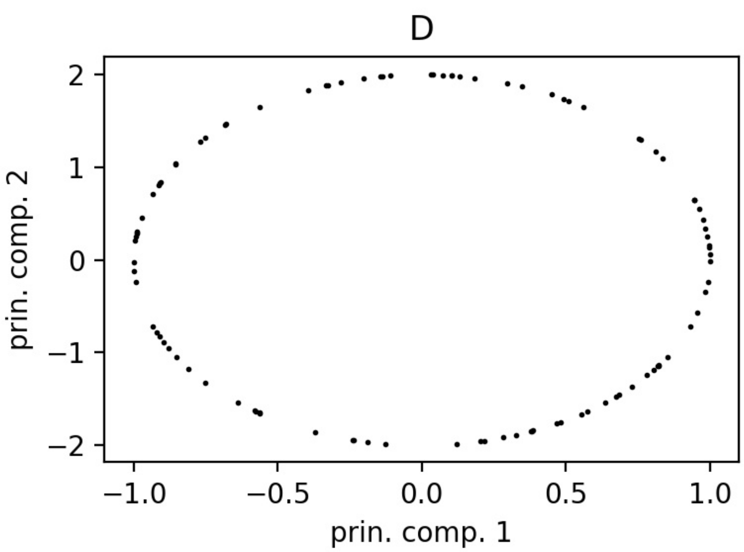
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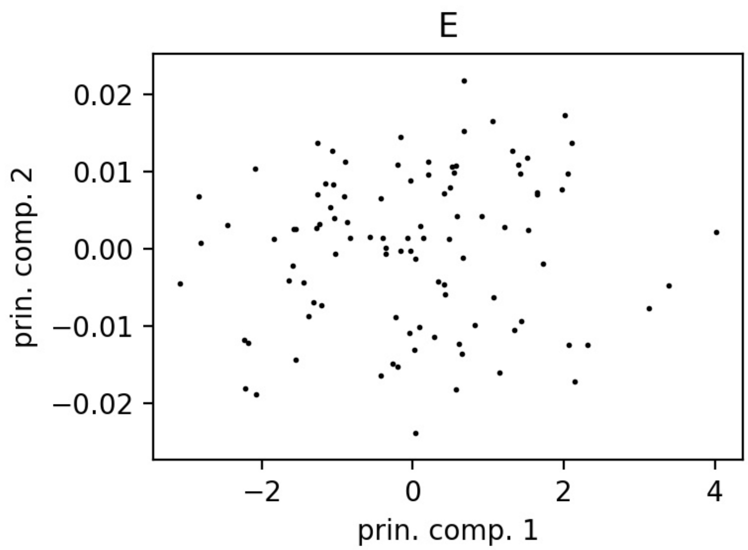
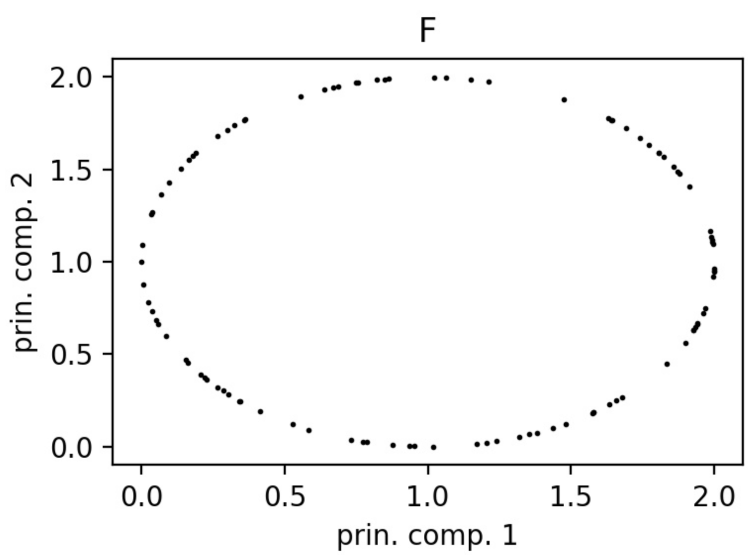
1.0
prin. comp. 1





Solution

a) False. A diagonal direction around 45 degrees has more variance.

b) False. PC2 not centered at zero.

c) True. This is a perfect circle which has rotational symmetry and therefore equal variance in any direction.

d) False. Variance in PC2 exceeds that in PC1.

e) True.

f) False. Not centered at zero.