DIFFERENCES BETWEEN ONE AND MULTI DIMENSIONAL SIGNAL PROCESSING

- More data for M-D signal processing.
  1. 1-D Speech → 10K samples per second.
  2. M-D Television → 500 × 500 pixels per frame, 30 frames a second, 7.5 Mega samples per second.
- Mathematics for M-D is not as complete as 1-D:
  1. 1-D systems are described by differential equations, M-D by partial differential equations.
  2. Fundamental theorem of algebra does not hold in M-D, but holds in 1-D → Factorability of polynomials in 1-D is guaranteed, but not in higer dimensions.
  3. This affects filter design, IIR filter stability, signal reconstruction, etc.
- Causality.

SEQUENCES

- Notation for 2-D sequences: $x(n_1, n_2)$.
- Unit sample sequence:

$$\delta(n_1, n_2) = \begin{matrix} 1 & n_1 = n_2 = 0 \\ 0 & 0 \end{matrix} \tag{1}$$

- Line impulse:

$$\delta(n_1) = \begin{matrix} 1 & n_1 = 0 \\ 0 & 0 \end{matrix} \tag{2}$$

- Line impulse $\delta(n_2)$ defined similarly.
- $\delta(n_1 - n_2)$ is 1 along $n_1 = n_2$.
- Step Sequence:

$$u(n_1, n_2) = u(n_1)u(n_2) = \begin{cases} 1 & n_1 \geq 0 \quad n_2 \geq 0 \\ 0 & 0 \end{cases} \tag{3}$$

- $u(n_1)$ and $u(n_2)$ and $u(n_1 - n_2)$ defined similarly.
- Exponential sequences: $a^{n_1} b^{n_2}$.

SEQUENCES (cont'd)

- **Definition:** Seperable sequences $x(n_1, n_2)$ are those that can be written as the product $x_1(n_1)x_2(n_2)$.
- Separable sequences are important because a large number of 1-D results can be applied to systems with separable response.
- Examples:

  1. $\delta(n_1, n_2) = \delta(n_1)\delta(n_2)$.
  2. $u(n_1, n_2) = u(n_1)u(n_2)$
  3. $a^{n_1}b^{n_2} + a^{n_1 + n_2} = a^{n_1}(b^{n_2} + a^{n_2})$.

- Peridic sequences:

$$x(n_1, n_2) = x(n_1 + N_1, n_2) = x(n_1, n_2 + N_2) \quad (4)$$

- Every sequence $x(n_1, n_2)$ can be expressed as:

$$x(n_1, n_2) = \sum_{k_1 = -\infty}^{+\infty} \sum_{k_2 = -\infty}^{+\infty} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2) \quad (5)$$

SYSTEMS

- Transformation of the input signal $x(n_1, n_2)$ into the output signal $y(n_1, n_2)$:

$$T[x(n_1, n_2)] = y(n_1, n_2) \qquad (6)$$

- Linearity:

$$T[ax_1(n_1, n_2) + bx_2(n_1, n_2)] =$$
$$aT[x_1(n_1, n_2)] + bT[x_2(n_1, n_2)] \qquad (7)$$

- System is Shift Invariance if

$$T[x(n_1, n_2)] = y(n_1, n_2) \qquad (8)$$

implies:

$$T[x(n_1 - k_1, n_2 - k_2)] = y(n_1 - k_1, n_2 - k_2) \quad (9)$$

- LINEAR SHIFT INVARIANT (LSI) SYSTEMS ARE OF UTMOST IMPORTANCE.

LSI SYSTEMS

- LSI systems can be uniquely specified by their *impulse response.* That is:

$$T[\delta(n_1, n_2)] = h(n_1, n_2)$$

- Knowing the impulse response enables one to determine the output uniquely for any given input.

- Input/output relationship for an LSI system with impulse response $h(n_1, n_2)$ is given by the *Convolution* sum:

$$\begin{aligned} y(n_1, n_2) &= \sum_{k_1}\sum_{k_2} x(k_1, k_2) T[\delta(n_1 - k_1, n_2 - k_2)] \\ & \sum_{k_1}\sum_{k_2} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2) \end{aligned} \tag{10}$$

- Notation for convolution:

$$y(n_1, n_2) = x(n_1, n_2) * h(n_1, n_2) \tag{11}$$

- An example of convolution.

SEPARABLE SYSTEMS

- A separable system is an LSI system whose impulse response is separable:

$$h(n_1, n_2) = h_1(n_1)h_2(n_2) \tag{12}$$

- Consider the number of multiplies involved in convolving an $N \times N$ input sequence $x(n_1, n_2)$ with an $M \times M$ impulse response of an LSI system $h(n_1, n_2)$. Assume $M << N$.,

  1. If x and h are not separable, then total number of multiplies goes as M^2N^2.
  2. If h is separable, then the number of multiplies goes as $2MN^2$.
  3. if both h and x are separable, then we need N^2 multiplies.

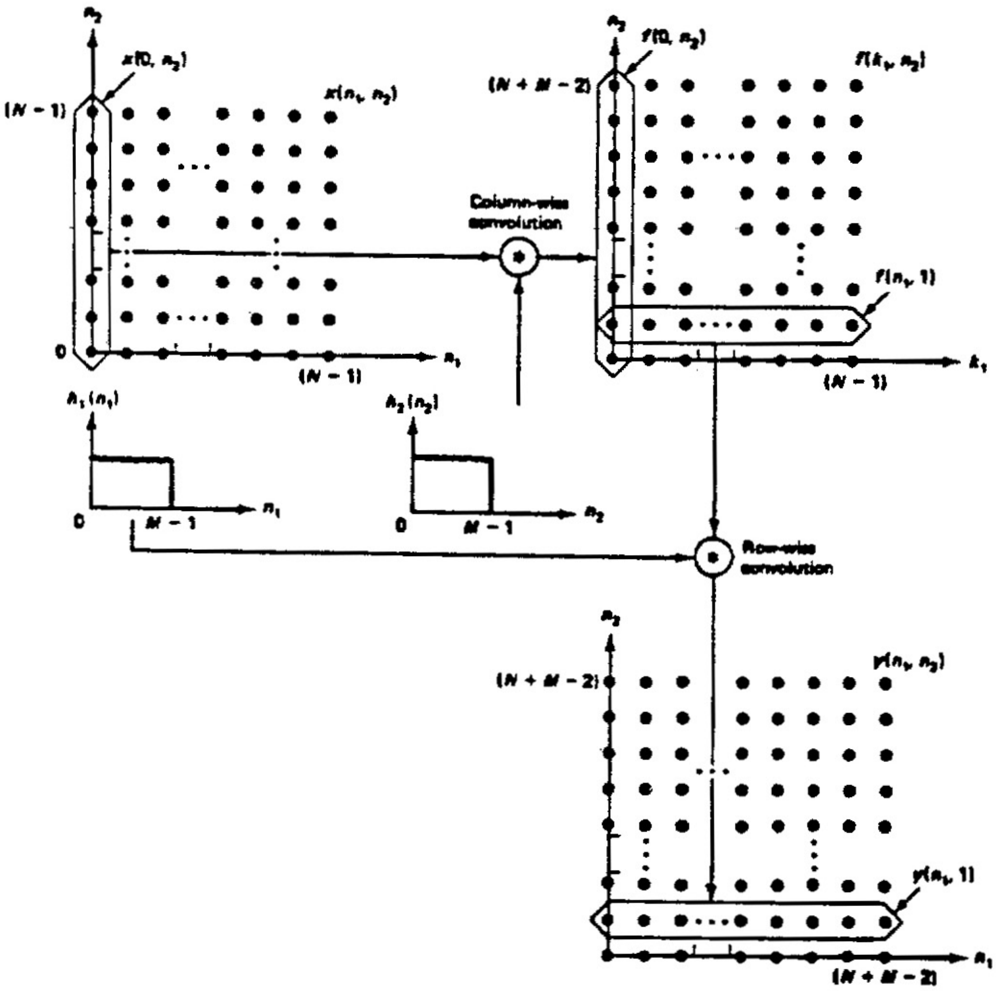

Figure 1.15 Convolution of $x(n_1, n_2)$ with a separable sequence $h(n_1, n_2)$.

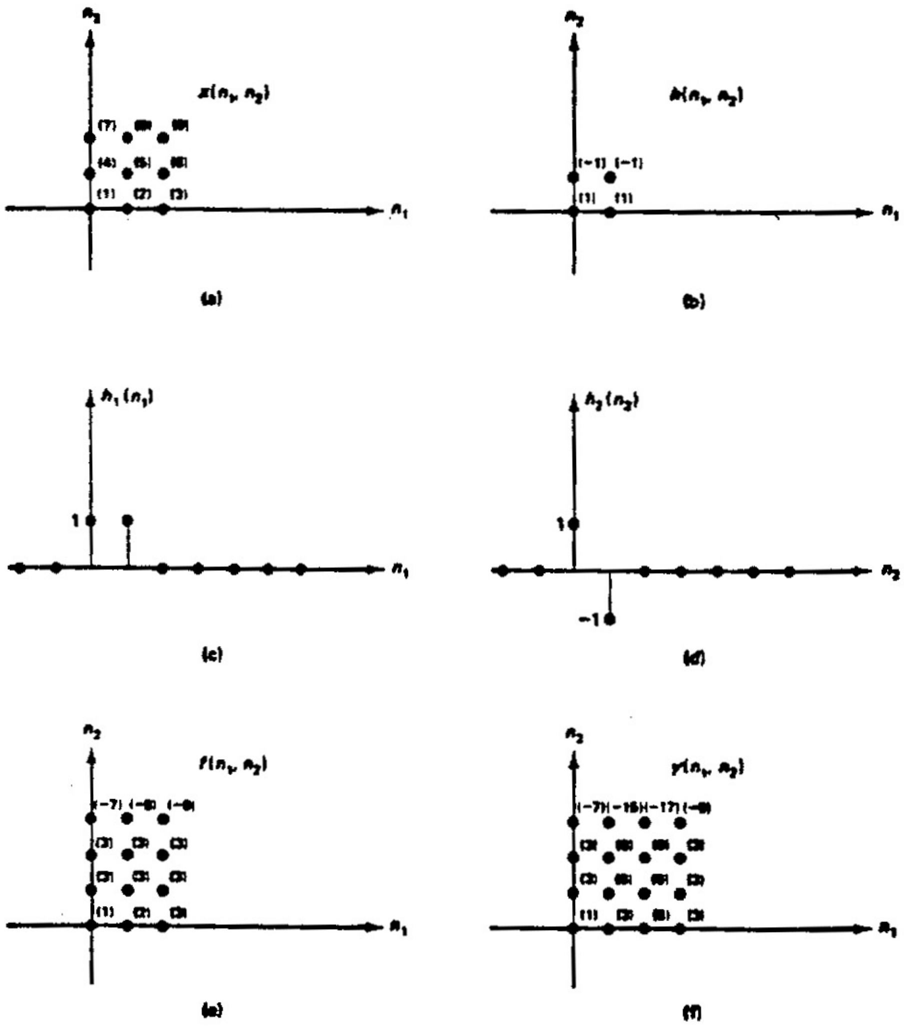

Figure 1.16 Example of convolving $x(n_1, n_2)$ with a separable sequence $h(n_1, n_2)$.