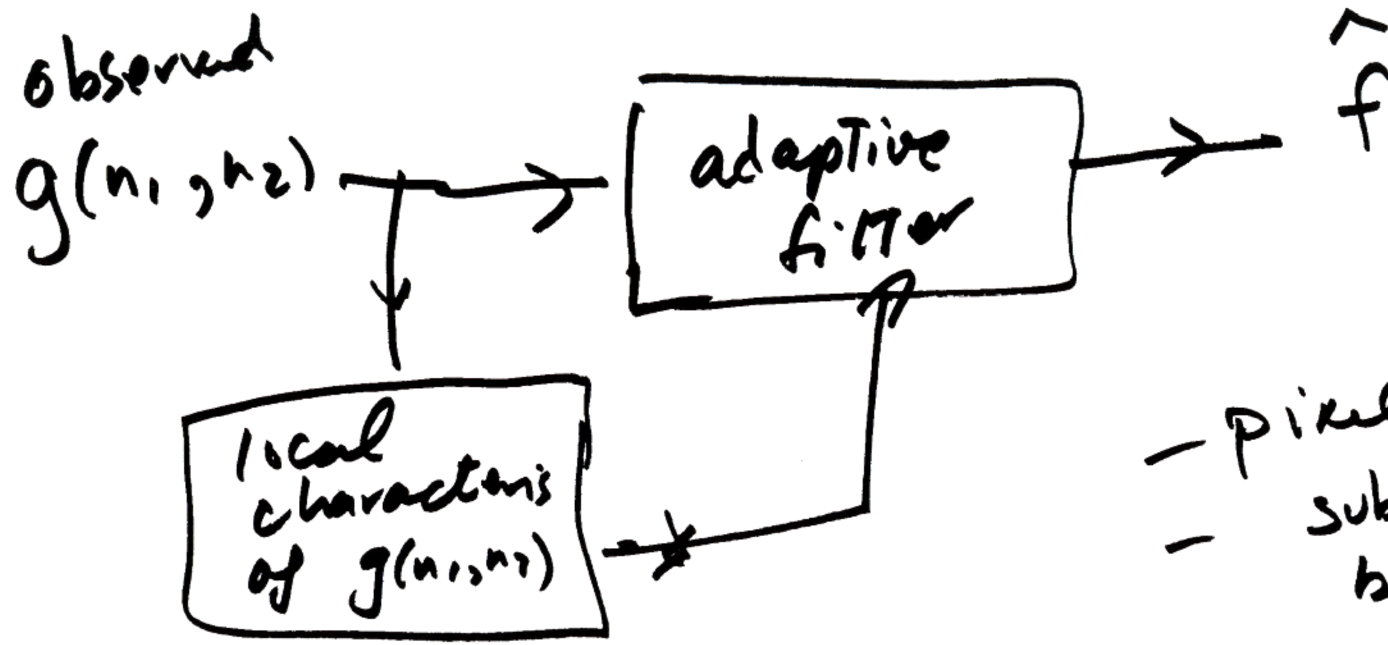
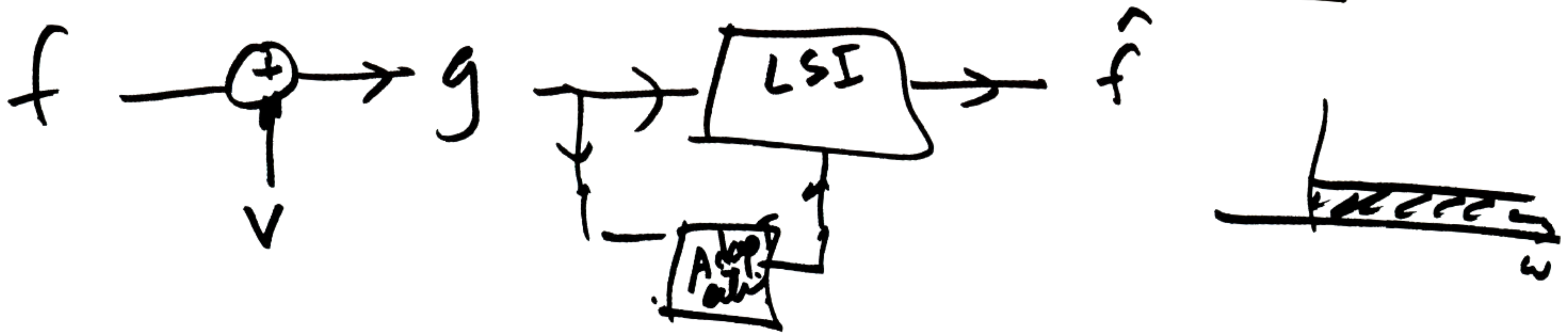
# Adaptive Weiner Filter :

**Basic idea** : no image stationary.




- pixel by pixel
- subimage by subimage
- block by block.

# One Possible Adaptive filter




- Assume, noise $v$ is white, zero mean, variance $\sigma_v^2$

$$P_v(\omega_1,\omega_2) = \sigma_v^2$$

- Assume signal satisfies the following model:

$$f(n_1,n_2) = m_f + \sigma_f\, w(n_1,n_2)$$

  - $w$ is a white noise, zero mean, unit variance process.

$\Rightarrow$ Weiner filter assuming zero mean input.

$$H(\omega_1,\omega_2) = \frac{P_f(\omega_1,\omega_2)}{P_f(\omega_1,\omega_2) + P_v(\omega_1,\omega_2)}$$

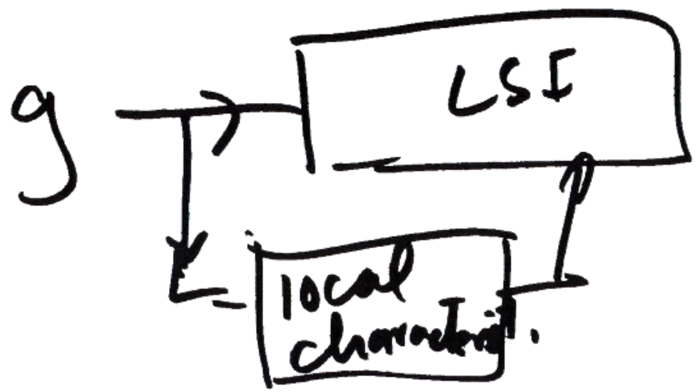
$$H(\omega_1,\omega_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2}$$

$$\Longrightarrow h(n_1,n_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \, \delta(n_1,n_2)$$

"Taking care of The mean"

$$\hat{f}(n_1,n_2) = m_f + \left(g(n_1,n_2) - m_f\right) * h(n_1,n_2)$$

$$\hat{f}(n_1,n_2) = m_f + \left(g(n_1,n_2) - m_f\right) * \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \delta(n_1,n_2)$$




$\left.\begin{matrix} m_f \\ \sigma_f^2 \end{matrix}\right\}$ are both fns of $(n_1,n_2)$

$$\ddot{f}(n_1,n_2) = m_f(n_1,n_2) + \left(g(n_1,n_2) - m_f(n_1,n_2)\right) \frac{\sigma_f^2(n_1,n_2)}{\sigma_f^2(n_1,n_2)+\sigma_v^2}$$

$$\boxed{\hat{f}(n_1,n_2) = m_f(n_1,n_2) + \left[g(n_1,n_2) - m_f(n_1,n_2)\right) \frac{\sigma_f^2(n_1,n_2)}{\sigma_v^2 + \sigma_f^2(n_1,n_2)}} \quad (*)$$

2 cases: (1) $\sigma_f^2 \ll \sigma_v^2 \Longrightarrow \hat{f} \simeq \underline{m_f(n_1,n_2)}$

flat parts of image. for $(n_1,n_2)$ on regions where image is "flat"

$\Longrightarrow$ output is just mean.

(2) $\sigma_f^2 \gg \sigma_v^2 \Longrightarrow \hat{f} \simeq g(n_1,n_2)$

Textured part of image

# How to estimate $m_f$?

$$f \longrightarrow \oplus \longrightarrow g$$

$$\uparrow$$

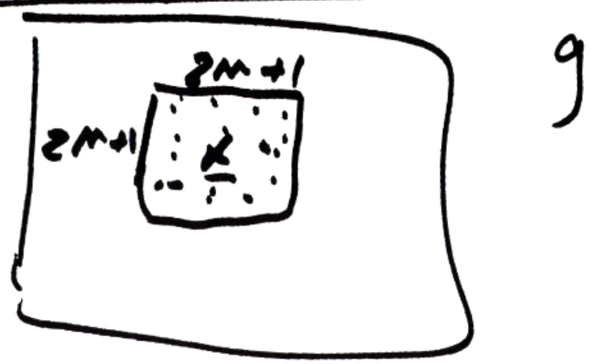
$$v$$

$$m_g = m_v + m_f \qquad \cdot\, m_v = 0$$

$$\Longrightarrow \hat{m}_f = \bar{m}_g = \cancel{\frac{1}{(2M+1)}}$$

$$\hat{m}_f(n_1, n_2) = m_g = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} g(k_1, k_2)$$

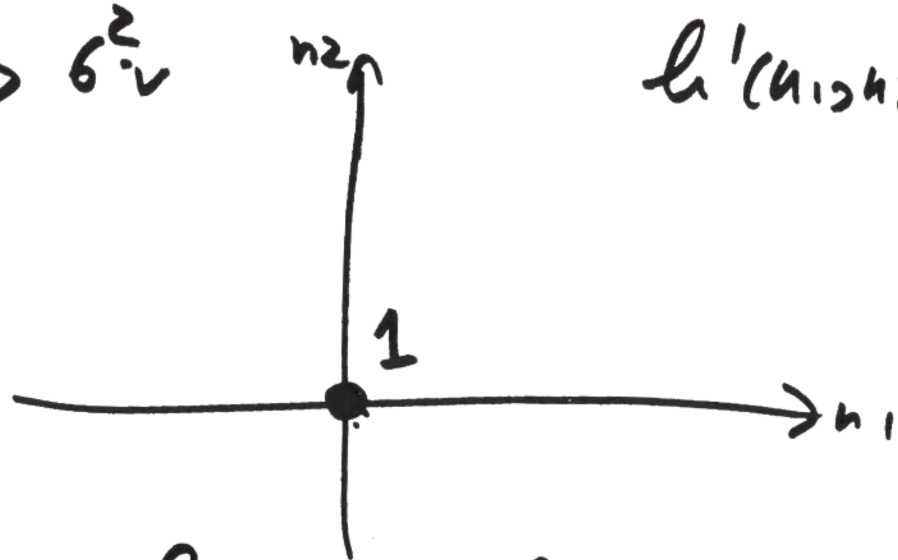
plug this into ⓚ



$$\hat{f} = g * h'$$

$$h'(n_1, n_2) = \begin{cases} \dfrac{\sigma_f^2 + \dfrac{\sigma_v^2}{(2M+1)^2}}{\sigma_f^2 + \sigma_v^2} & n_1 = n_2 = 0 \\[2ex] \dfrac{\dfrac{\sigma_v^2}{(2M+1)^2}}{\sigma_f^2 + \sigma_v^2} & |n_1| \leq M,\ |n_2| \leq M \text{ except } n_1 = n_2 = 0 \\[2ex] 0 & \text{otherwise} \end{cases}$$

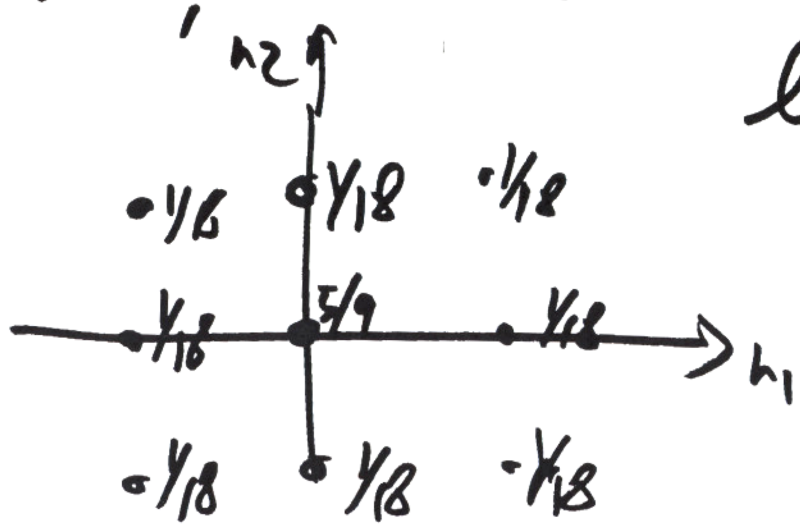
3 cases ① $\sigma_f^2 \gg \sigma_v^2$ $\Longrightarrow$




$h'(n_1, n_2)$

If energy/variance of f is much larger than noise, let the signal through.

② $\sigma_f^2 \simeq \sigma_v^2$, $M=1 \Longrightarrow$

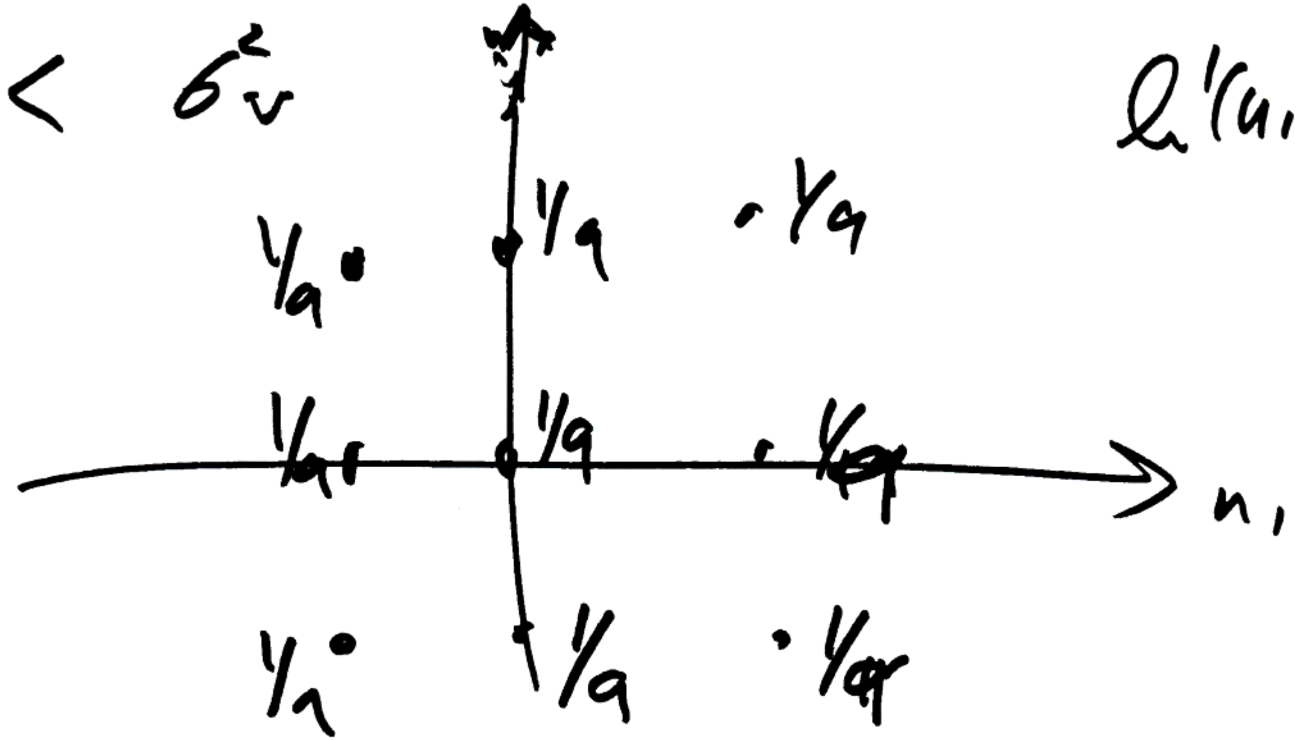


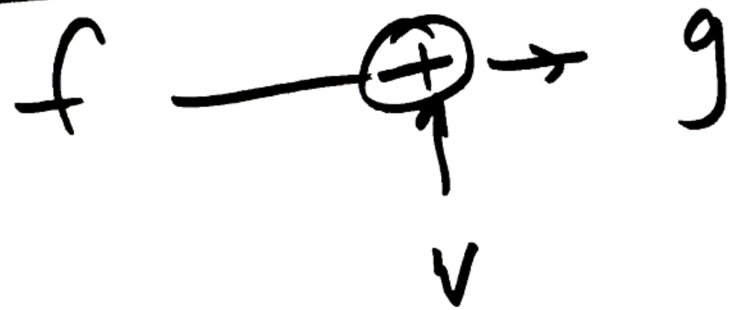

$h'$

③ $\sigma_f^2 \ll \sigma_v^2$

$M=1$

$h^1(n_1,n_2)$




f, v are independent

$$g = v + f \implies \sigma_g^2 = \sigma_v^2 + \sigma_f^2$$

$$\implies \cancel{\hat{\sigma}_f^2 = \sigma_g^2} - \sigma_v^2 \quad ??$$

$$\hat{\sigma}_f^2 = \begin{cases} \sigma_g^2 - \sigma_v^2 & \text{only if } \sigma_g^2 > \sigma_v^2 \\ 0 & \text{otherwise.} \end{cases}$$

$$\hat{\sigma}_g^2$$

$$\hat{\sigma}_g^2(n_1, n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} \left[ g(k_1, k_2) - \hat{m}_f(n_1, n_2) \right]^2$$

show figure 9.11 of J. Lim

Alternative To Wiener

$$f \rightarrow \boxed{d} \rightarrow \oplus \rightarrow g \rightarrow \boxed{???} \rightarrow \hat{f}$$

$$\uparrow$$

$$w$$

$$(*) \quad \| d * \hat{f} - g \|^2 \lesssim \sigma_w^2 \longleftarrow \text{ideally}$$

__Approach__ find a "smooth" signal such that (*) is satisfied.

Define $c(n_1, n_2)$ as high pass filter.

minimize $\| c * \hat{f} \|^2$

subject to (*)

$\longrightarrow$ Constrained Least Squares.

$$H_{res}(\omega_1,\omega_2) = \frac{D^*(\omega_1,\omega_2)}{\left(D^*(\omega_1,\omega_2)\,D(\omega_1,\omega_2)\right) + \alpha\, C^*(\omega_1,\omega_2)\,C(\omega_1,\omega_2)}$$

$\alpha$ = ideally ~~B~~ ~~so~~ chosen to satisfy *

→ regularization parameter.

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Example of $c$ → 2D laplacian operator

$$\begin{array}{ccc} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{array}$$

Fig 8 → Bicubic

Fig 5.30 G/W.

# Iterative Filters

motivation.

1. Actively control Trade off between ringing + blurring.
2. Can incorporate a priori constraint about your signal.

$$\hat{f}_{i+1}(n_1,n_2) \leftarrow \hat{f}_i(n_1,n_2) + \beta \left[ g(n_1,n_2) - d * f_i \right]$$

Show converge if $|1-\beta D(w_1,w_2)| < 1$

$\forall (w_1,w_2)$

Assuming $|D(\omega_1, \omega_2)| \leq 1$

Then convergence

$0 < \beta < 2$ provided $D(\omega_1, \omega_2) > 0$

$$\lim_{i \to \infty} \hat{f}_i = h_{inv} * g$$

Fig 9 Lagrel. / Biemond.

If D(u,v) does become zero, then use:

$$\hat{f}_{i+1}(n_1, n_2) = \left(\delta(n_1, n_2) - \alpha\beta c(-n_1, -n_2) * c(n_1, n_2)\right) * \hat{f}_i\,(n_1, n_2) +$$
$$+ \beta d(-n_1, -n_2) * \left(g(n_1, n_2) - d(n_1, n_2) * \hat{f}_i\,(n_1, n_2)\right)$$

Apriori knowledge: We know image is positive. $\longrightarrow$ Apply a projection operator:

$$P\left[\hat{f}(n_1,n_2)\right] = \begin{cases} \hat{f}(n_1,n_2) & \text{if } \hat{f}(n_1,n_2) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{f}_{i+1}(n_1,n_2) \leftarrow P\left[\hat{f}_i + \beta(g - d * f_i)\right]$$

$$\hat{f}_{i+1} \leftarrow P_1 P_2 P_3 \left[ \cdots \right]$$

**POCS** = Projection Onto Convex Sets.

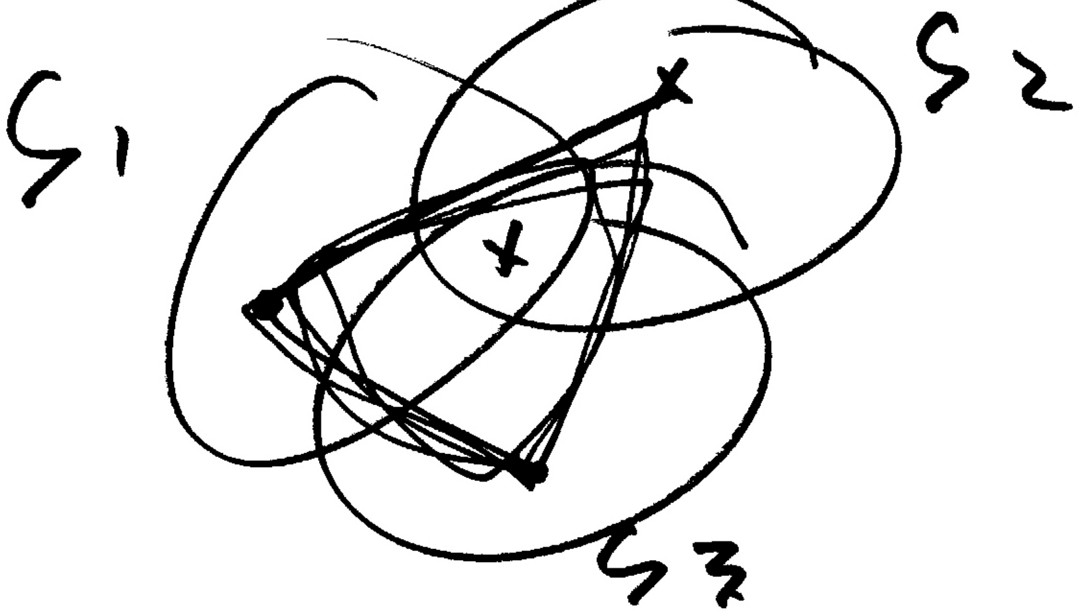
Convex Set : $S$

$a \in S \qquad b \in S$.

$\forall \epsilon \qquad a\epsilon + (1-\epsilon)b$ is also $\in S$

**Example** : set of positive image is convex.

$S_1, S_2, S_3 \longrightarrow$ convex




Want to find <u>any</u> point in the intersection

Start at an arbitrary ~~and~~ point, keep projecting $\longrightarrow$ eventually converge to the intersection.