Discussion 1
Friday, August 28, 2020  12:47 AM

Agenda:
1) Systems engineering diagram
2) Nature of this course
3) Highlight some resources, keeping track of links
4) Course survey
5) break
6) Notation and Proofs
7) Group work round 1
8) Group work round 2

What's a "dynamic model"?

Resources that may help with 221A:

Text: Linear Algebra Done Right by Axler:
  linear.axler.net
Solns to linear algebra done right:
  http://linearalgebras.com/7b.html

Video series: Blue1Brown
  https://www.youtube.com/c/3blue1brown/playlists?view=50&sort=dd&shelf_id=20

Professor Brunton video series
  www.youtube.com/playlist?list=PLMrJAkhleNNR20Mz-VpzgfQs5zrYi085m
EE211A Section 1  
August 28, 2020

1 Preface 
Systems engineering flowchart, nature of this course

2 Review of Math Notation

- Sets: \( \mathbb{Z} \), \( \mathbb{R} \), \( \mathbb{N} \), \( \mathbb{C} \)
- Logic: \( \exists \), \( !\exists \), \( \not\exists \), \( \forall \)
- Logic: \( s.t., \exists, \mid \), \( \iff \), \( \Rightarrow \)
- Acronyms: WTS, WLOG, RHS, LHS, NBNS, "memorize"

3 Methods of proof

1. Direct proof
   \( A \Rightarrow B \): Logically derive the conclusion by combining the definitions, assumptions, lemmas, theorems etc.

2. Proof by contrapositive
   Establish the conclusion "\( A \Rightarrow B \)" by showing \( \neg B \Rightarrow \neg A \).

3. Proof by contradiction
   Assume \( A \) and \( \neg B \): First assume that the givens hold and the conclusion is false, and then derive a logical contradiction. Therefore, the conclusion must be true.

4. Proof by construction
   Construct an example that shows the fallacy or validity of a statement. Usually useful for disproving an assertion such as "all X are Y" or confirming a statement such as "there exists a W such that Z."

5. Proof by induction (if the conclusion involves the natural number \( n \in \{1, 2, 3, \cdots \} \))
   First prove that the conclusion is true for the initial condition, such as when \( n = 1 \). Then, show that if the conclusion is true whenever \( n \leq k \), the conclusion is true for \( n = k + 1 \).

Definition 1. A number $n \in \mathbb{Z}$ is even if $n = 2k$ for some $k \in \mathbb{Z}$.

Definition 2. A number $q$ is rational if there exist $a, b \in \mathbb{Z}$ with $b \neq 0$ such that $q = \frac{a}{b}$.

Problem 1 (Direct method). Prove that the sum of two odd numbers is even.

Problem 2 (Direct method). Prove that the product of an even number and any other number is even.

Problem 3 (Proof by contrapositive). Show that, if $x^2$ is even, then $x$ is even.

Problem 4 (Proof by contrapositive). If $ab$ is even, then either $a$ or $b$ is even.

Problem 5 (Proof by contradiction). Prove that $\sqrt{2}$ is an irrational number.

Problem 6 (Proof by construction). Prove that not all odd numbers are prime.

Problem 7 (Proof by induction). Prove that $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$.