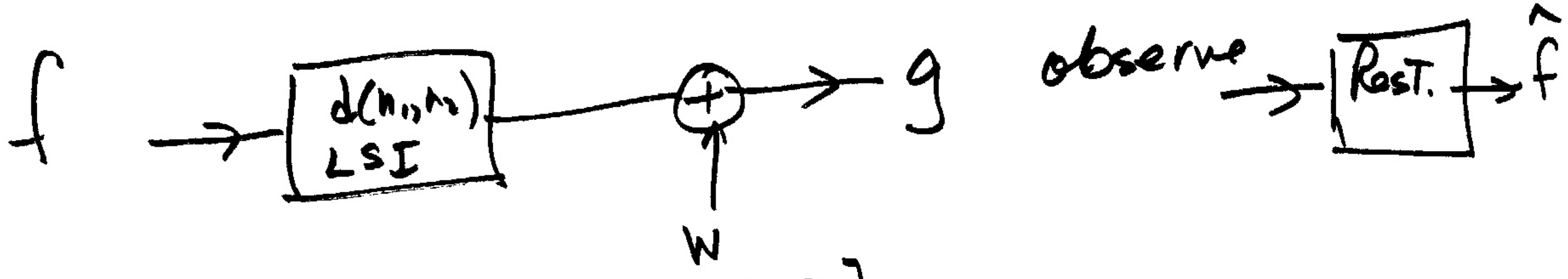
Image Restoration



Minimize $E[(f-\hat{f})^2]$.

3 methods:

1. Inverse filters.
2. Least Squares filters
   - Weiner
   - Constraint least square.
3. Iterative Technique.

Key Assumption : We know d.

↙ bluring fn.

How about if we don't know d.

⟹ Blind Deconvolution.

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2 Step process : ① Esitimate d

② use "classical" Restoration.

How To do this?

① Estimate parameters of The blur function

Knowing what kind of Blur

→ Atmospheric

→ out of focus

→ motion.

Approach: look at F.T of g.

Zeros of $G(\omega_1, \omega_2)$ → Deduce the parameters of blur fn.

→ R in out of focus

→ L, ϕ → motion Blur

Show figure 11 of Lagendijk paper

② Maximum Likelihood Blur Estimation.

ML → used for parameter estimation.

Given observation, find parameter of the model for that observation that maximizes the likelihood.

~~Blind Deconvolution~~

Parameter in our case: $\theta = \{\sigma_w^2, d(n_1,n_2), \sigma_v^2, a_{ij}\}$

a_{ij}, σ_v^2 relate to the autoregressive model for f:

$$f(n_1,n_2) = \underline{a_{01}}\, f(n_1, n_2-1) + \underline{a_{11}}\, f(n_1-1, n_2-1) + \underline{a_{10}}\, f(n_1-1, n_2) + v(n_1, n_2)$$

$v(n_1,n_2) \rightarrow \sigma_v^2$ ⇝ Gauss.

σ_w^2 = variance of the added noise w ⇝ Gauss.

Maximize log likelihood function:

$$\text{㊀}\quad L(\theta) = -\sum_{\omega_1}\sum_{\omega_2}\left(\log P(\omega_1,\omega_2) + \frac{|G(\omega_1,\omega_2)|^2}{P(\omega_1,\omega_2)}\right)$$

~~P~~ where $P(\omega_1,\omega_2) = \sigma_v^2 \dfrac{|D(\omega_1,\omega_2)|^2}{|1 - A(\omega_1,\omega_2)|^2} + \sigma_w^2$

$A(\omega_1,\omega_2) \longrightarrow$ 2D DTFT of a_{ij}

Issues associated with solving ©

① Must apply regularization Techniques to make it "well conditioned".

Add constraints.

(a) Energy conservation.

$$\sum\sum d(n_1, n_2) = 1$$

i.e D.C value $D(\omega_1, \omega_2)$ is 1

$$[D(\omega_1, \omega_2)]_{(\omega_1, \omega_2) = (0,0)} = 1$$

⇒ Blurring was passive process

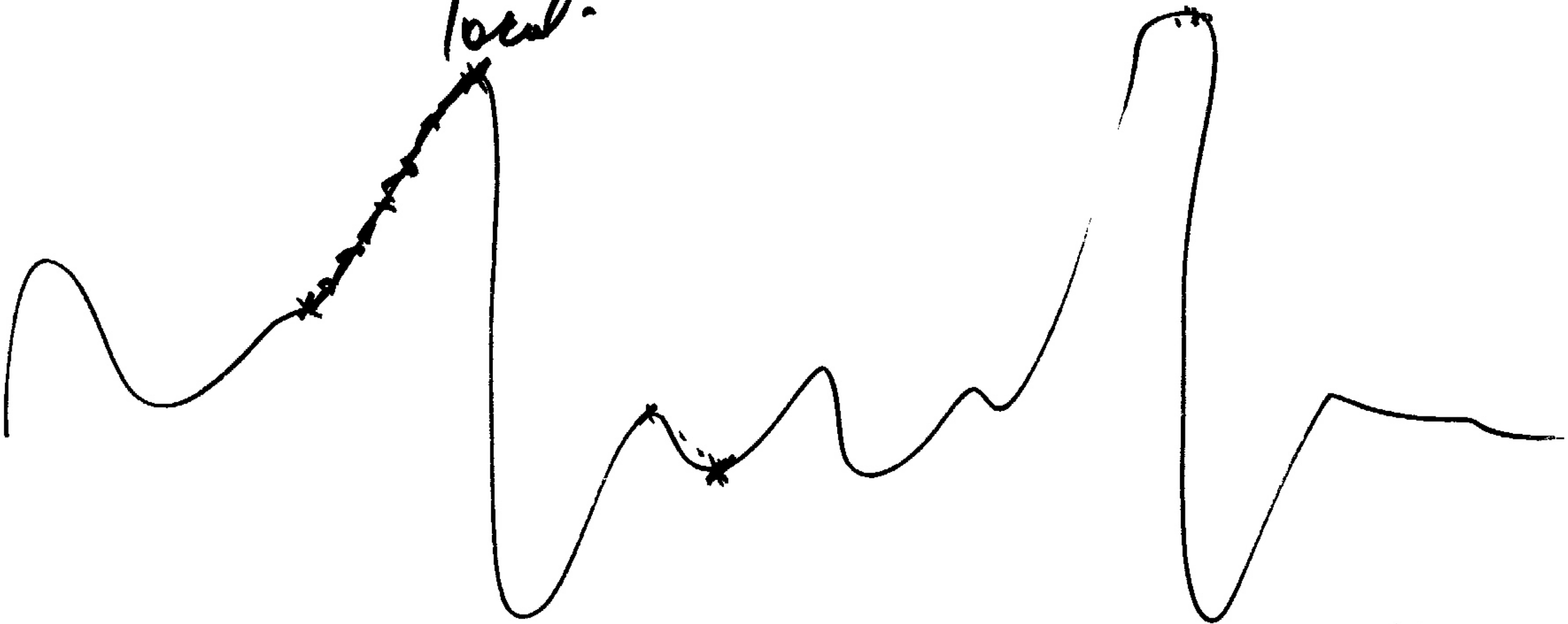
no energy was either generated nor absorbed.

(b) $d(n_1, n_2) = d(-n_1, -n_2)$, i.e. blur p.s.f is symmetric.

② optimization of a nonlinear fn.

→ steepest descent ⇒ local minima.

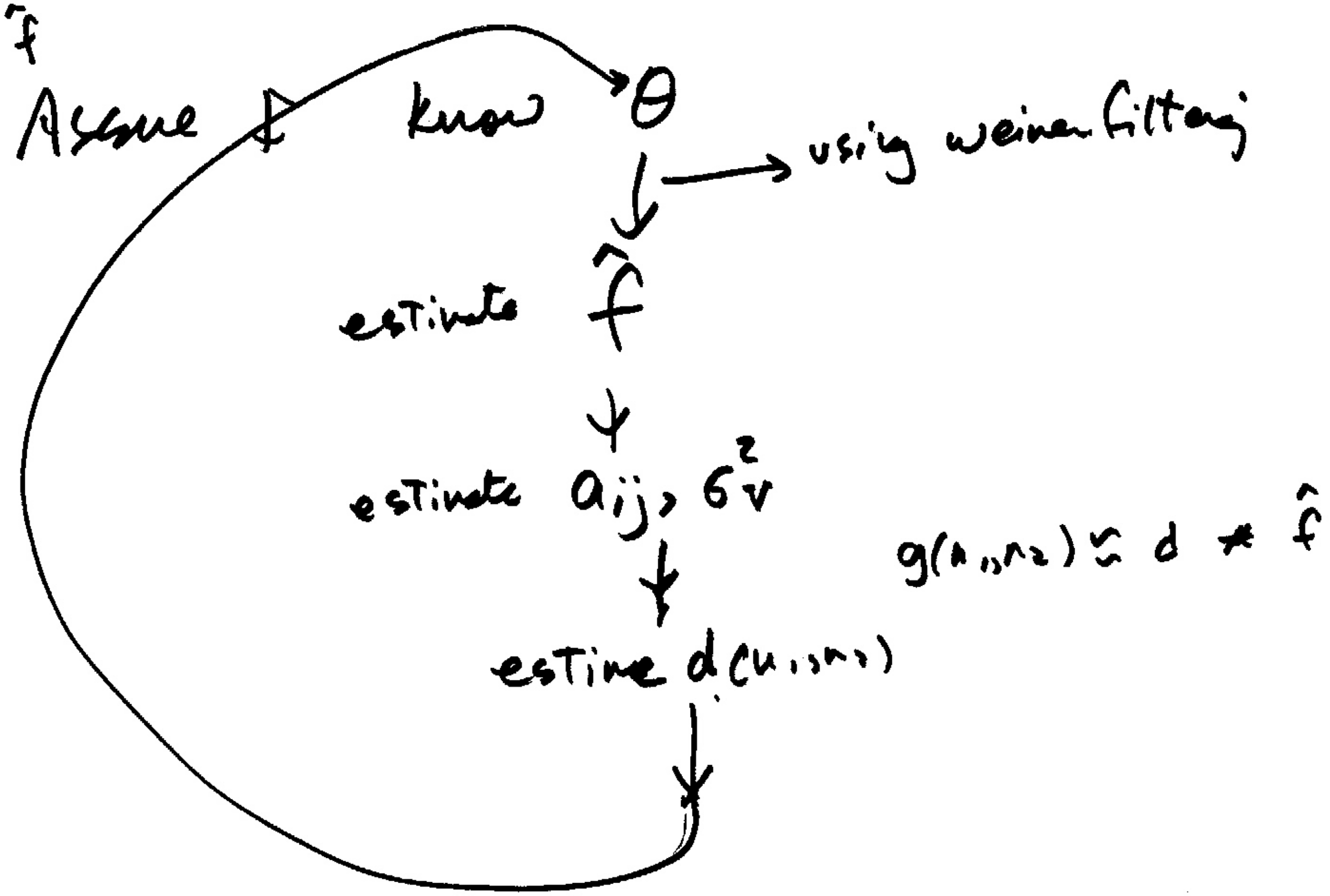
⇒ initial condition

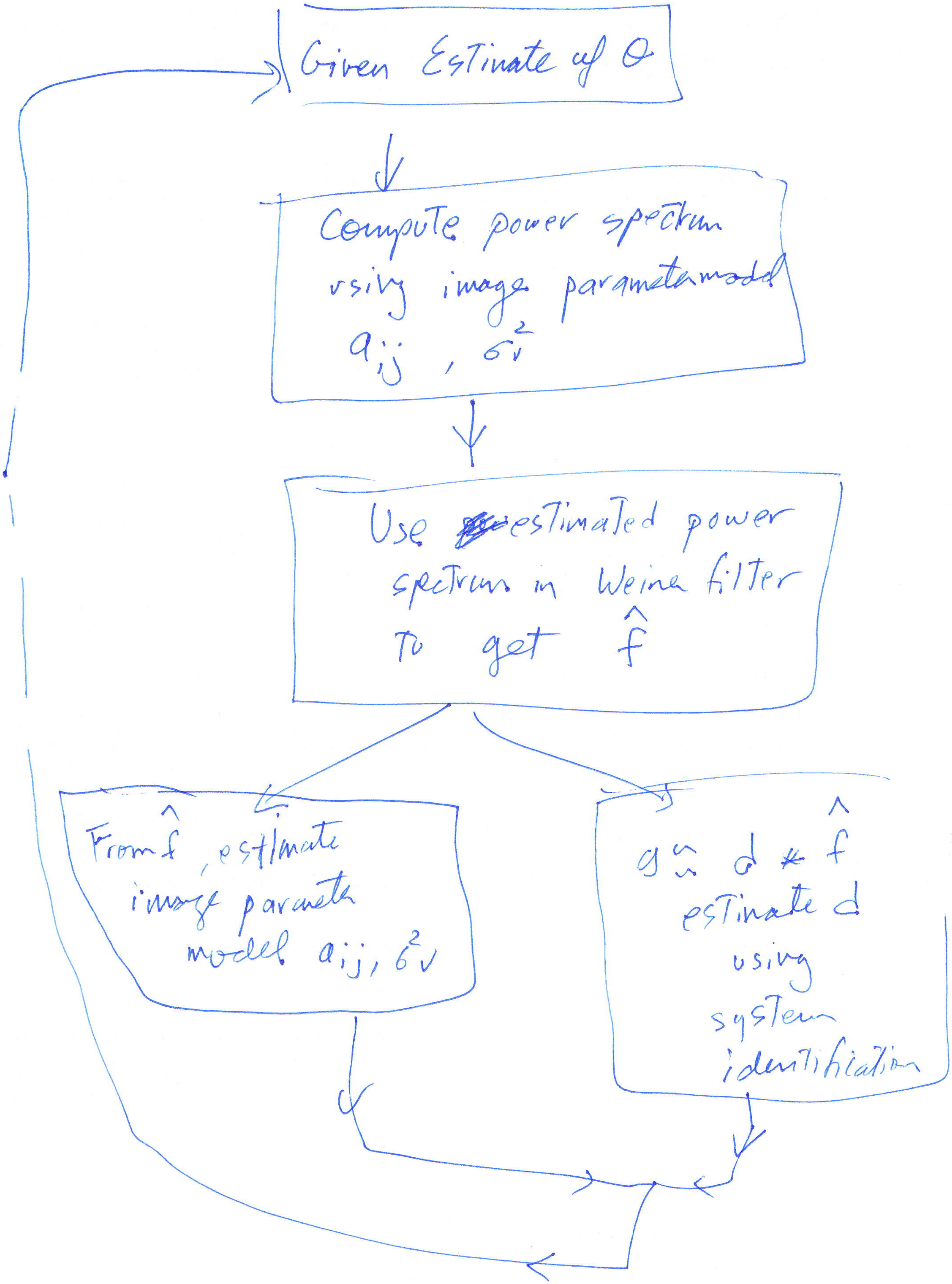


One. gradient descent technique → EM

EM = Expectation Maximization.

G given θ, $\hat{f}$




Given Estimate of θ
Compute power spectrum using image parametermodel a_{ij} , σ_v^2
Use estimated power spectrum in Weiner filter To get $\hat{f}$
From $\hat{f}$, estimate image paramete model a_{ij}, σ_v^2
$g \approx d * \hat{f}$ estimate d using system identification

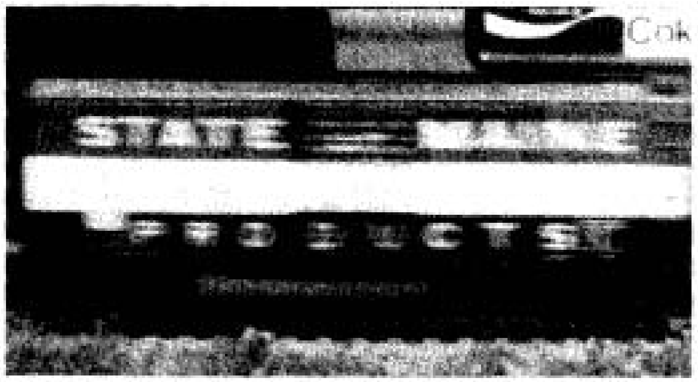
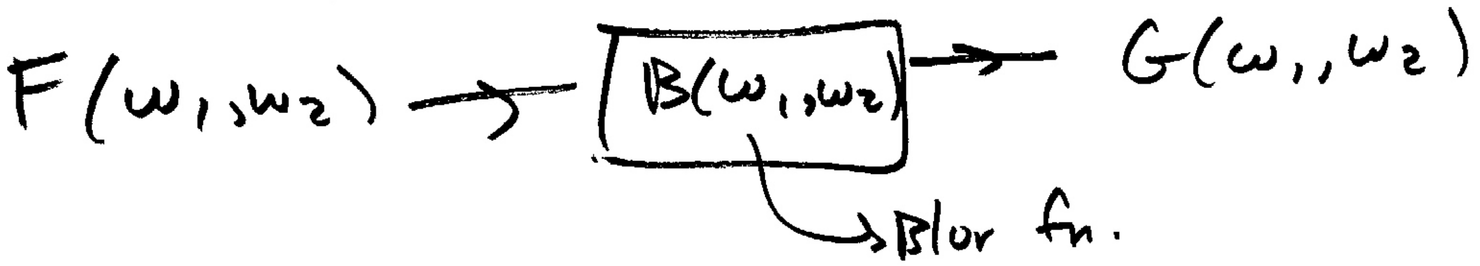


Fig. 7. Blur introduced by real motion.

Fig. 8. Blurred section of Fig. 7, and its restoration results using identified parameters.

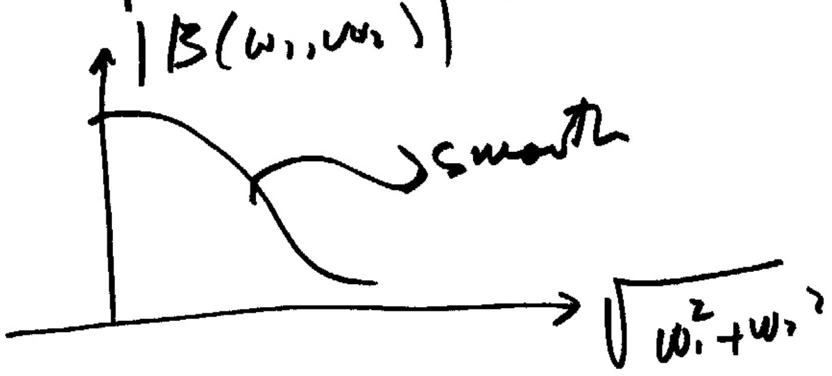
Blind deconvolution



Assumption $|B(\omega_1,\omega_2)|$ is smooth.

$$G(\omega_1,\omega_2) = F(\omega_1,\omega_2)\, B(\omega_1,\omega_2)$$

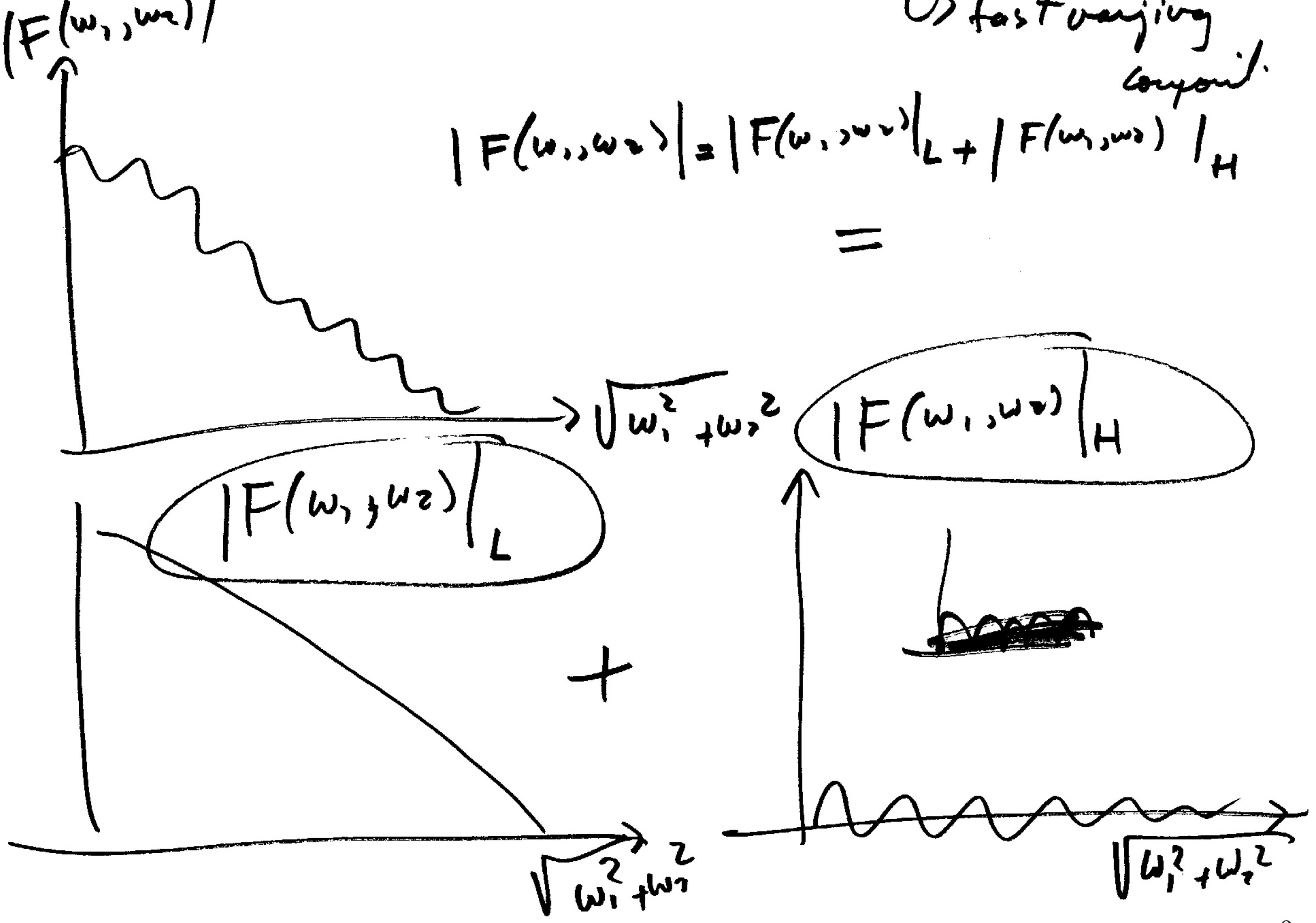
$$|G(\quad)| = |F(\quad)|\,|B(\quad)|$$

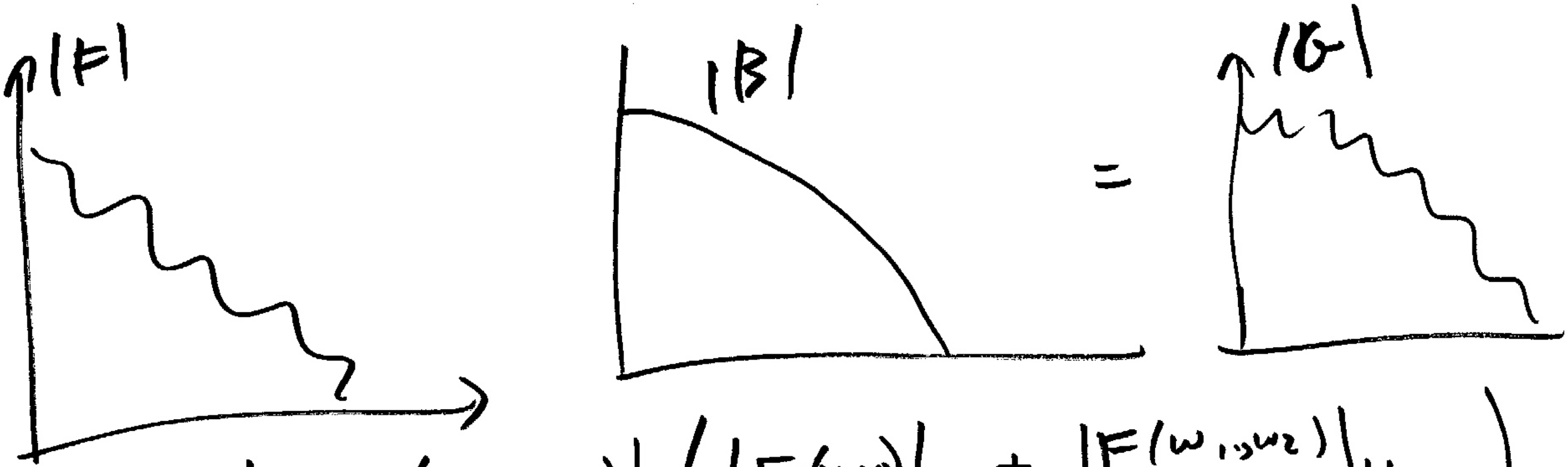


Decompose. $|F(\omega_1,\omega_2)|$ into → slowly varying.

→ fast varying component.

$$|F(\omega_1,\omega_2)| = |F(\omega_1,\omega_2)|_L + |F(\omega_1,\omega_2)|_H$$



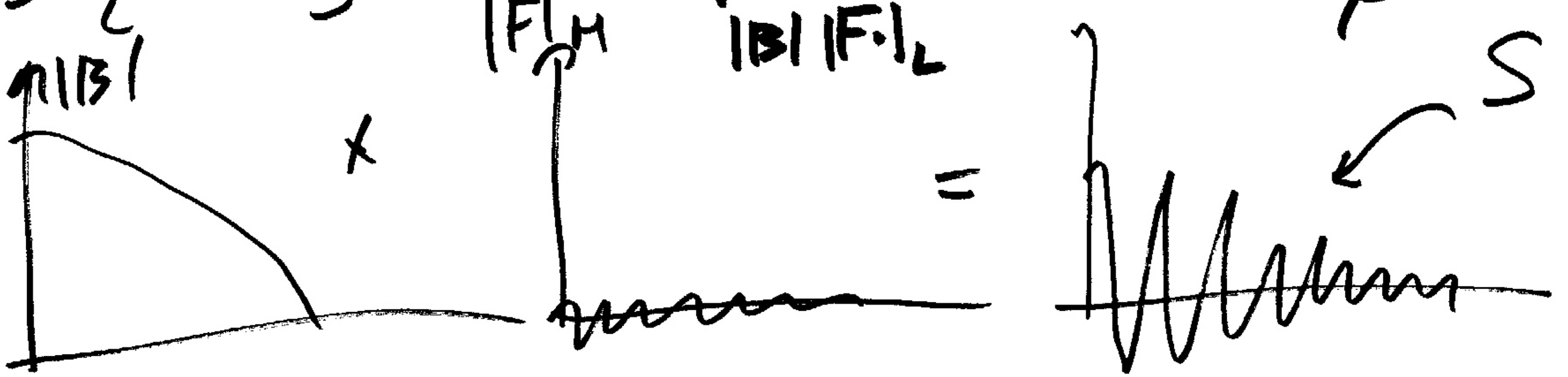


$$|G(\omega_1,\omega_2)| = |B(\omega_1,\omega_2)|\left(|F(\omega_1,\omega_2)|_L + |F(\omega_1,\omega_2)|_H\right)$$

$$|G| = |B|\,|F|_L + |B|\,|F|_H$$

Apply "Smoothing Operator" to Both sides → S

$$S\{|G|\} = \underbrace{S\{|B|\,|F|_L\}}_{|B|\,|F\cdot|_L} + \underbrace{S\{|B|\,|F|_H\}}_{\emptyset}$$



$$S\{|G|\} \simeq |B|\ |F|_L$$

$$\Longrightarrow |B| \simeq \frac{S\{|G|\}}{|F|_L}$$

Estimate $|F|_L$ by using an ensemble of images $|F'|_L$

$$|B(\omega_1, \omega_2)| \simeq \frac{S\{|G|\}}{|F'|_L}$$

How about Phase.

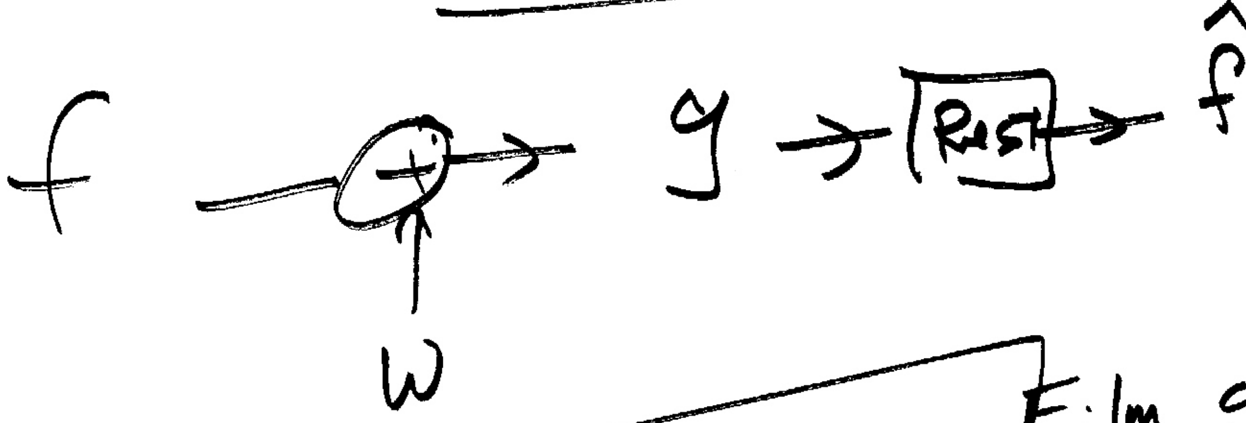
$$\angle B(\omega_1, \omega_2) \simeq 0 \longrightarrow \text{zero phase (Iter.}$$

Fig 9.21 ⟶ shows Blind deconvolution To estimate B + Iterative.)

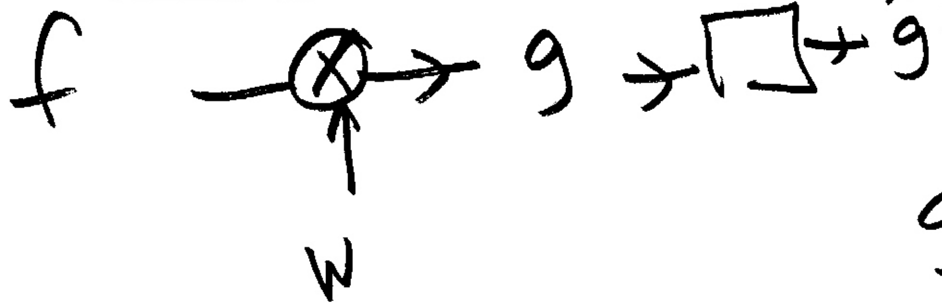
$$\hat{f}_{K+1}(n_1,n_2) = \hat{f}_K(n_1,n_2) + \lambda\Big(g(n_1,n_2) - \hat{f}_K(n_1,n_2) * b(n_1,n_2)\Big)$$

Show figure 9.21 of J. Lim

Homomorphic Processing



Film grain noise is multiplicat.



$$g \simeq f \cdot w.$$

$$\log(g) \simeq \log(f) + \log(w)$$

$$g' \simeq f' + w'$$

Apply any additive noise Restoration
Alg to restore $\hat{f}$

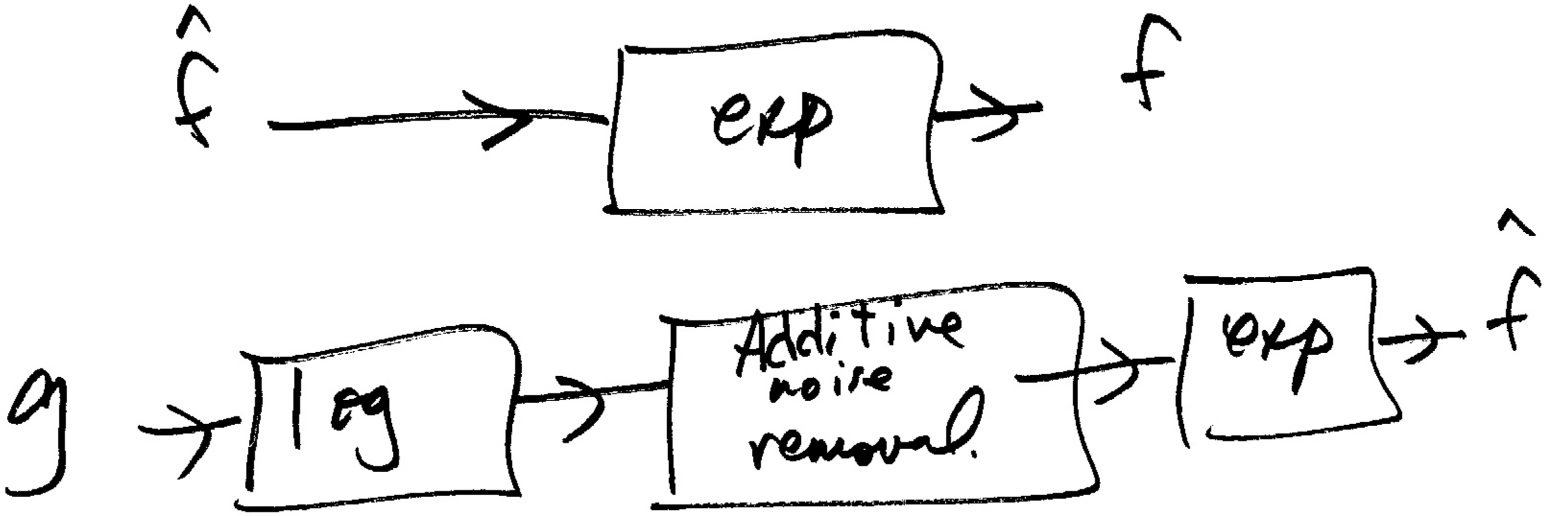
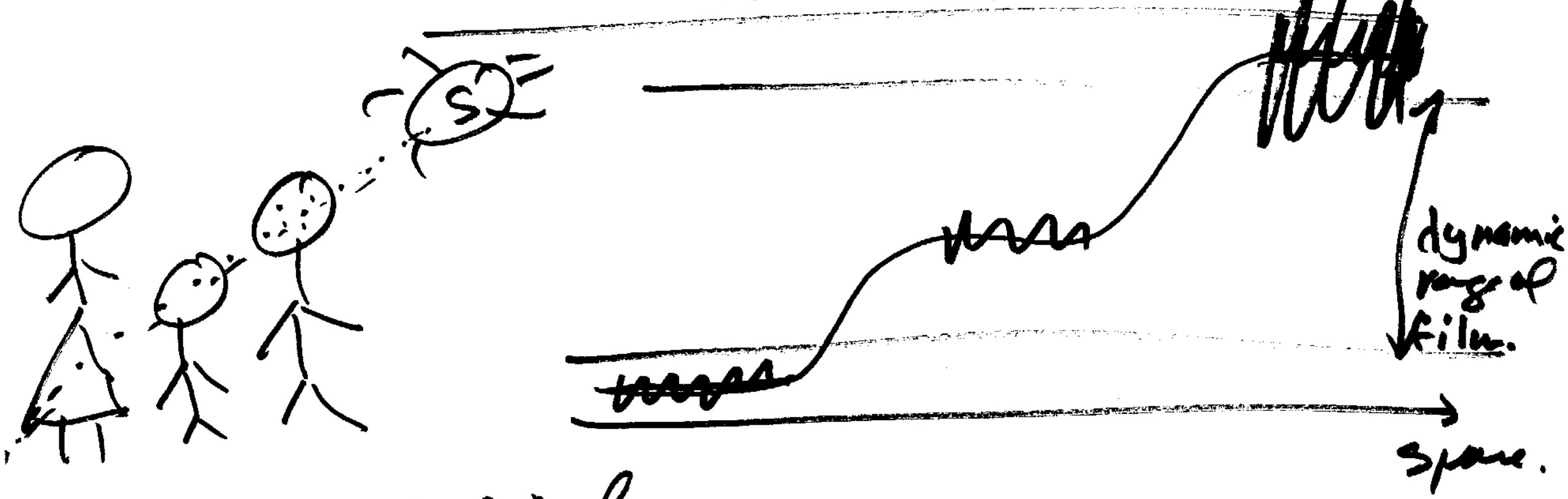


Fig 9.27 S. Lin

Another App of Homomorphic filtering

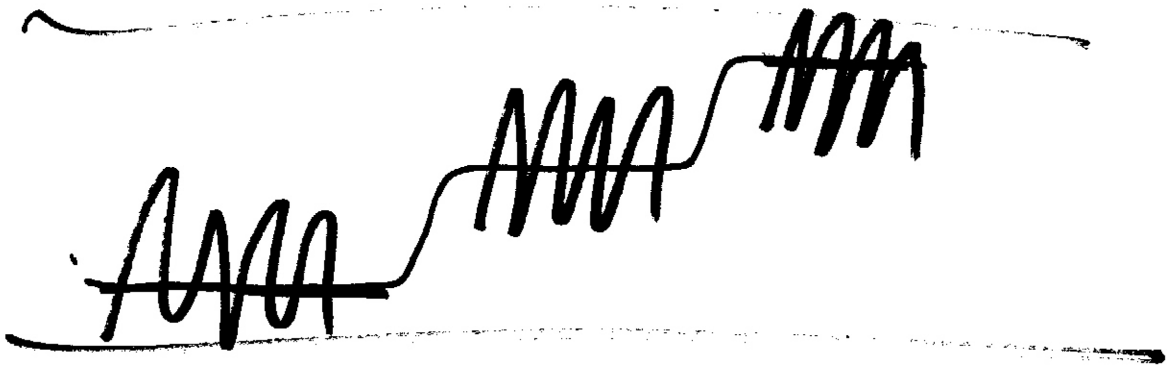


recorded signal

$$f(n_1,n_2) = r(n_1,n_2) \cdot i(n_1,n_2)$$

r → reflectance due to objects → varies fast

i → illumination due to light source. Slowly varying as a fn of space. ⟹ dynamic range local Contrast enhancement + dynamic range reduction.



$$\log f(n_1,n_2) = \underbrace{\log r(n_1,n_2)}_{\text{rapidly varying}} + \underbrace{\log i(n_1,n_2)}_{\downarrow\ \text{slowly varying}}$$

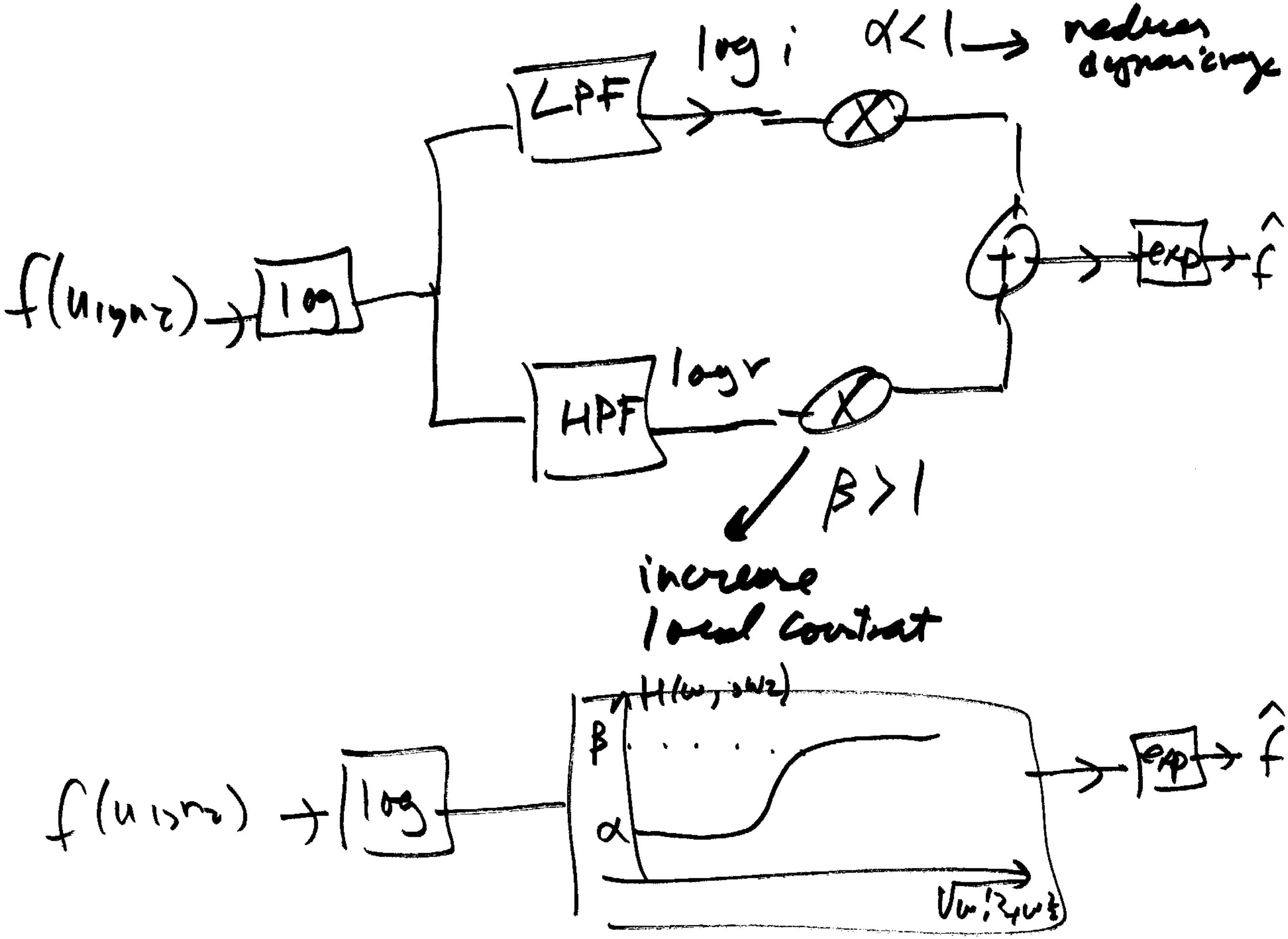

f(n1,n2)
log
LPF
HPF
log i
log r
α<1 → reduce dynamic range
β>1
increase local contrast
exp
f̂
H(ω1,ω2)
β
α
√(ω1²+ω2²)

Fig 8.11 J. Lim