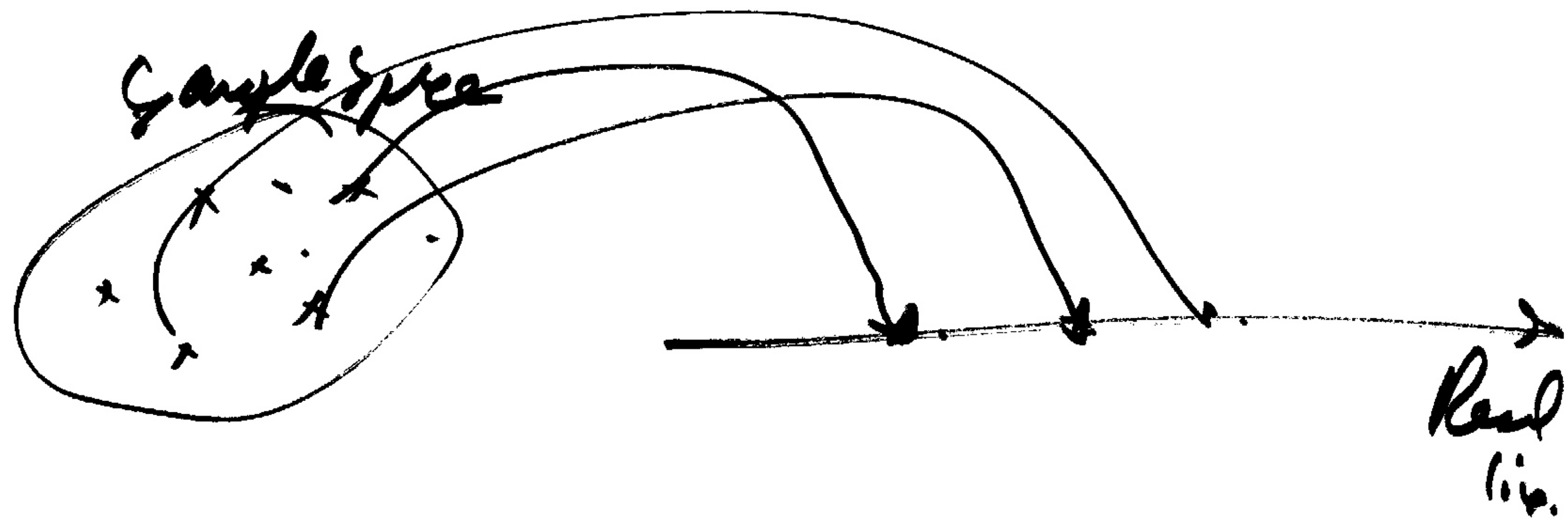
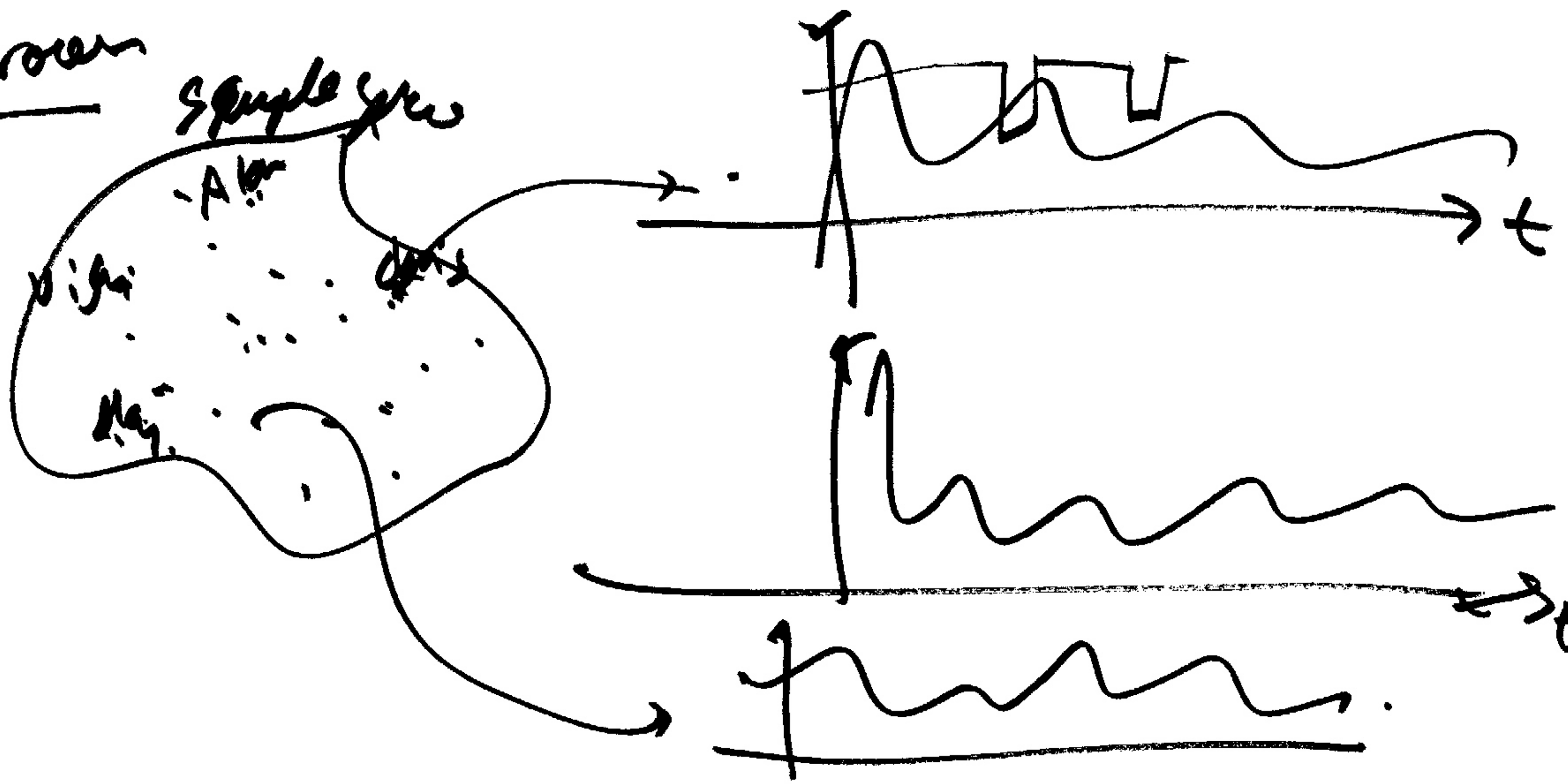
R.V



R.Process



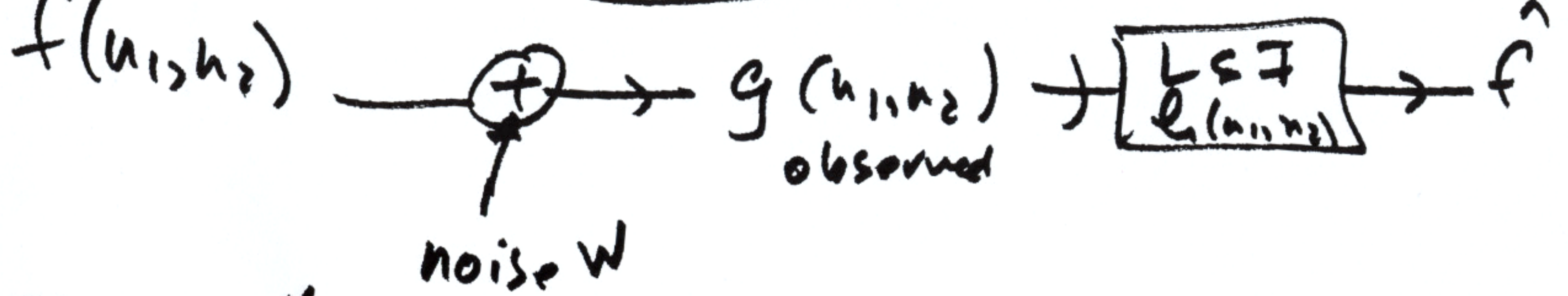
Stationarity

$$P_{x(t_1)\,x(t_2)\ldots\ldots x(t_N)}\left(X_1, X_2, X_3 \ldots, X_N\right)$$

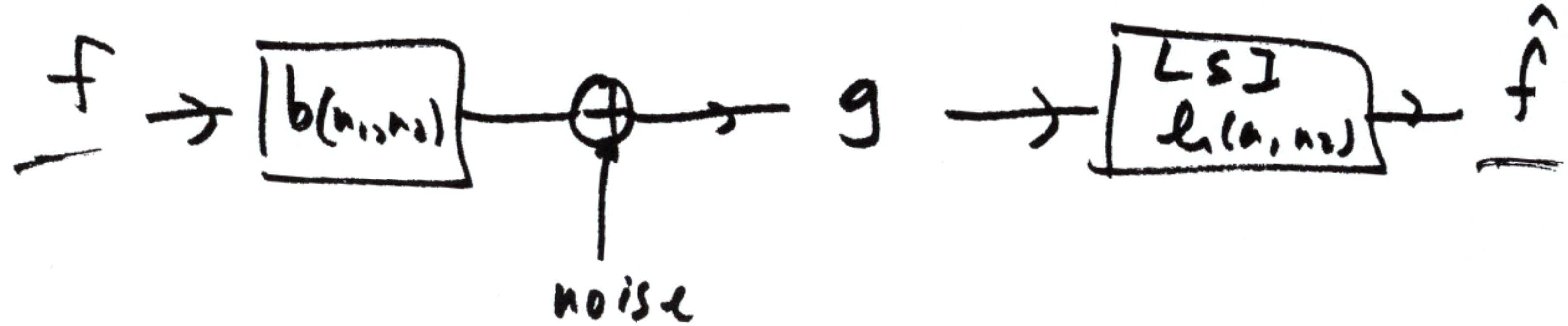
$$=$$

$$P_{x(0),\, x(t_2-t_1),\, \ldots\, x(t_N-t_1)}\left(X_1, X_2, \ldots X_N\right)$$

Weiner filtering



more generally



f sample of a zero mean stationary random process.

w " " " " " " " " "

f, w are independent of each other

Define metric : $\hat{f}$ as close as possible to f.

$$E\left[(f-\hat{f})^2\right] \longrightarrow \text{linear least square error.}$$

$\perp$

<u>orthogonality principle</u> :

least square error is achieved when "error orthogonal to observation"

$\perp$

$$e = f - \hat{f} \perp g \Rightarrow$$

$f - \hat{f}$ must be uncorrelation with g.

$$f \longrightarrow \oplus \longrightarrow g \longrightarrow \boxed{h} \longrightarrow \hat{f}$$

w

Goal: Design h so that $f - \hat{f} \perp g$.

$$E\left[\left(f(n_1,n_2) - \hat{f}(n_1,n_2)\right) \cdot g(m_1,m_2)\right] = 0$$

$$\forall (n_1,n_2),\ (m_1,m_2).$$

$$\Longrightarrow E\left[f(n_1,n_2)\, g(m_1,m_2)\right] = E\left[\hat{f}(n_1,n_2)\, g(m_1,m_2)\right]$$

find h.

$$g * h = \hat{f}$$

$$E\left[f(n_1,n_2)\,g(m_1,m_2)\right] =$$

$$E\left[\left(\sum_{k_1}\sum_{k_2} h(k_1,k_2)\,g(n_1-k_1,\, n_2-k_2)\right) g(m_1,m_2)\right]$$

$\Longrightarrow$

cross correlation $= R$.

cross correlation f,g.

$$R_{fg}(n_1-m_1,\, n_2-m_2) =$$

$$\sum_{k_1}\sum_{k_2} h(k_1,k_2)\, R_g(n_1-k_1-m_1,\, n_2-k_2-m_2)$$

auto correlation of g with itself.

chg of variable

$$\Longrightarrow \quad R_{fg}(n_1,n_2) = \sum_{k_1}\sum_{k_2} h(k_1,k_2)\, R_g(n_1-k_1,\, n_2-k_2)$$

$$R_{fg}(n_1,n_2) = h(n_1,n_2) * R_g(n_1,n_2)$$

$\downarrow$ F.T.

$$P_{fg}(\omega_1,\omega_2) = H(\omega_1,\omega_2)\, P_g(\omega_1,\omega_2)$$

Weiner filter

$$\boxed{H(\omega_1,\omega_2) = \frac{P_{fg}(\omega_1,\omega_2)}{P_g(\omega_1,\omega_2)}}$$

$$R_{fg}(n_1,n_2) \triangleq E\left[f(k_1,k_2)\quad g(k_1-n_1,k_2-n_2)\right]$$

$$g = f + w.$$

$$R_{fg}(n_1,n_2) = E\left[f(k_1,k_2)\left(f(k_1-n_1,k_2-n_2) + w(k_1-n_1,k_2-n_2)\right)\right]$$

$$R_{fg}(n_1, n_2) = E\left[f(k_1, k_2)\, f(k_1 - n_1, k_2 - n_2)\right] + \underbrace{E\left[f(k_1, k_2)\, w(k_1 - n_1, k_2 - n_2)\right]}_{0}$$

f, w are indep.

$$R_{fg}(n_1, n_2) = R_f(n_1, n_2)$$

$$R_f(n_1, n_2) = h(n_1, n_2) * R_g(n_1, n_2)$$

$\downarrow$ F.T.

Weiner $\rightarrow$ $$\boxed{H(\omega_1, \omega_2) = \frac{P_f(\omega_1, \omega_2)}{P_g(\omega_1, \omega_2)}}$$

$$R_g(n_1,n_2) = E\left[g(k_1,k_2)\,g(k_1-n_1,k_2-n_2)\right]$$

$$= E\left[\left(f(k_1,k_2)+w(k_1,k_2)\right)\left(f(k_1-n_1,k_2-n_2)+w(k_1-n_1,k_2-n_2)\right)\right]$$

$$= E\left[f(k_1,k_2)\,f(k_1-n_1,k_2-n_2)\right] + \underbrace{E\left[f(k_1,k_2)\,w(k_1-n_1,k_2-n_2)\right]}_{0} + \underbrace{E\left[w(k_1,k_2)\,f(k_1-n_1,k_2-n_2)\right]}_{0} + E\left[w(k_1,k_2)\,w(k_1-n_1,k_2-n_2)\right]$$

f, w indeped.

$$R_g = R_f(n_1,n_2) + R_w(n_1,n_2)$$

$\Big\downarrow$ F.T.

$$P_g(\omega_1,\omega_2) = P_f(\omega_1,\omega_2) + P_w(\omega_1,\omega_2)$$

$$H(\omega_1, \omega_2) = \frac{P_f(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2) + P_W(\omega_1, \omega_2)}$$

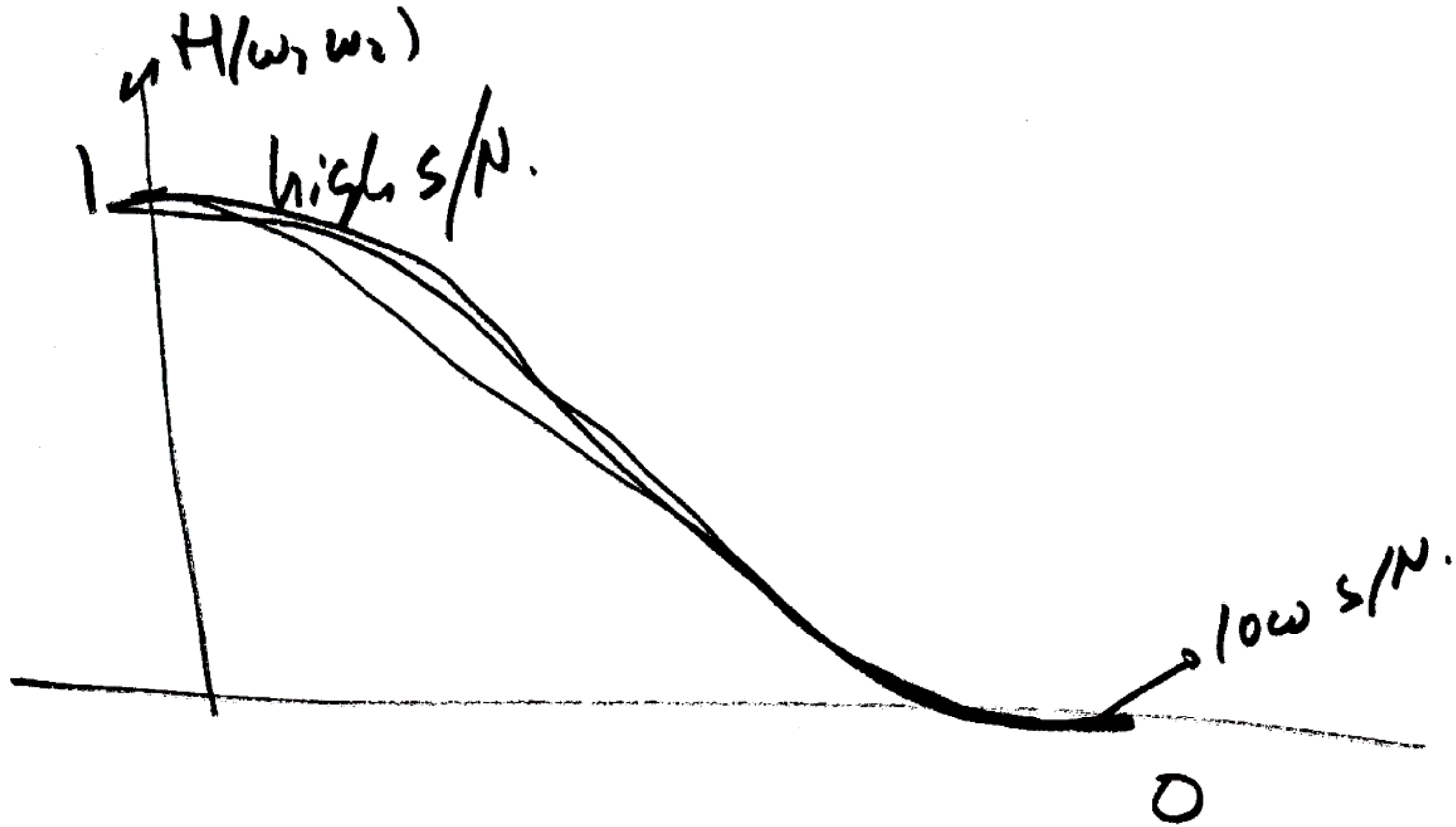
Weiner filter.

most natural image.

Consider 2 cases:

1. $P_f \gg P_w \Rightarrow$
   denominator $\simeq P_f \Rightarrow H(\omega_1, \omega_2) \simeq 1$
   $\Longrightarrow$ signal gets thru.

2. $P_f \ll P_w \Longrightarrow$
   $H(\omega_1, \omega_2) \simeq \dfrac{P_f}{P_w} \simeq 0$
   $\Longrightarrow$ Nothing gets thru.


$H(\omega_1, \omega_2)$
1
high S/N.
low S/N.
0

<u>Problem</u> How to find P_f, P_w?

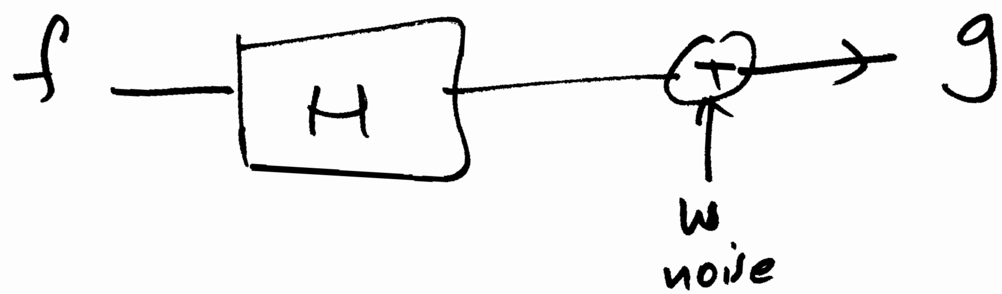
1. f is just a sample of R.P.

   Average $|F_i(\omega_1, \omega_2)|$ over a lot of natural imagery.

2. Assume model P_f, estimate parameter of P_f by observing g.

$\Rightarrow$ Another problem: Images are not really globally stationary, locally stationary.

For The Case:



$$H_{weiner}(\omega_1, \omega_2) = \frac{H^*(\omega_1, \omega_2)}{|H(\omega_1, \omega_2)|^2 + \dfrac{P_w(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2)}}$$

– obs 3 : For (ω_1, ω_2)

$P_w(\omega_1, \omega_2) \ll P_f(\omega_1, \omega_2)$

(noise is much smaller than signal)

Then weine filter approaches Inverse filter.

For (ω_1, ω_2) $P_w(\omega_1, \omega_2) \gg P_f(\omega_1, \omega_2)$

$H_{weine} \to 0$ frequency rejection filter

How to estimate P_f

① $P_f = P_g - P_w$

Model W as a white noise with variance σ_w^2

$$P_f(\omega_1, \omega_2) = P_g(\omega_1, \omega_2) - \sigma_w^2$$

$$= \frac{1}{N_1 N_2} G(\omega_1, \omega_2) G^*(\omega_1, \omega_2) - \sigma_w^2$$

Periodogram

(2) Set of representative image

(3) model Based: 2D Causal autoregressive model.

$$f(n_1, n_2) = a_{01} f(n_1, n_2 - 1) + a_{11} f(n_1 - 1, n_2 - 1) + a_{10} f(n_1 - 1, n_2) + v(n_1, n_2)$$

white noise with some sort of variance.

Cameraman

$a_{01} = .709$

$a_{11} = -0.467$

$a_{10} = .739$

$\sigma_v^2 = 231$

Fig 6 in Biemond/Langendijk paper.

Show figure 5.29 of G&W

$$P_f(\omega_1, \omega_2) = \frac{\sigma_v^2}{\left|1 - a_{0,1} e^{-j\omega_1} - a_{1,1} e^{-j\omega_1 - j\omega_2} - a_{1,0} e^{-j\omega_2}\right|}$$