# Image Restoration

1. Enhancement → image "looks" better

    ↙

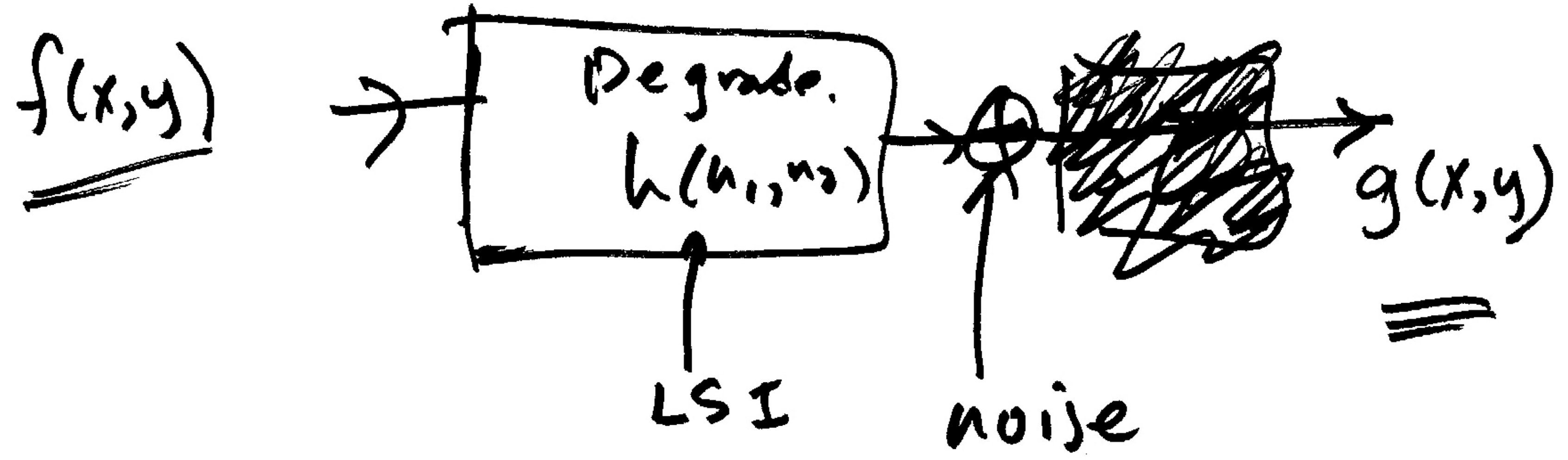
    subjective.
    improvement.

2. Restoration -

    Image has been degraded by something

    - noise
    - blur
    - Atmospheric Turbulence.

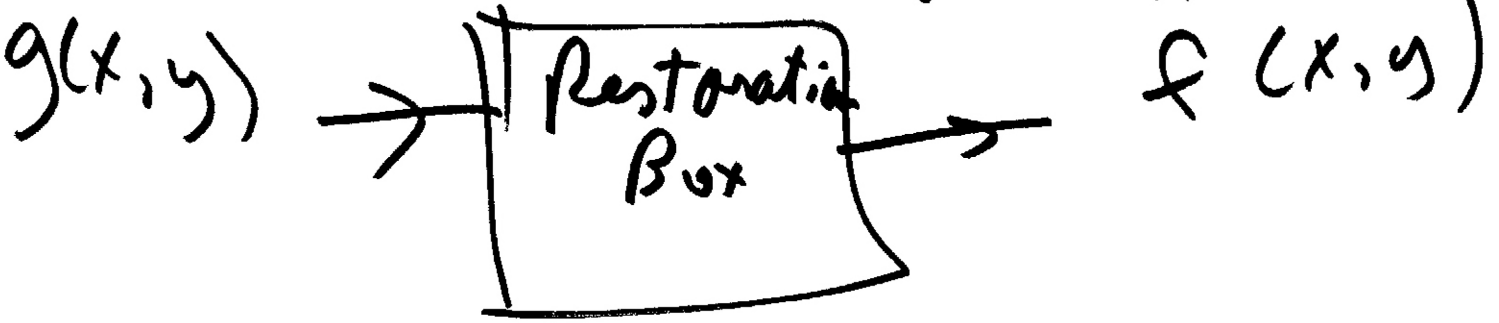
    objective models → Error → MSE



$$g(x,y) = h * f + \eta$$

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

noise



minimize $E\left[\left(f(x,y) - \hat{f}(x,y)\right)^2\right]$

---

Today.

$H$ is identity.

only corruption → noise.

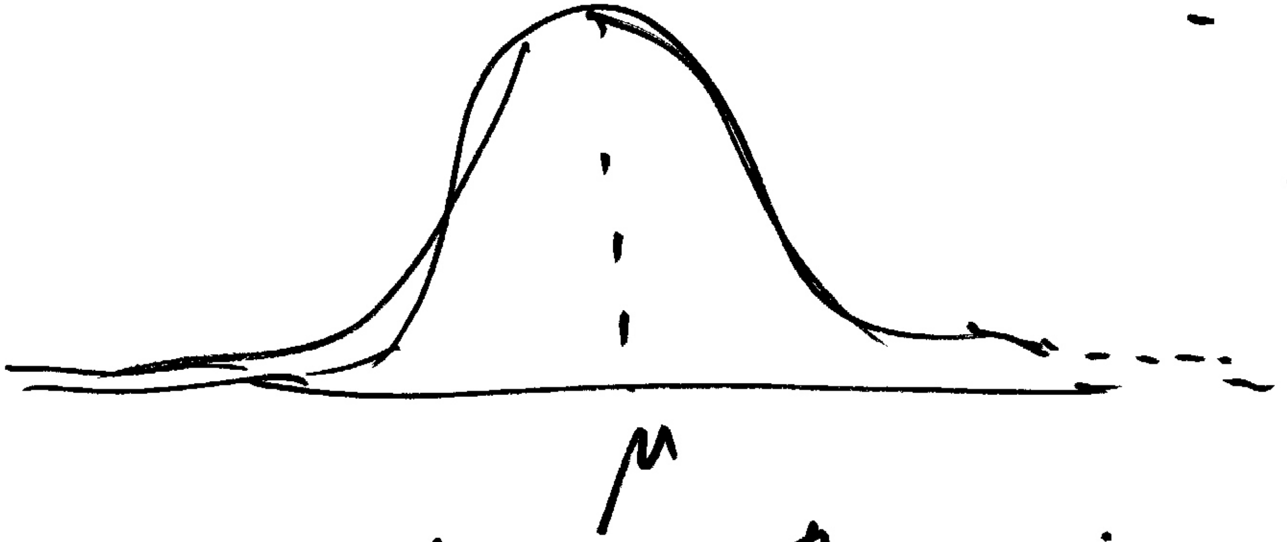
$$\underset{\text{observed.}}{g(x,y)} = \underset{\text{original image}}{f(x,y)} + \underset{\text{noise}}{\eta(x,y)}$$

Assume noise is independent of spatial coordinates
uncorrelated w.r.t. image itself.

① Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\pi\sigma^2}}$$

$var = \sigma^2$ mean: $\mu$



- electronic noise

- sensor noise due to poor illumination or high temperature

-

70 % of values are within one sigma $[\mu - \sigma, \mu + \sigma]$

95 % within 2 sigma.

② Rayleigh.

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

$$\mu = a + \sqrt{\frac{b\pi}{4}} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

noise in range imaging apps.

③ Erlang (Gamma) noise.

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1} e^{-az}}{(b-1)!} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\sigma^2 = \frac{b}{a^2}$$

laser imaging.

$$\mu = b/a$$

④ Exponential special case of Erlang.

$$p(z) = \begin{cases} a e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Networking

(5) Uniform.

$$p(z)=\begin{cases}\frac{1}{b-a} & a\le z\le b\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu = \frac{a+b}{2} \qquad \text{Var} = \frac{(b-a)^2}{12}$$

random # generation.

(6) Impulse, salt/pepper noise.

quick transit, faulty switching.

$$p(z)=\begin{cases}P_a & z=a\\ P_b & z=b\\ 0 & \text{otherwise.}\end{cases}$$

if $b>a$, $b$ show up as light dots

$a$ " " " dark dot.

## ① Mean Filters

1. Arithmetic Mean.

$S_{xy} \rightarrow$ window.

$m \times n$.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

2. Geometric Mean.

similar to arithmetic but loose less detail $\leftarrow$

$$\hat{f}(x,y) = \left[ \prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

3. Harmonic mean filter.

works for salt noise.
fails for pepper

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

4. Conta Harmonic.

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}.$$

Q>0 → remove pepper noise

Q<0 → remove salt noise

# order statistics filter

1. Median

look at $S_{xy}$. find median

replace $(x,y)$ with the

median value.

2. Max $\hat{f}(x,y) = \max_{S_{xy}} g(s,t)$

also removes some dark pixel.

3. Min $\hat{f}(x,y) = \min_{S_{xy}} g(s,t)$

also remove some white pixel

4. Midpoint $\frac{1}{2}\{\max + \min\}$.

# Alpha Trimed mean filter

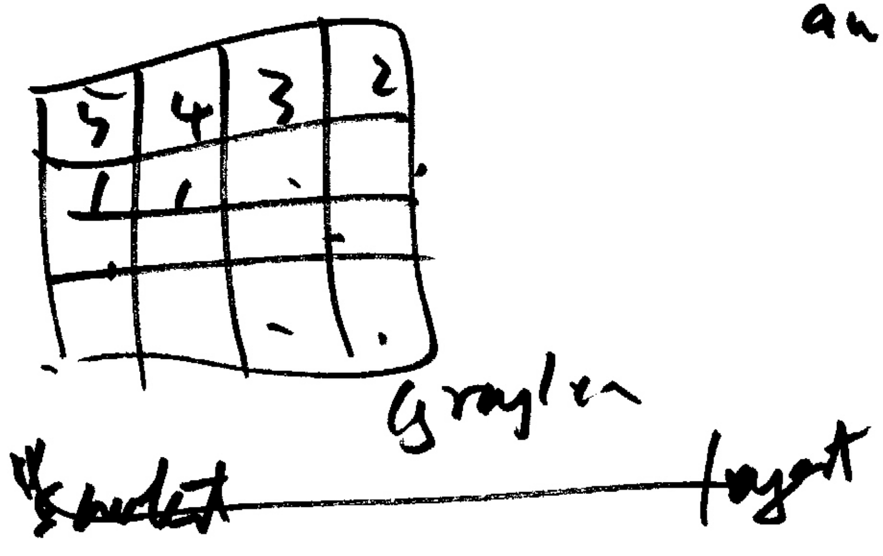
$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t) \in S_{xy}} g_{\boxed{r}}(x,y)$$

$S_{xy}$ $m \times n$.

$g_r(s,t) \longrightarrow g(s,t)$ excloding.

$d/2$ brightest graylevel

and $d/2$ darkest " "



# Adaptive local noise Reduction

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_N^2}{\sigma_L^2}\left[ g(x,y) - m_L \right)$$

$m_L$ = local mean $= \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$

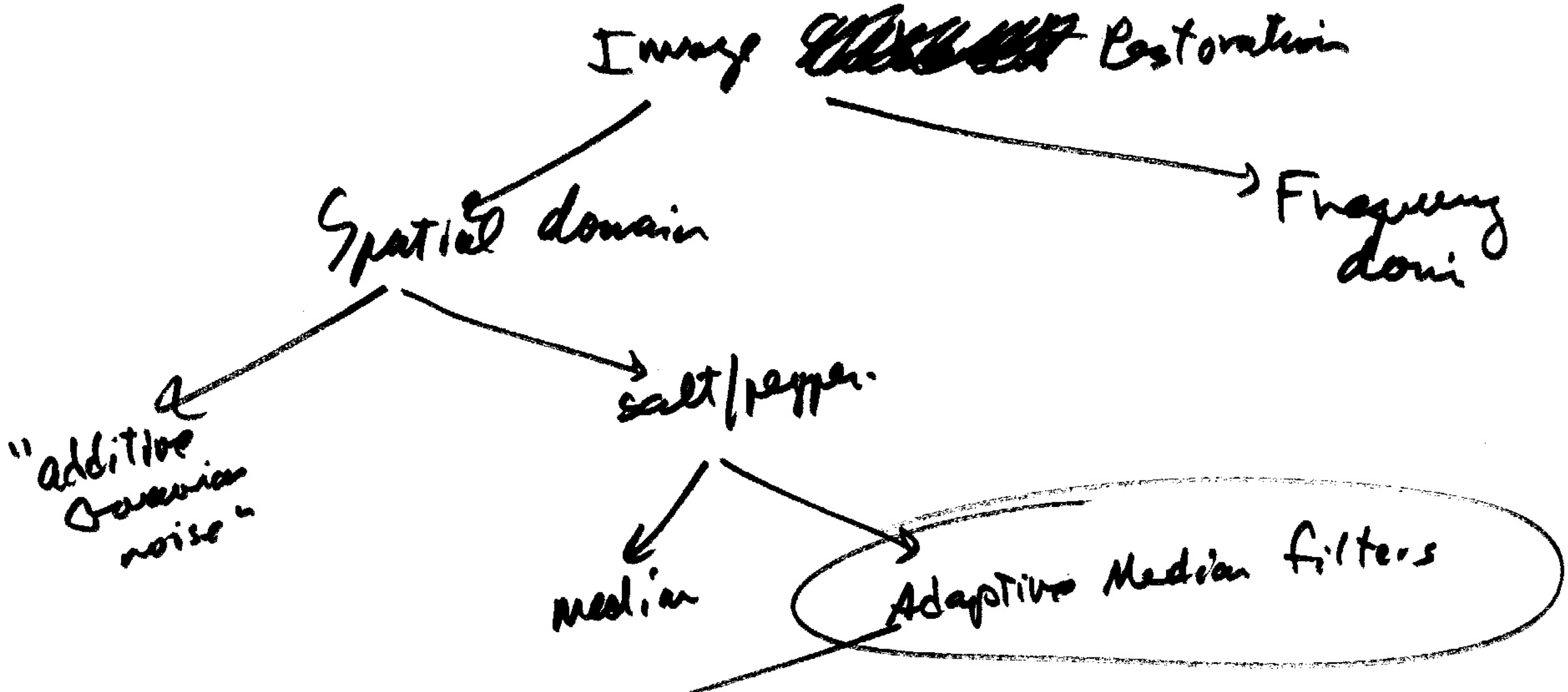
$\sigma_L^2$ = local variance.

$\sigma_N^2$ = noise variance.

if $\sigma_N^2 << \sigma_L^2 \longrightarrow \hat{f} \simeq g$ good

if $\sigma_N^2 \gg \sigma_L^2$ then approximate $\frac{\sigma_N^2}{\sigma_L^2} \simeq 1$

$\longrightarrow \hat{f} \simeq m_L$



## Adaptive Median Filter

- looking at neighborhood $S_{x,y}$ around pixel $(x,y)$
- $z_{xy}$ = pixel value at center of $S_{x,y}$
- $z_{max}$ = max value of intensity over $S_{xy}$

- $z_{min}$ = min value in $S_{xy}$
- $z_{med}$ = median value in $S_{xy}$

---

## outline

- Keep increasing window size until $z_{med}$ is not an impulse.

  $Z_{min} < Z_{med} < Z_{max}$

- When this happen, check $z_{xy}$.
  - If $z_{xy}$ is not an impulse → output $z_{xy}$
  - If $z_{xy}$ is an impulse → output $z_{med}$

    since $z_{med}$ is not an impulse.

## Psuedo code

```
Part A     if        $z_{min} < z_{med} < z_{max}$
                     Then go To part B.          → $z_{med}$ is not an impulse.

           else. if  windowsize $< S_{max}$
                     window ← window + 1 , goTo part A
           else.     output $z_{xy}$.

part B.    If        $z_{min} < z_{xy} < z_{max}$      → $z_{xy}$ is not an impulse
                     output $z_{xy}$

           else.     output $z_{med}$
```