Tiling of the time frequency plane
Figure 6.3

Basis vectors (for N = 16) of some commonly encountered transforms: (a) Fourier basis (real and imaginary parts), (b) discrete Cosine basis, (c) Walsh-Hadamard basis, (d) Slant basis, (e) Haar basis, (f) Daubechies basis, (g) Biorthogonal B-spline basis and its dual, and (h) the standard basis, which is included for reference only (i.e., not used as the basis of a transform).
How to determine time & frequency extent of a function $h(t)$ in the time-frequency plane.

Let $P_h(t) = \frac{|h(t)|^2}{\|h(t)\|^2}$ probability density fn.

Mean: $\mu_t = \frac{1}{\|h(t)\|^2} \int_{-\infty}^{\infty} t |h(t)|^2 \, dt$

Variance: $\sigma_t^2 = \frac{1}{\|h(t)\|^2} \int_{-\infty}^{\infty} (t - \mu_t)^2 |h(t)|^2 \, dt$

For $\int h(t)^2 = H(f)$
\[ p_H(f) = \frac{|H(f)|^2}{\|H(f)\|^2} \] - probability density function for \( H(f) \)

\[ \text{mean} \quad \mu_f = \frac{1}{\|H(f)\|^2} \int_{-\infty}^{\infty} f |H(f)|^2 \, df \]

\[ \text{variance} \quad \sigma_f^2 = \frac{1}{\|H(f)\|^2} \int_{-\infty}^{\infty} (f - \mu_f)^2 |H(f)|^2 \, df \]
(a) Basis function localization in the time-frequency plane. (b) A standard basis function, its spectrum, and location in the time-frequency plane. (c) A complex sinusoidal basis function (with its real and imaginary parts shown as solid and dashed lines, respectively), its spectrum, and location in the time-frequency plane.
Recall Haar basis fn:

\[ \psi_{s,t}(t) = 2^{s/2} \psi(2^s t - t) \quad t, s \text{ integer} \]

\[ \psi_{j,k}(x) = 2^{j/2} \psi(2^j x - k) \]

\[
\text{F.T. } \left\{ \psi(2^s t) \right\} = \frac{1}{2^s} \psi \left( \frac{f}{2^s} \right)
\]

For \( s > 0 \) \quad \rightarrow \text{spectrum is stretched}

For \( s < 0 \) \quad \rightarrow \text{spectrum is compressed}
Time and frequency localization of 128-point Daubechies basis functions.