Computing DFT

$$X(k_1,k_2) = \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} x(n_1,n_2)\, e^{-j\frac{2\pi n_1 k_1}{N}}\, e^{-j\frac{2\pi n_2 k_2}{N}}$$

- 3 ways of computing this sum.
  - Direct method.
  - Row/col. Decomposition.
  - Vector. Radix.

Factors affect speed:

- # of arithmetic ops
- # I/O access, memory.
- language code is written in.

Direct Method:

- For each k_1 and k_2 :
  - assign $e^{j2\pi n_1 k_1 / N}$; $e^{j2\pi n_2 k_2 / N}$

are all pre-computed & pre stored. because They are signal independent.

And To compute $\cos\theta$, $\sin\theta$ one should avoid power series computation.

$\Rightarrow$ DO: Table look up.

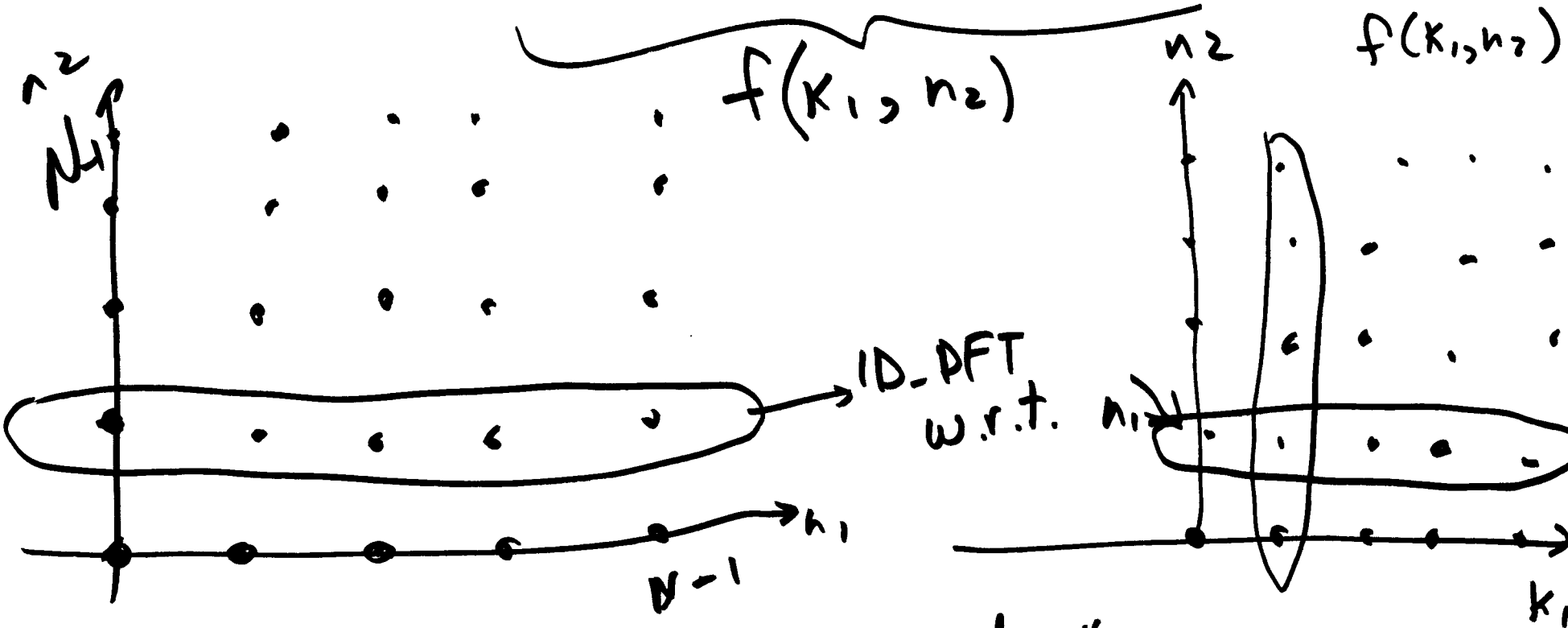
N^2 mult
N^2 adds

$\Rightarrow$ N^4 mult
N^4 adds.

N^2 values of k_1 & k_2

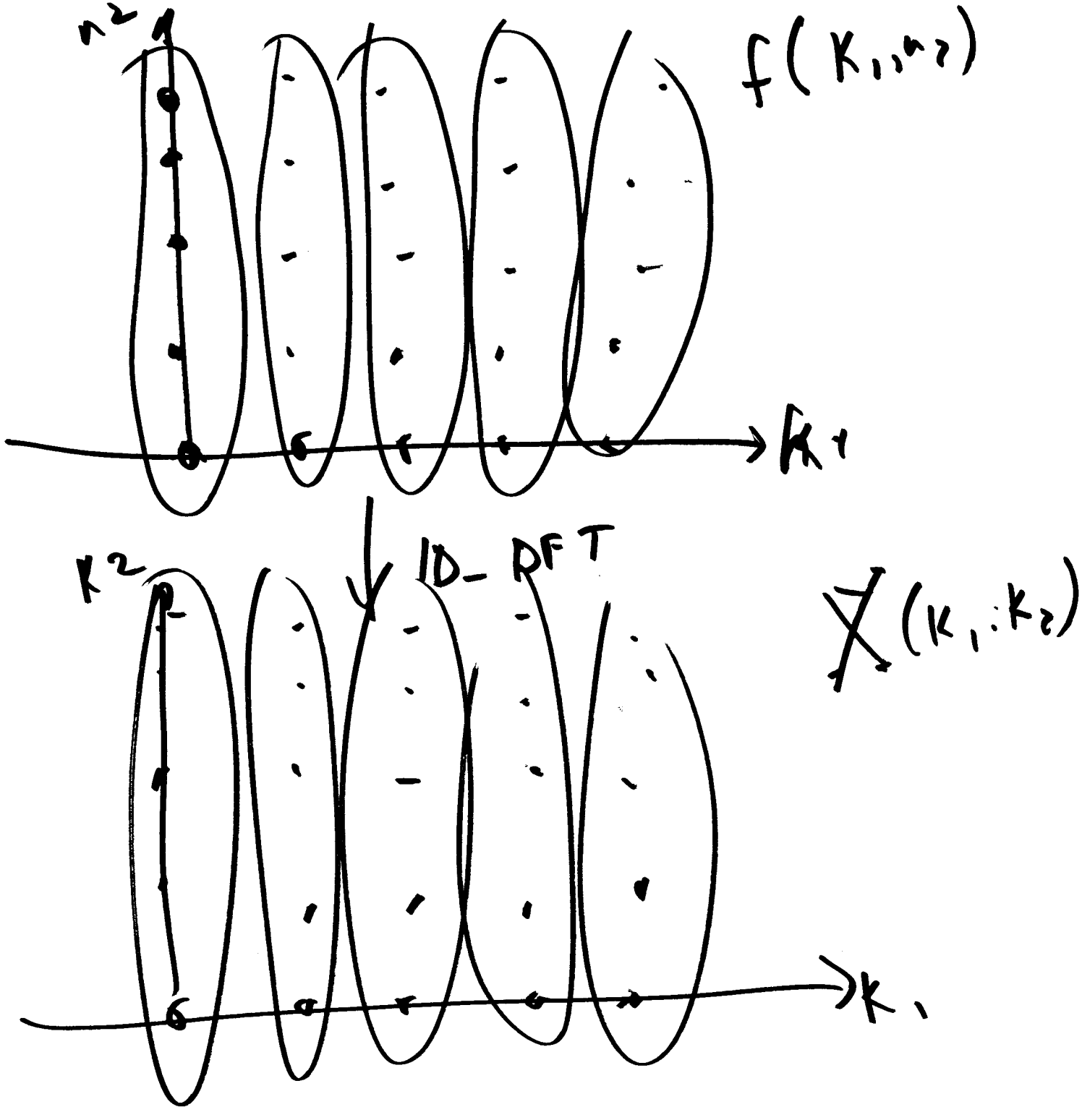
Row/Col. Decomposition

$$X(k_1,k_2)=\sum_{n_2=0}^{N-1}\left(\underbrace{\sum_{n_1=0}^{N-1}x(n_1,n_2)\,e^{-j\frac{2\pi n_1 k_1}{N}}}_{f(k_1,n_2)}\right)e^{-j\frac{2\pi n_2 k_2}{N}}$$



(1) N - one Dimensional DFT of rows to get $f(k_1,n_2)$
each 1-D DFT, N point seq.

(2) N - one Dimension DFT of coloms to get final
each one of these N pt.


n2
f(K1,n2)
K1
1D- DFT
K2
X(K1,K2)
K1

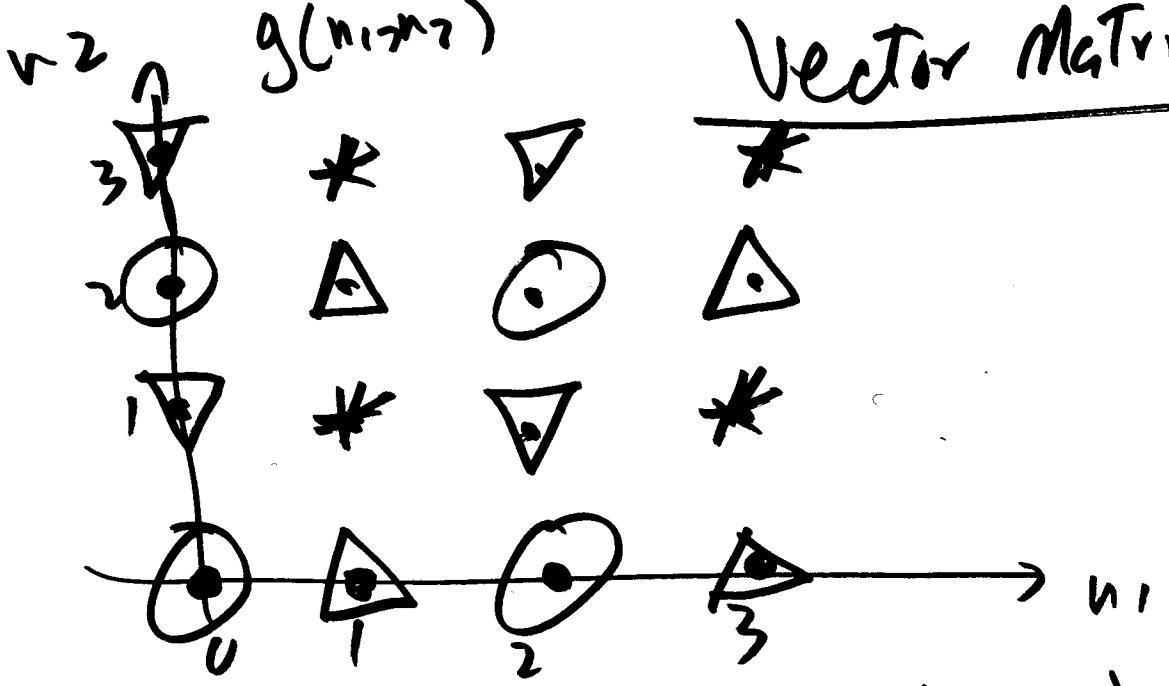
Complexity of 1-D FDFT.

- Direct method $\sum_n x(n) e^{-j\frac{2\pi nk}{N}}$ $\Longrightarrow$ N^2 mult, N^2 adds.
- FFT (Fast Fourier Transform) $\Longrightarrow$ $N\log_2 N$ mult, $\frac{N}{2}\log_2 N$ adds
  - Divide & Conquer.

Row/Col Decay:

- Direct 1-D DFT $\longrightarrow$ $2N$ DFT. N^2 mult / N^2 adds $\longrightarrow 2N^3$ mult/adds
- FFT - my 1-D DFT $\longrightarrow$ $2N$ DFT, $N\log_2 N$ mult, $\frac{N}{2}\log_2 N$ adds $\longrightarrow O(N^2 \log_2 N)$ mult/add

Vector Matrix FFT



4X4 ⇒

- 2x2 $g_{ee}(n_1,n_2)$
- 2x2 $g_{eo}(n_1,n_2)$
- 2x2 $g_{oe}(n_1,n_2)$
- 2x2 $g_{oo}(n_1,n_2)$

⊙ even index in g_{ee} n_1 & n_2 - (n_1,n_2)

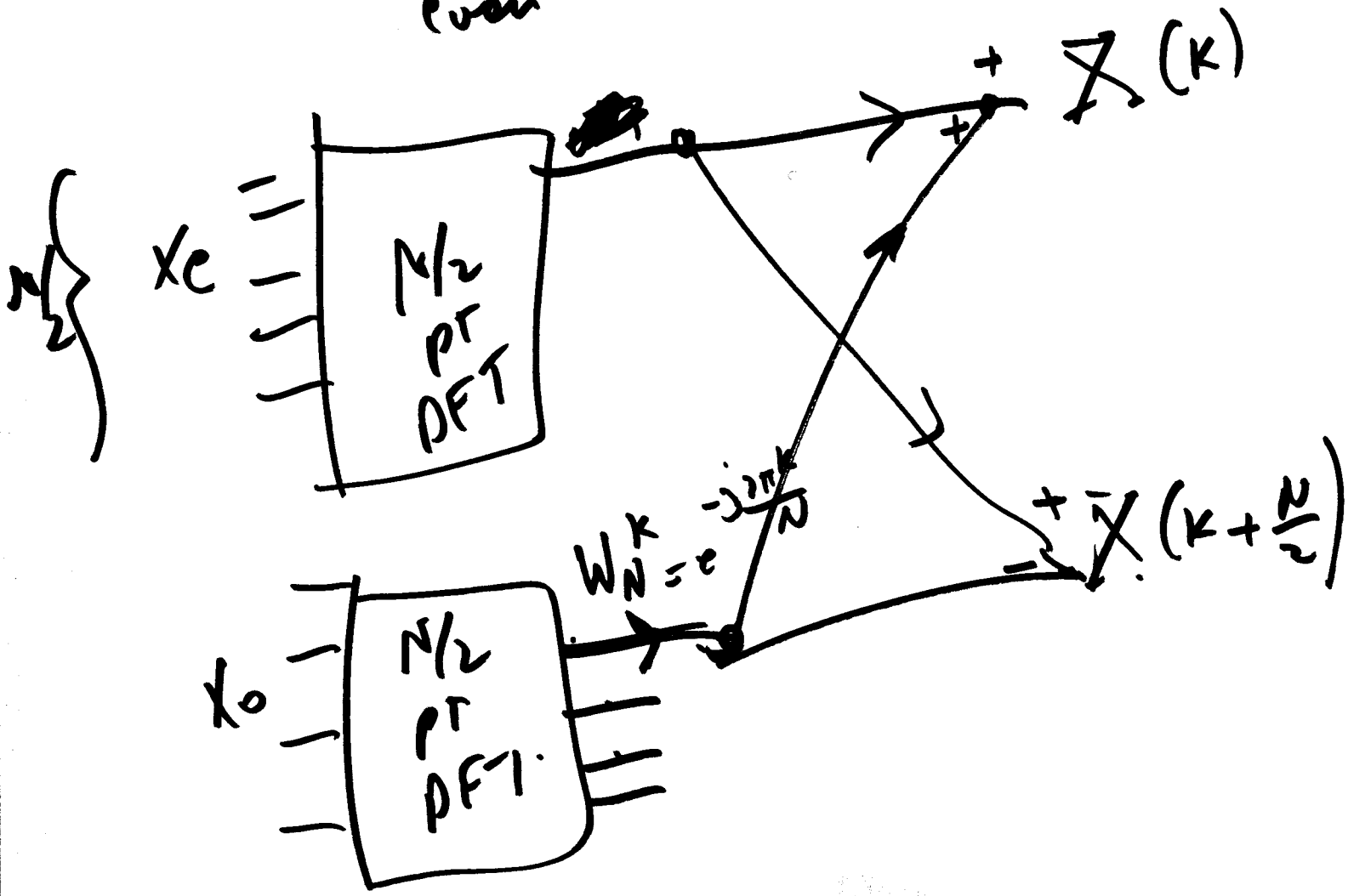
△ even in n_2 $g_{oe}(n_1,n_2)$ odd in n_1

▽ even in n_1 $g_{eo}^{(n_1,n_2)}$ odd in n_2

✱ odd in n_1 $g_{oo}^{(n_1,n_2)}$ // // n_2

$x(n) \rightarrow x_e \quad x_o$

even odd



$$X(k_1,k_2) = G_{ee}(k_1,k_2) + G_{eo}(k_1,k_2)\, e^{-j\frac{2\pi k_2}{N}} + G_{oe}(k_1,k_2)\, e^{-j\frac{2\pi k_1}{N}} + G_{oo}(k_1,k_2)\, e^{-j\frac{2\pi k_1}{N}} e^{-j\frac{2\pi k_2}{N}}$$

where $G_{ee}(k_1,k_2)$ is $\frac{N}{2} \times \frac{N}{2}$ pt DFT of $g_{ee}(n_1,n_2)$

G_{eo} — — — g_{eo}

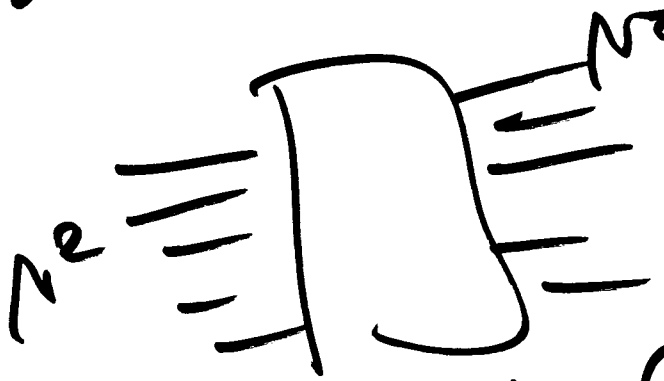
G_{oe} — — — g_{oe}

G_{oo} — — — g_{oo}

Show Fig 3.23, 3.24, 3.25 J.S. Lim

op cont for Vector Radix

- # of stages for NXN image.
  - $\log_2 N$
  - Each stage



$\Rightarrow \frac{N^2}{4}$ Butterflies.

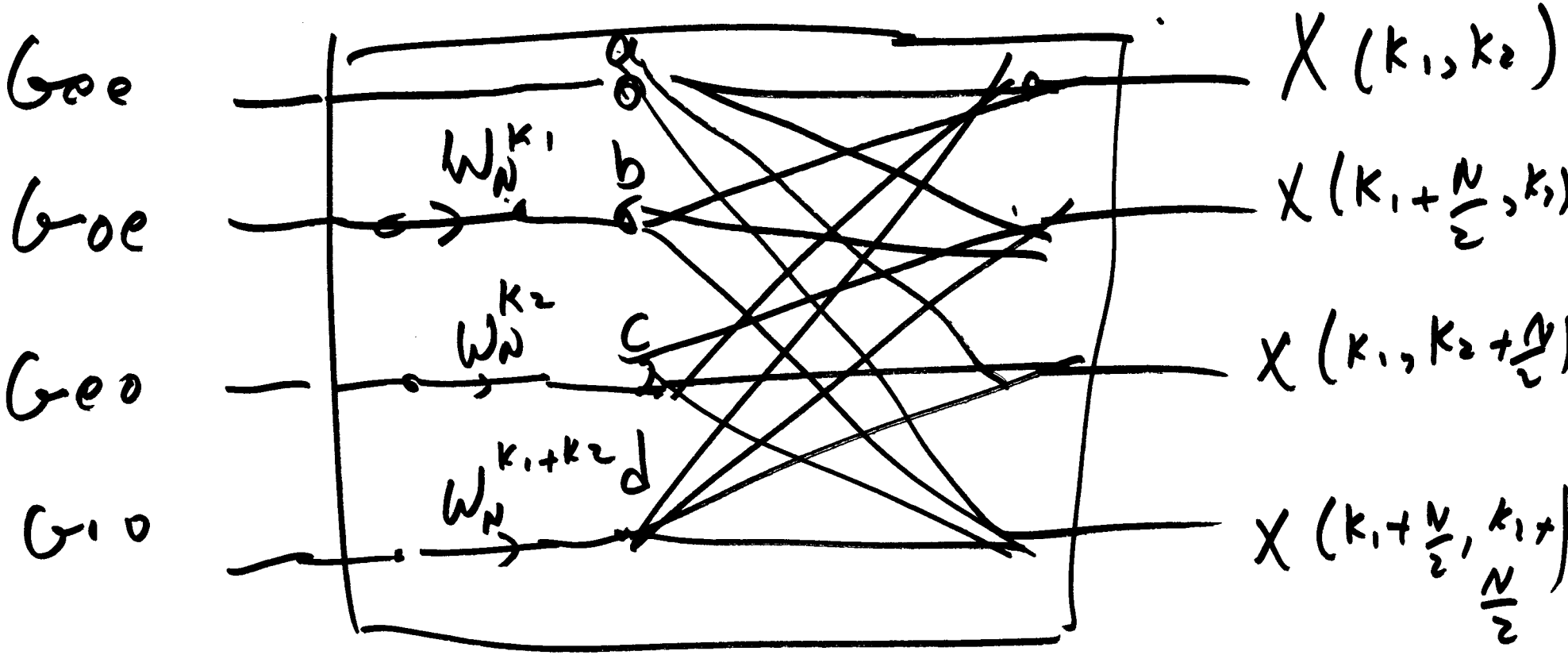
- Each butterfly.
  - 3 mults
  - 12 adds.

(Trick 16 → 8)

→ Total mult.: $3\frac{N^2}{4}\log_2 N$

adds: $4N^2 \log_2 N$

Q: In each butterfly: 8 instead of 12 adds.



Simple way: 12 adds:

Define

$A = a+b$

$B = a-b$

$C = c+d$

$D = c-d$

$\Longrightarrow$

$X(k_1,k_2) = A+C$

$X(k_1+\frac{N}{2},k_2) = B+D$

$X(k_1,k_2+\frac{N}{2}) = A-C$

$X(k_1+\frac{N}{2},k_2+\frac{N}{2}) = B-D$

Can