EE224A Fall 2009: Homework #5

Assigned: 10/13/2009 --- Due: 10/22/2009 in class *before* lecture

Problem 1

[10] 1. In a BPSK system, the clock that specifies the sampling the receive pulse output is offset from the optimum sampling time by 10%.

a) If the signal pulse used is rectangular, determine the loss in SNR

b) Determine the energy of the intersymbol interference introduced

Problem 2

[10] 2. A voice-band telephone channel operates on the baseband up to 3 kHz. It is desired to design a modem that transmits at a symbol rate of 4800 symbols/s, with the objective of achieving 9600 bits/s. Select an appropriate PAM constellation and the roll-off factor of a raised cosine pulse that utilizes the entire frequency band. Sketch the spectrum of the transmitted signal pulse and determine the important frequencies.

Problem 3

[60] 3. This problem asks you to build a rudimentary communication system, adding progressively more components in each part. It is helpful to draw a system diagram for the system designed in each part of the question. Please also explain carefully what you have done in each part. Exhibit trial runs of your system in each step to make sure it works as intended.

a) Implement on MATLAB a BPSK modulation and demodulation scheme over a baseband $[-100Hz, 100Hz]$. You can assume that the information sequence is i.i.d. equiprobable to be 0 or 1. Use sinc pulses and transmit symbols at Nyquist rate, but to make your system have a finite delay, you have to truncate the pulses. Note that the truncated pulses are no long orthogonal and so there is possibly inter-symbol interference. Explore the impact of the truncation on the signal to ISI ratio at the sample times. Also, explore the effect of mis-synchronization, where the sampling times are off by a constant offset. Document your experiments. (You can ignore the additive noise in this part of the question.)

b) Redo part (a) using raised cosine pulses. Choose an appropriate roll-off factor. In what ways are these pulses better than sinc pulses? What is your symbol rate? What is your data rate?

c) In part (b) we considered only uncoded transmission. Now consider the use of an orthogonal code to transmit the information bits. (This orthogonal code is used in the CDMA cellular standard.) The information bit stream are partitioned into k-bit blocks. Each k-bit block is mapped into a $M = 2^k$ length orthogonal vector. The M possible orthogonal vectors are the columns of a M by M matrix H_M. (called the Hadamard matrix). H_M is defined recursively for each M:

$$H_1 = [1] \qquad H_M = \begin{bmatrix} H_{M/2} & H_{M/2} \\ H_{M/2} & -H_{M/2} \end{bmatrix}, \quad M \geq 2.$$

Each vector is then transmitted as M symbols.

i) What is the data rate of the system in terms of k?

ii) Explain why the columns of H_M are orthogonal.

iv) Implement this code on your system in (b) for $k = 4$ and also design and implement a ML detector for the detection of each k-bit block.

Problem 4

[10] 4. In class, we derive the zero-forcing equalizer for estimating $x[0]$ and $x[1]$ for the channel with two taps. Continue the process and derive the zero-forcing filter for estimating $x[3], x[4], \ldots$ and for general $x[m]$. Under what condition on the channel response does the filter converge to a linear time-invariant filter as $m \to \infty$? If it does converge, what is the LTI filter and what is the SNR at the output of that filter?

Problem 5

In this exercise, we will go a bit deeper into the matched filter, ZFE and MMSE equalizers. The model of the ISI channel is a simple 2-tap one:

$$y[n] = h_0 x[n] + h_1 x[n-1] + w[n], \quad n \geq 1$$

where h_0, h_1 are fixed (not changing with n) real numbers known to the receiver. Time instants begin at unity and you can suppose that $x[0] = 0$. The additive noise $w[n]$ is Gaussian (zero mean with variance σ^2) and independent across time instants. The transmission is simple bit-by-bit signaling. At each time instant n, we send one of $\pm\sqrt{\mathcal{E}_b}$ voltages depending on the bit that is needed to be communicated at time instant n.

1. Focus on the input symbol at time 1. This symbol $x[1]$ is seen at two received time instants and we thus focus on $y[1]$ and $y[2]$ to make our decision on what $x[1]$ is. In class, we have derived simple expressions for the matched filter and ZFE. Now suppose that $h_0 = 1$ and $h_1 = 0.5$. Plot the SNRs at the output of the matched filter and ZFE (on the same graph) as a function of the *input* SNR: $\frac{\mathcal{E}_b}{\sigma^2}$.

2. Using a MATLAB simulation evaluate numerically the SNR at the output of the MMSE receiver. Plot the output SNR as a function of the input SNR: $\mathcal{E}_b/\sigma^2$. Juxtapose this plot with the corresponding plots for the matched filter and ZFE (from the previous part). Comment on the performance of the matched filter and ZFE in terms of the output SNR as compared with the MMSE filter (which, by definition is the best *linear* filter in terms of the output SNR).

Problem 6

Consider the L-tap ISI channel model:

$$y[n] = \sum_{l=0}^{L-1} h_l x[n-l] + w[n], \quad n \geq 1,$$

and suppose plain bit-by-bit signaling over this channel. So, $x[n]$ is i.i.d. in time n and equally likely to be $\pm\sqrt{E}$. As usual, $w[n]$ is additive white Gaussian noise (with energy per symbol equal to σ^2) and is independent of the input $x[n]$. So, the average energy of the input signal per symbol is E. Calculate the average energy, $\mathbb{E}\left[y^2[n]\right]$, of the output signal per symbol. Conclude from your calculation that ISI increases the average SNR of the channel, i.e., the more the ISI, the better the SNR is.