Spatial Filtering

LSI $\longrightarrow$ FIR.

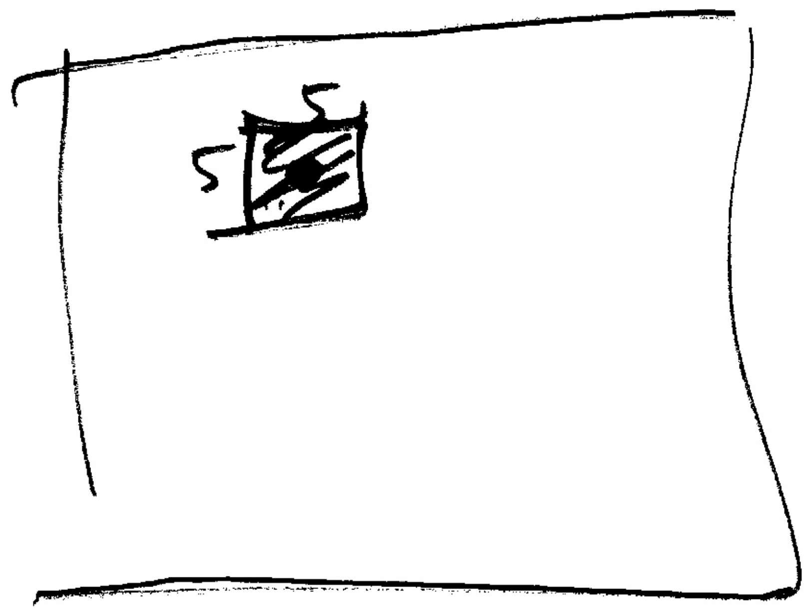
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{+b} w(s,t)\, f(x+s, y+t)$$

filter.

If $w(s,t) \longrightarrow$ LPF $\longrightarrow$ blurring → blurs edges → remove noise

$w(s,t) \longrightarrow$ HPF $\longrightarrow$ sharpening → sharpen edge → accentuate noise

Order Statistics:



→ Median

→ Max

→ Min.

Salt + Pepper noise

Impulse noise.

Image Enhancement

- unsharp masking + high boost filtering

- unsharp Masking.

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

processed   original   LPF = blurred.

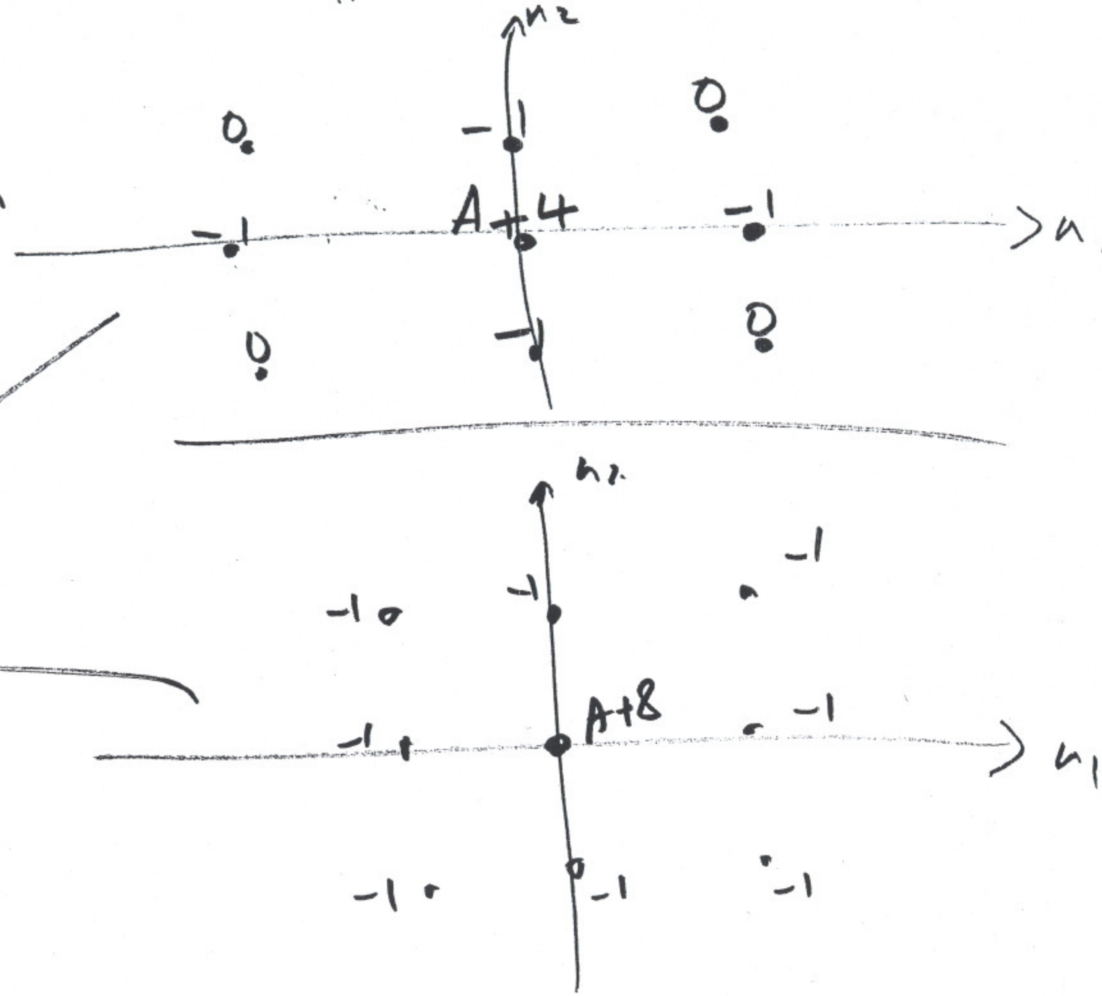
- General verison unsharp masking. : highboost filtering

$$f_{hb}(x,y) = A\, f(x,y) - \bar{f}(x,y)$$

scaling factor.
$A > 1$

$$f_{hb}(x,y) = (A-1)\,f(x,y) + f_s(x,y).$$

hb = linear operator

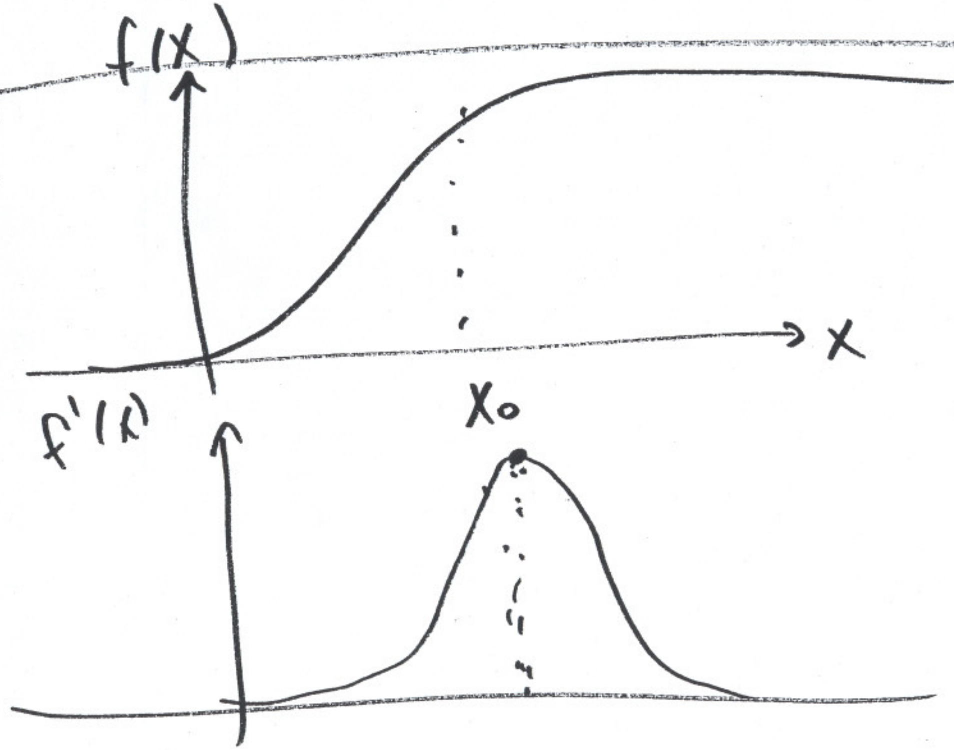


possible impulse response
HB filtering.

Edge Detection

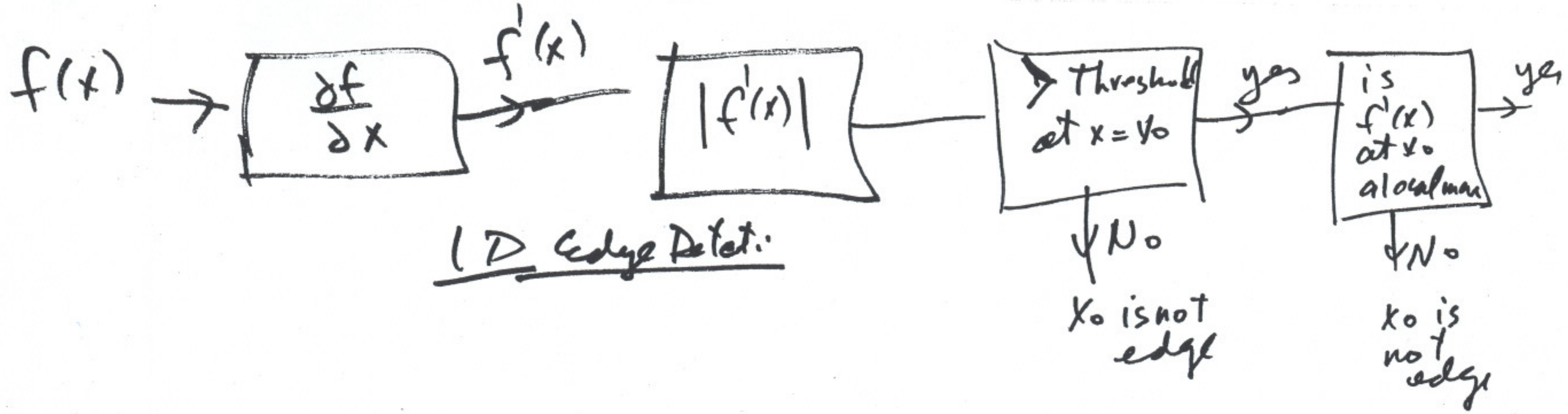
- Gradient Method.
- Laplacian $\nabla^2 f$
- L of G = Laplacian of Gaussian: Hildreth/Marr.

1-D



Derivative

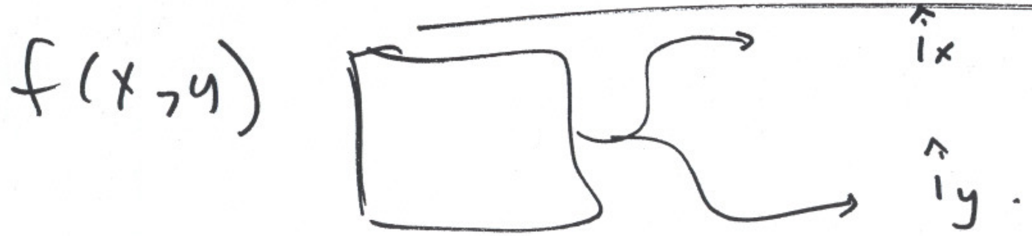
if $\frac{\partial f}{\partial x}$ large $\longrightarrow$ likely an edge.



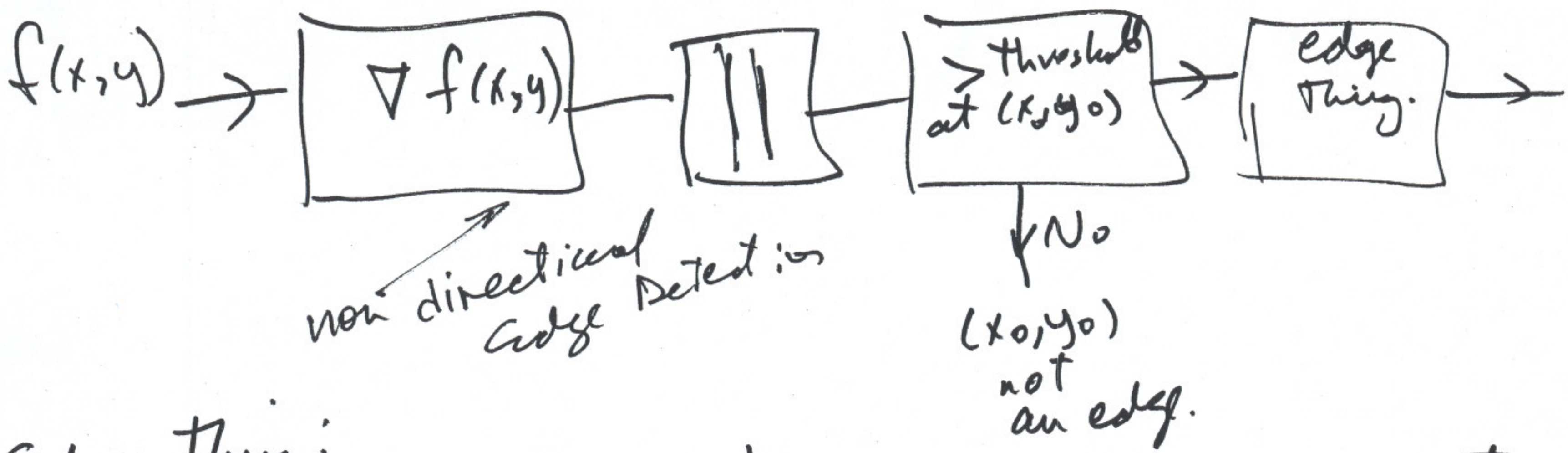
1D Edge Detect:

show 10.7 of G/W

Extension To 2D



$$\nabla f(x,y) = \frac{\partial f(x,y)}{\partial x}\,\hat{i}_x + \frac{\partial f(x,y)}{\partial y}\,\hat{i}_y$$



Edge Thing:

(a) If $|\nabla f(x,y)|$ has a local max at (x_0,y_0) in horizontal direction, but not vertical, we declare (x_0,y_0) an edge when

$$\left|\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)} > K\left|\frac{\partial f(x,y)}{\partial y}\right|_{(x_0,y_0)}$$

$K \simeq 2$

(b) If $|\nabla f(x,y)|$ has a local max at (x_0, y_0) in vertical direction but not horizontal, declare (x_0, y_0) an edge. when

$$\left| \frac{\partial f}{\partial y}(x,y) \right|_{(x_0,y_0)} > K \left| \frac{\partial f}{\partial x}(x,y) \right|_{(x_0,y_0)}$$

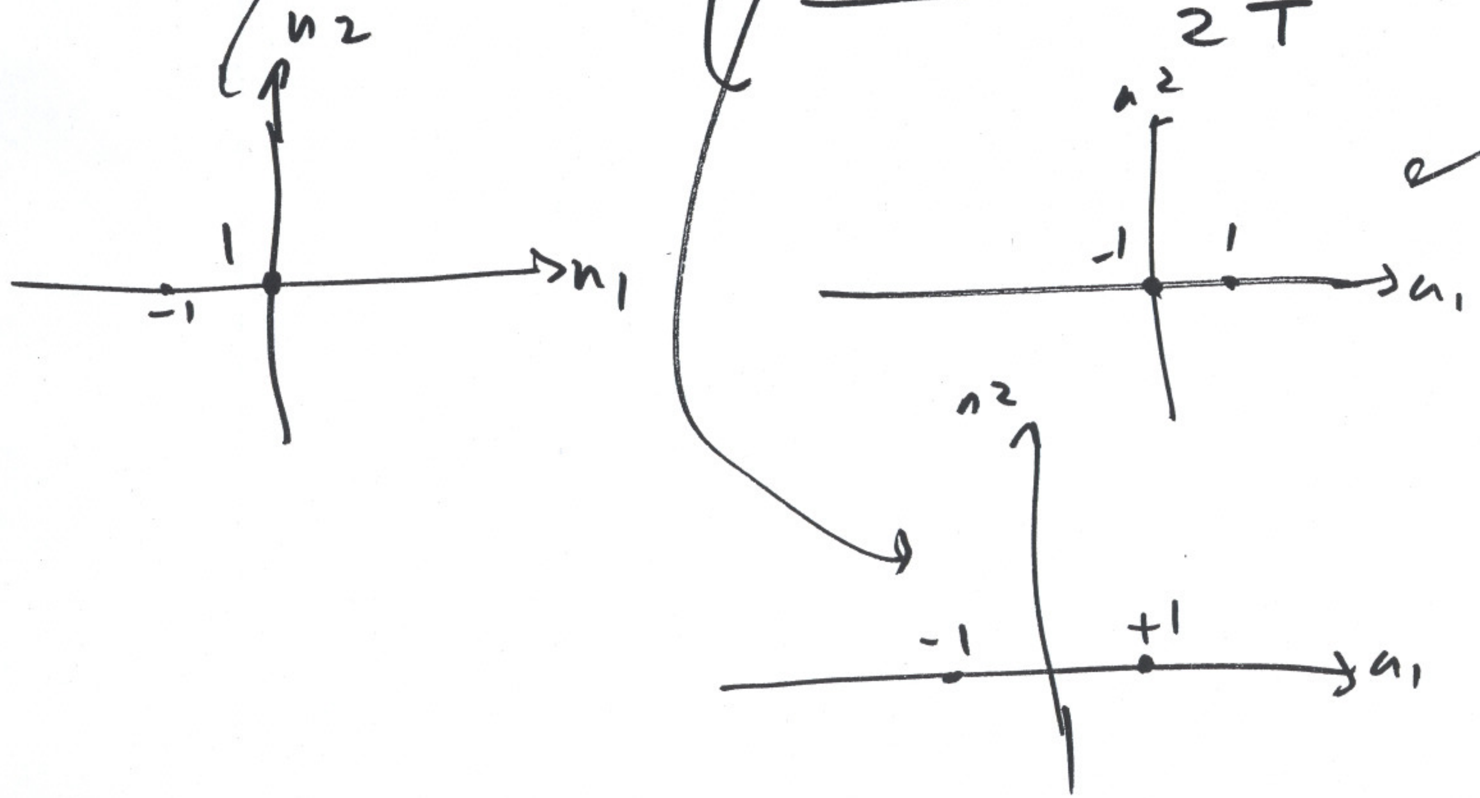
Directional Gradient Edge Detectors

- Bias Towards a particular direction.

compute vertical edge: $\frac{\partial f(x,y)}{\partial x}$

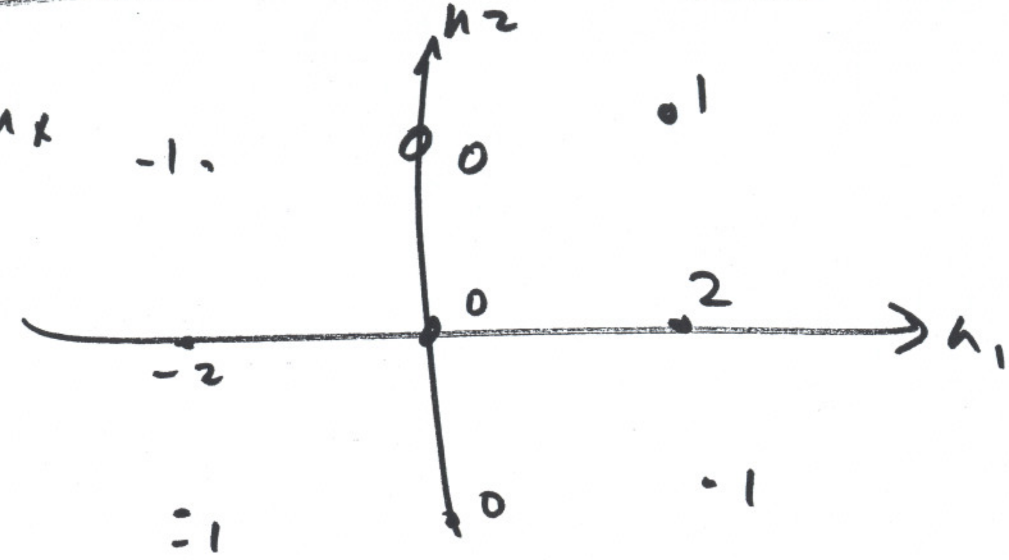
horizontal edge $\frac{\partial f(x,y)}{\partial y}$

$$\frac{\partial f(x,y)}{\partial x} = \begin{cases} \dfrac{f(x,y) - f(x-1,y)}{T} \\[2ex] \dfrac{f(x+1,y) - f(x,y)}{T} \\[2ex] \dfrac{f(x+1,y) - f(x-1,y)}{2T} \end{cases} \qquad x \overset{}{\underset{T}{\longleftrightarrow}} x$$



Famous Directional Edge Detection — SoBel

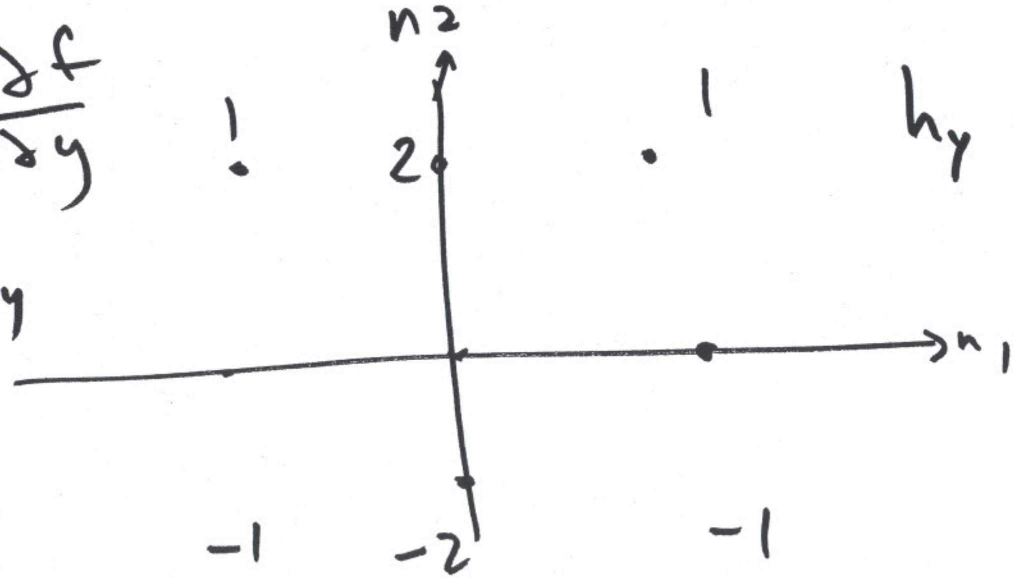
$\frac{\partial f}{\partial x}$ $f * h_x$



$= \frac{\partial f}{\partial y}$

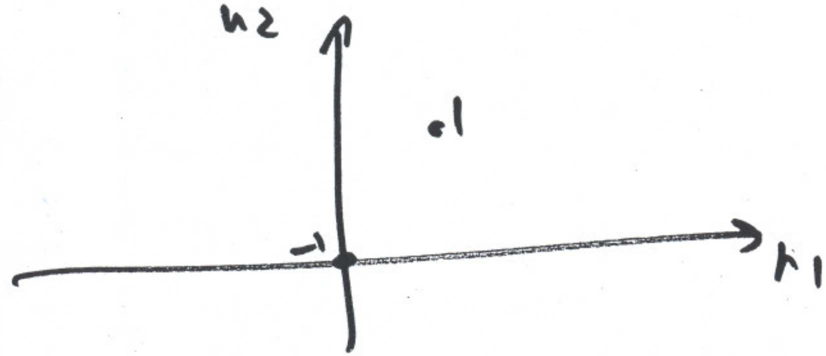
$= f * h_y$

h_y

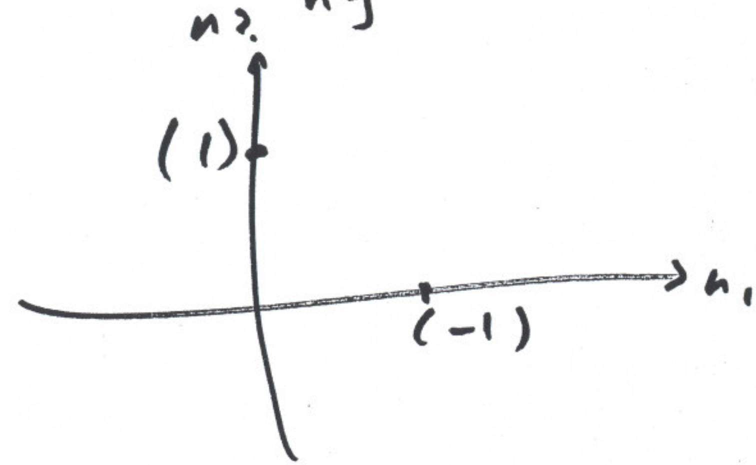


Roberts's edge Detector

h_x



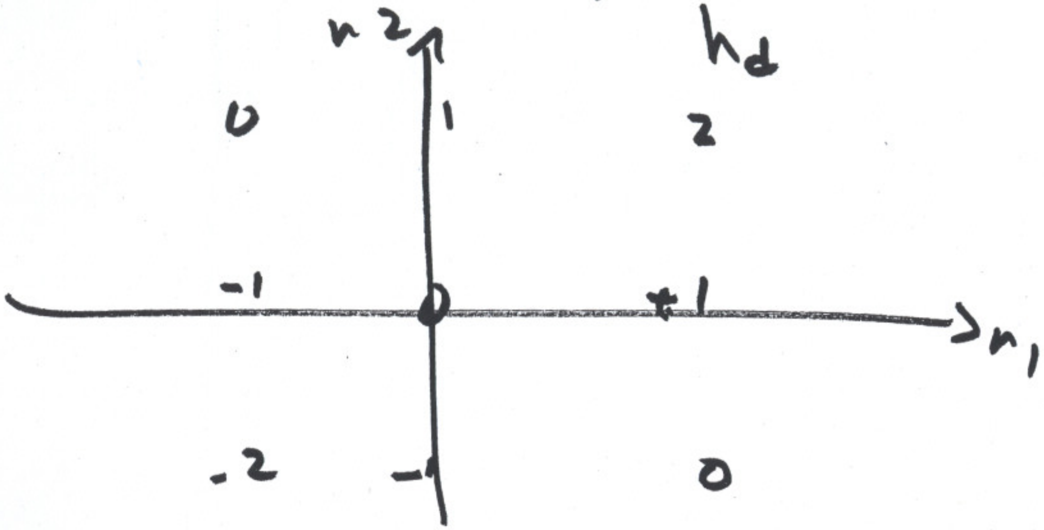
h_y

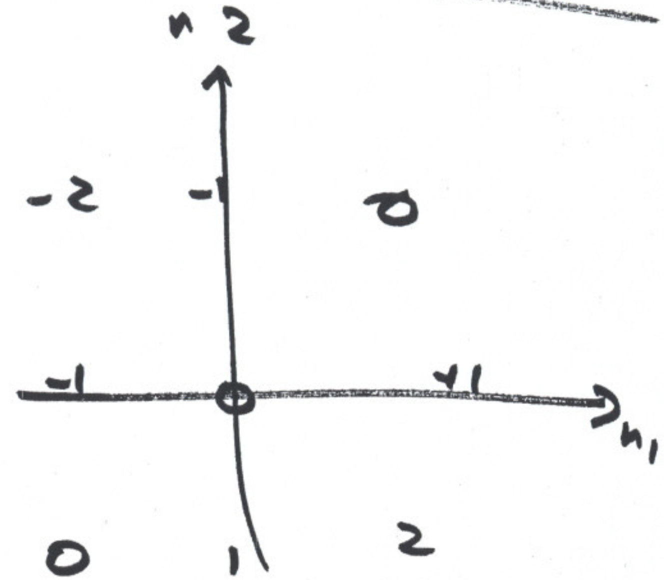


Diagonal Directional Gradient filters

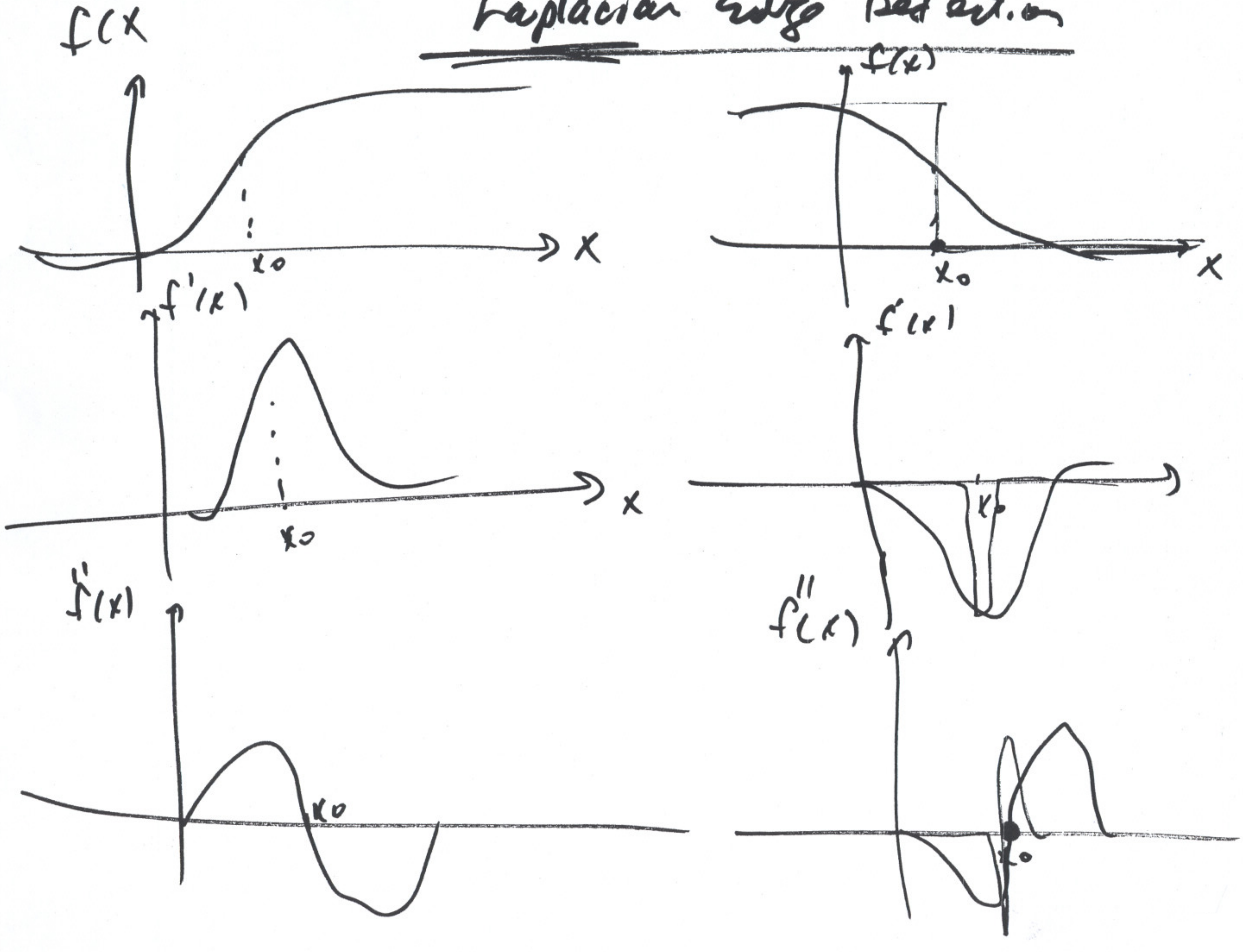
Sobel $\rightarrow$ Diagonal.

h_d





Laplacian Edge Detection



- Zero crossing of $f''(x) \longrightarrow$ edge.

Extension To 2D

$$\nabla^2 f(x,y) = \nabla\left(\nabla f(x,y)\right)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

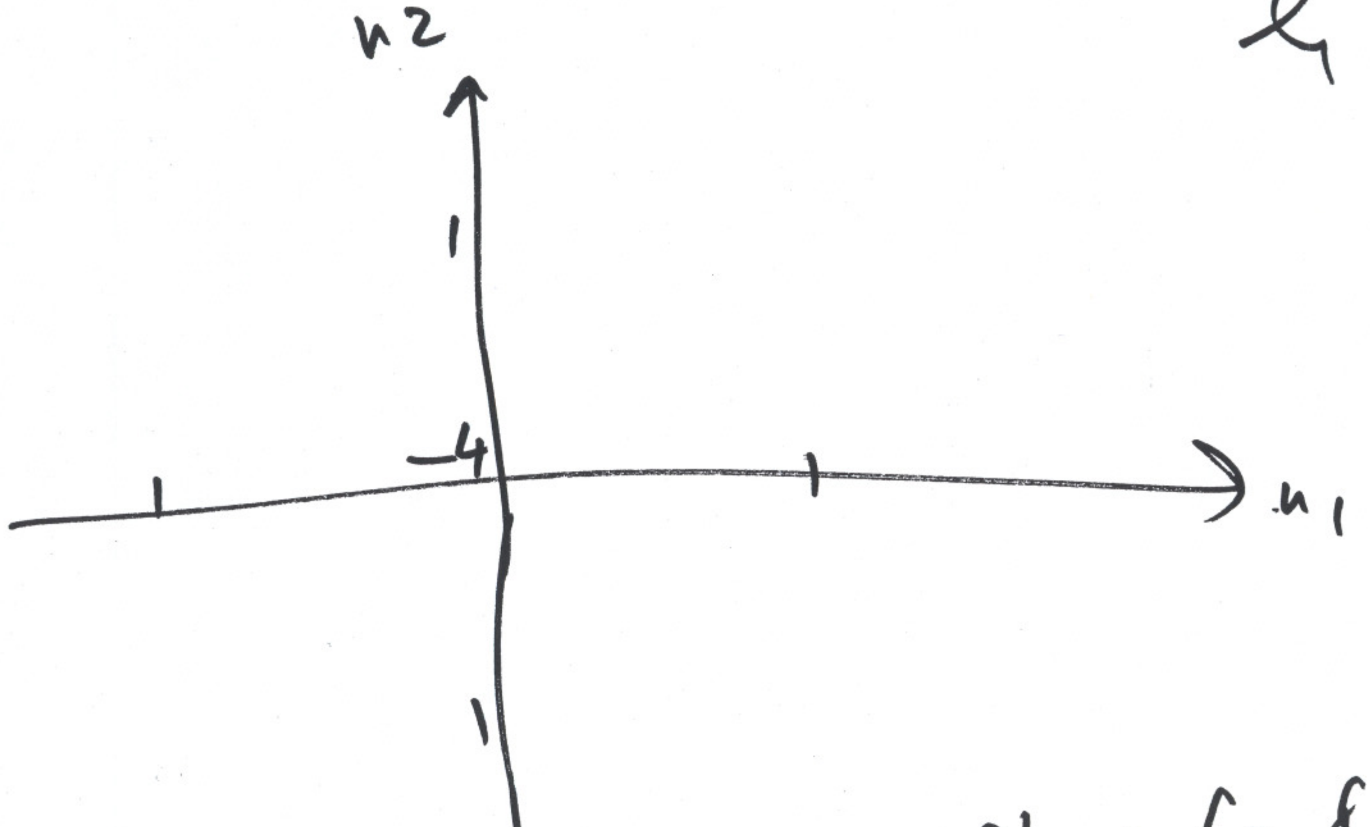
$$\left.\begin{aligned} \frac{\partial f}{\partial x} &\longrightarrow f(n_1+1,n_2) - f(n_1,n_2) \\ \frac{\partial^2 f}{\partial x^2} &\longrightarrow f_x(n_1,n_2) - f_x(n_1-1,n_2) \end{aligned}\right\} \Longrightarrow$$

$$\frac{\partial^2 f}{\partial x^2} \Longrightarrow f(n_1+1,n_2) - 2f(n_1,n_2) + f(n_1-1,n_2)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \longrightarrow f(n_1+1, n_2) + f(n_1-1, n_2) + f(n_1, n_2+1) + f(n_1, n_2-1) - 4f(n_1, n_2)$$

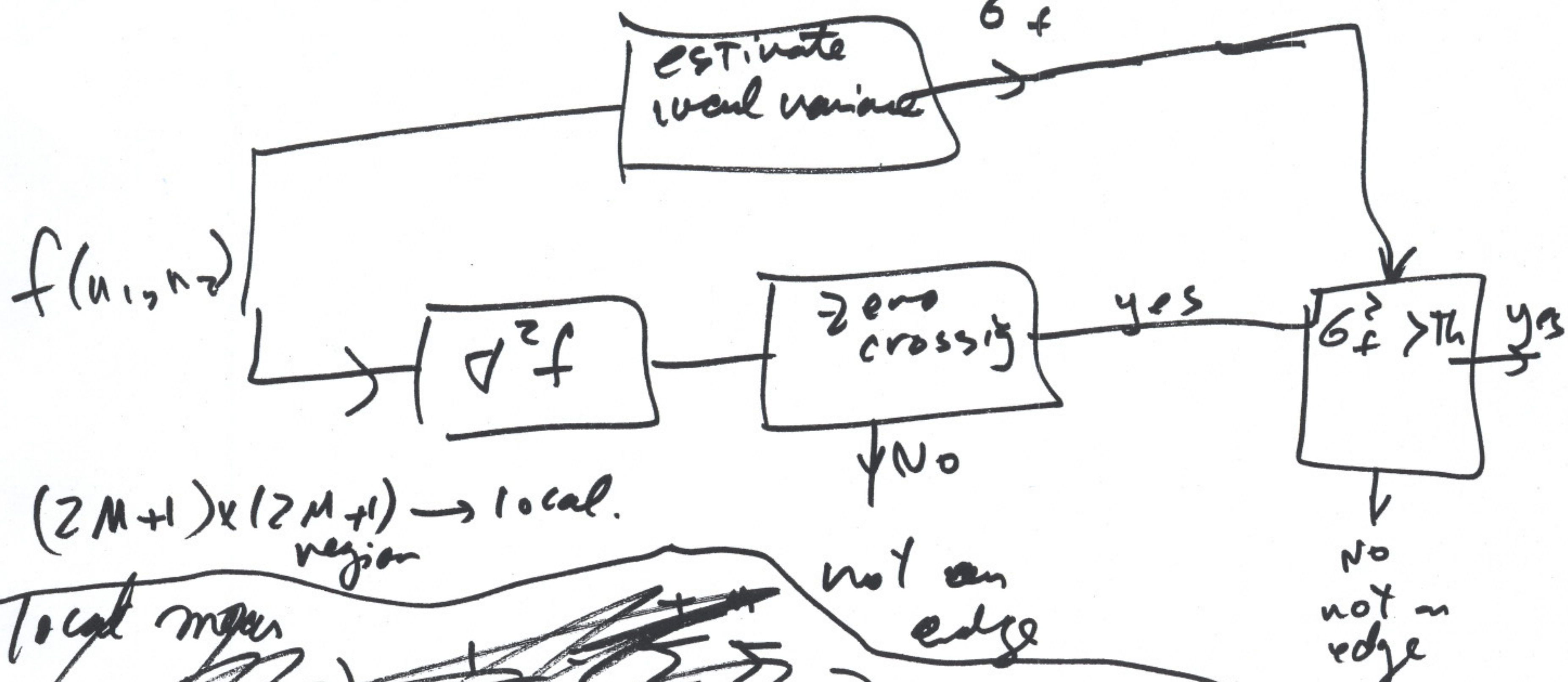
$$= f * h$$

h



Show fig 8.33
J Lim

How To use $\nabla^2 f$ operator to detect edges?



$(2M+1) \times (2M+1) \rightarrow$ local. region

~~local mean~~

~~local variance~~

local mean

$$m_f(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} f(k_1,k_2)$$

local variance.

$$\sigma_f^2(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} \left(f(k_1,k_2) - m_f(k_1,k_2)\right)^2$$

Window $(2M+1) \times (2M+1)$

centered around (n_1, n_2)

Edge Detection Laplacian of Gaussian

Gaussian $h(x,y) = e^{-\frac{(x^2+y^2)}{2\pi\sigma^2}}$

$$H(\Omega_x, \Omega_y) = 2\pi^2 \sigma^2 e^{-2\pi\sigma^2 \frac{(\Omega_x^2 + \Omega_y^2)}{2}}$$

"Laplacian of Gaussian" = LoG.

$$\nabla^2 \left[f(x,y) * h(x,y) \right] \equiv$$

Gaussian

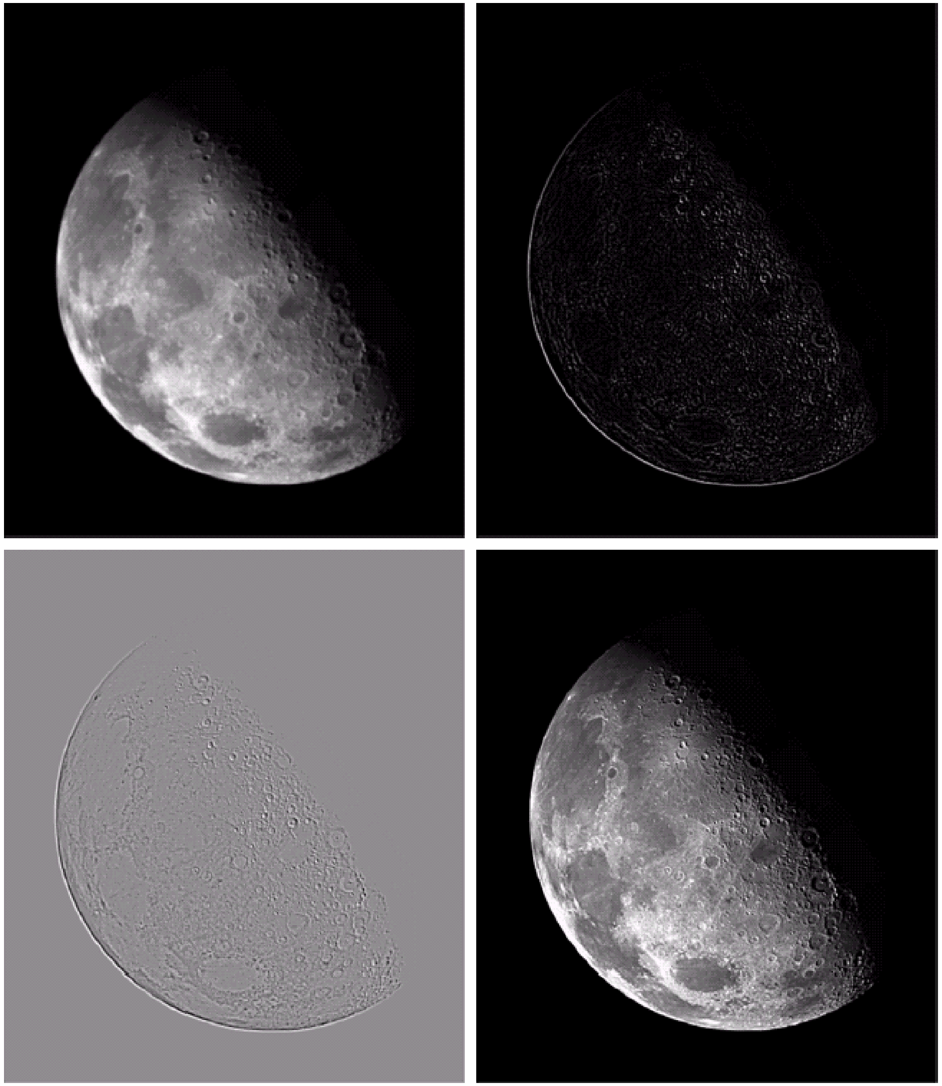
$$= f(x,y) * \left[\nabla^2 h(x,y) \right]$$

$$= f(x,y) * \left[\frac{\partial^2 h(x,y)}{\partial x^2} + \frac{\partial^2 h(x,y)}{\partial y^2} \right]$$

$$\nabla^2 h(x,y) = \frac{e^{-(x^2+y^2)/2\pi\sigma^2}}{(2\pi\sigma^2)^2}\,(x^2+y^2-2\pi\sigma^2)$$

$$F\{\ \}= -2\pi^2\sigma^2\, e^{-\pi\sigma^2(\Omega_x^2+\Omega_y^2)/2}\,(\Omega_x^2+\Omega_y^2)$$

Fig. 8.36 J. Lim



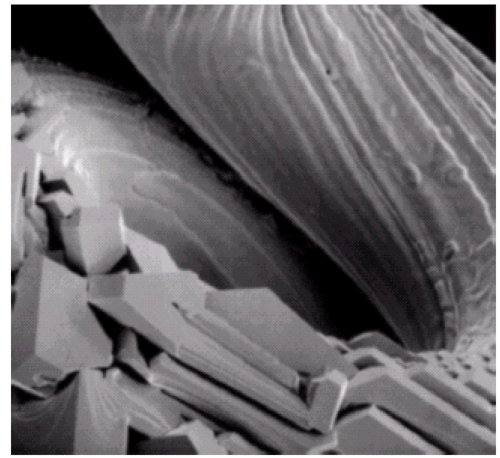
a b
c d

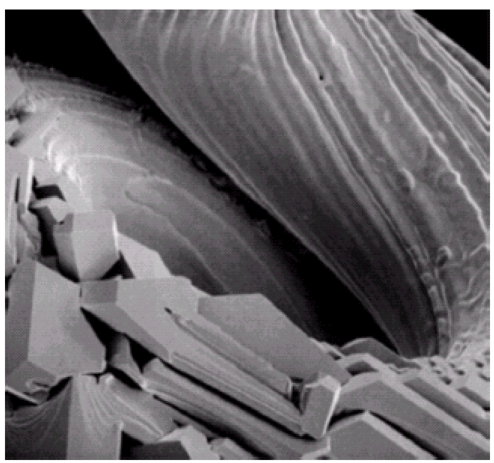
FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

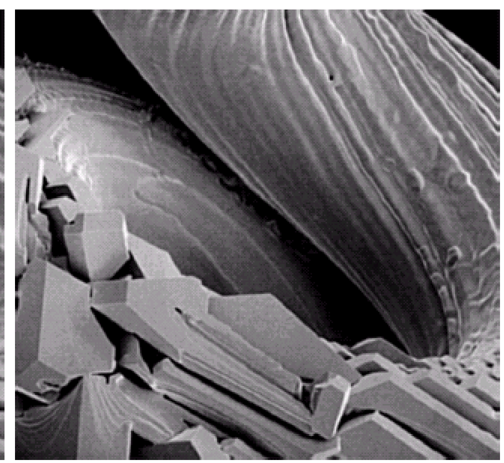
Chapter 3
Image Enhancement in the Spatial Domain

0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1







a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

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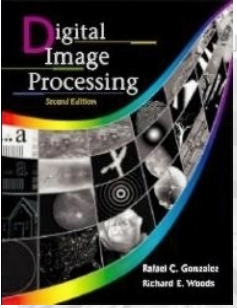
Image Enhancement in the Spatial Domain

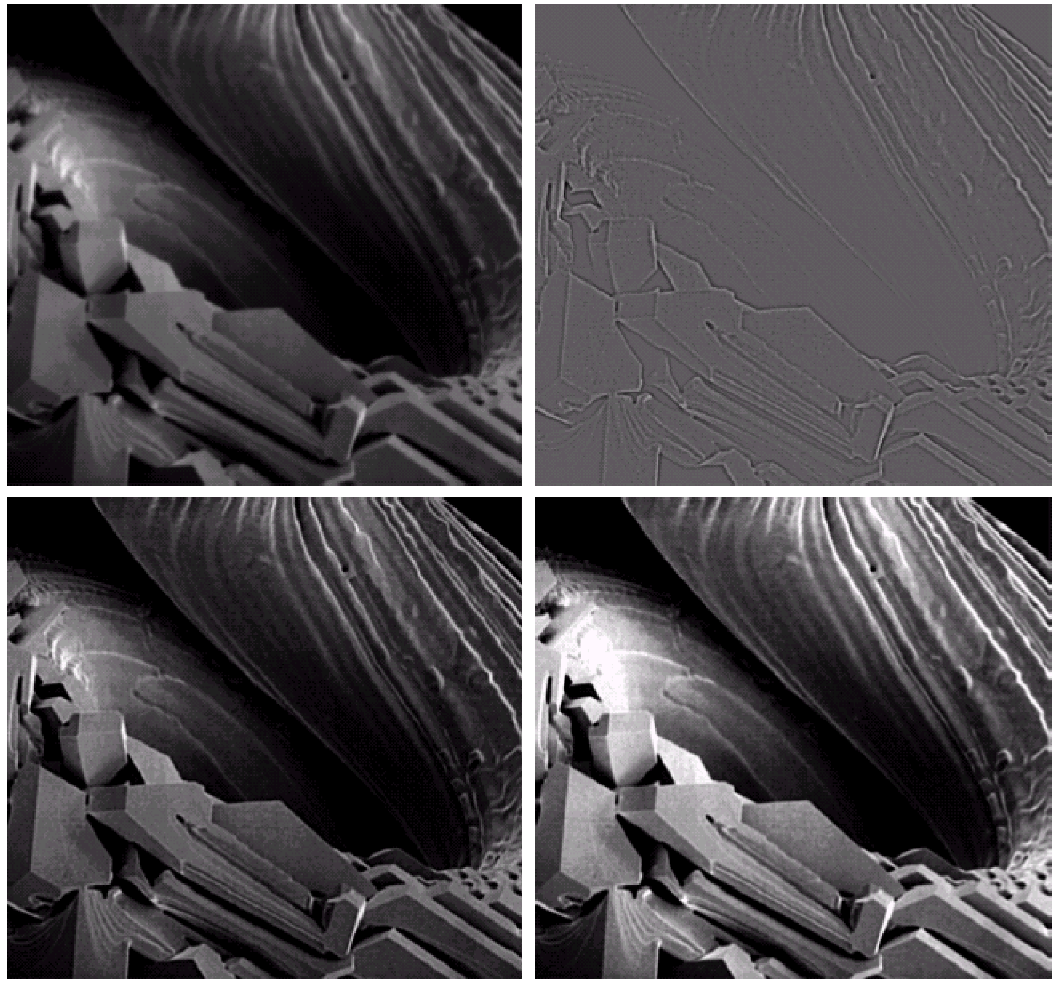
0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

a b

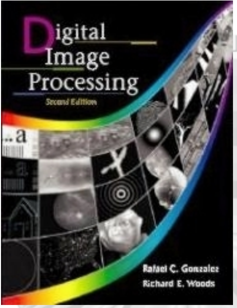
FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.





a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



Chapter 3
Image Enhancement in the Spatial Domain

a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

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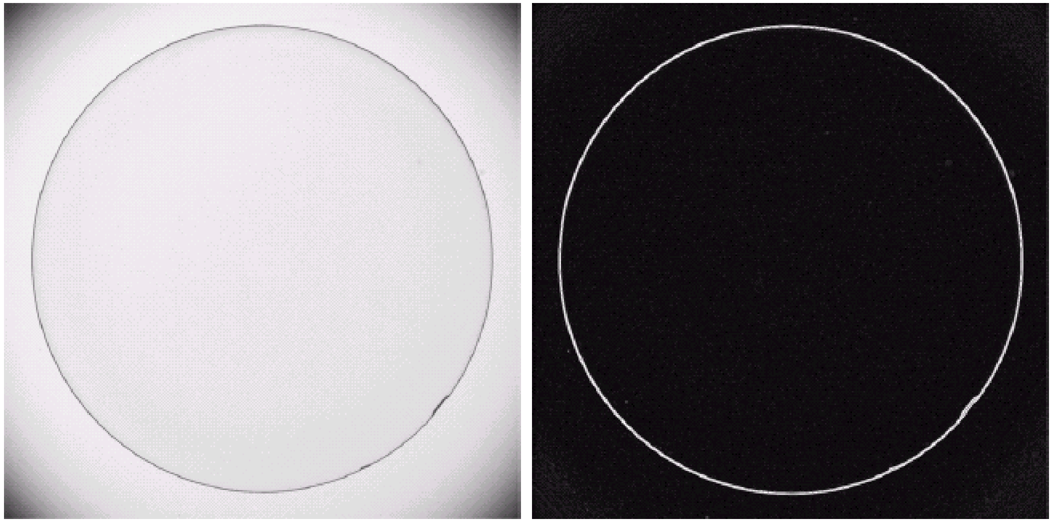

a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)