Lecture 11. Lossy Image Compression

EL512 Image Processing

Dr. Zhu Liu
zliu@research.att.com

Note: Part of the materials in the slides are from A. K. Jain's Fundamentals of Digital Image Processing

Lecture Outline

- Introduction
- Lossy predicative coding
- Transform coding
- JPEG
- JPEG 2000

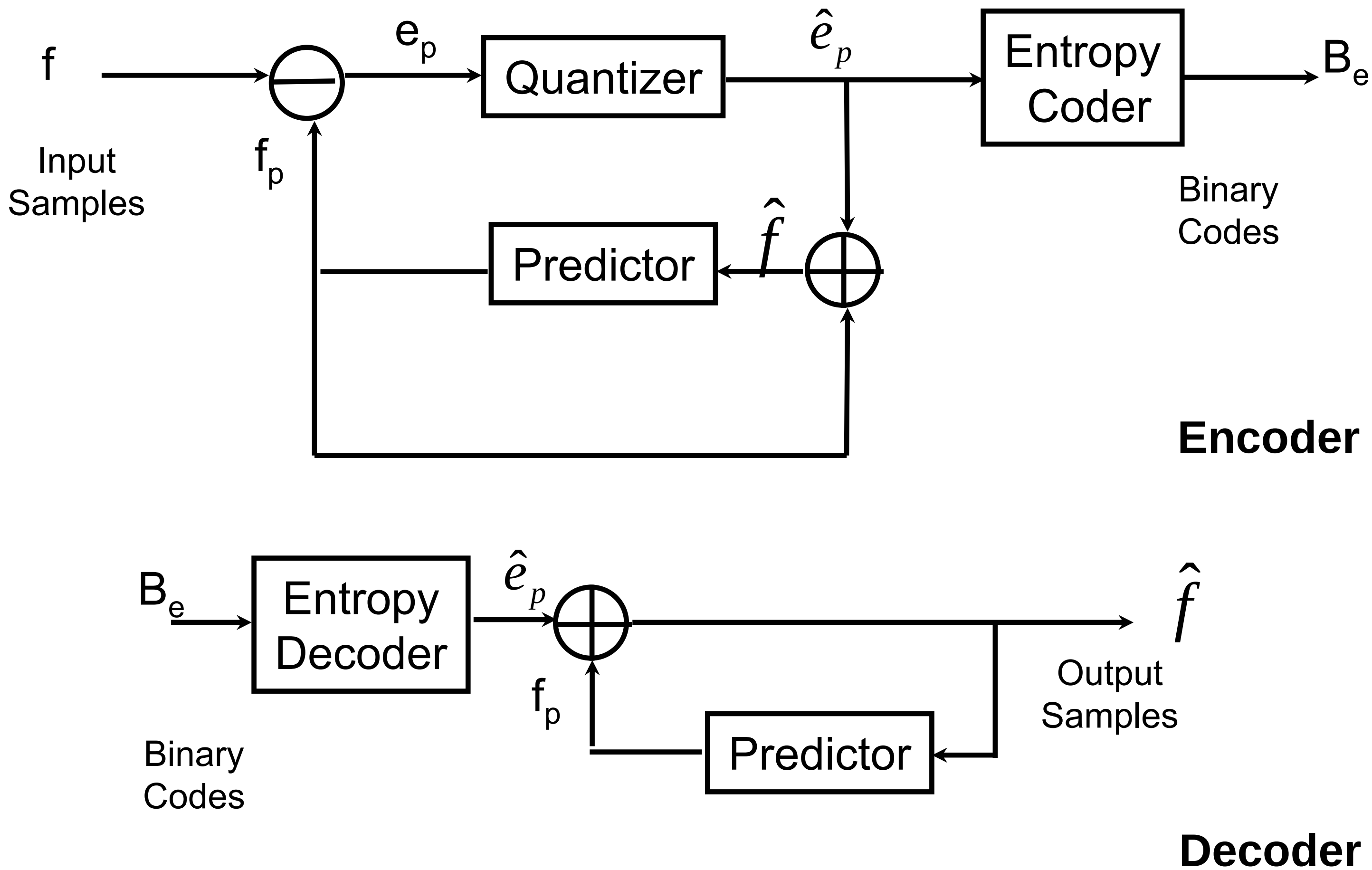
Spatial Prediction

A	B	C	D
E	F	G	H
I	J	K	L

Linear Estimator: $\hat{f}_K = a\,f_F + b\,f_G + c\,f_H + d\,f_J$

Optimal MSE Predictor: The coefficients are determined to minimize the mean square prediction error.

A Typical Predictive Coder



Closed-loop prediction

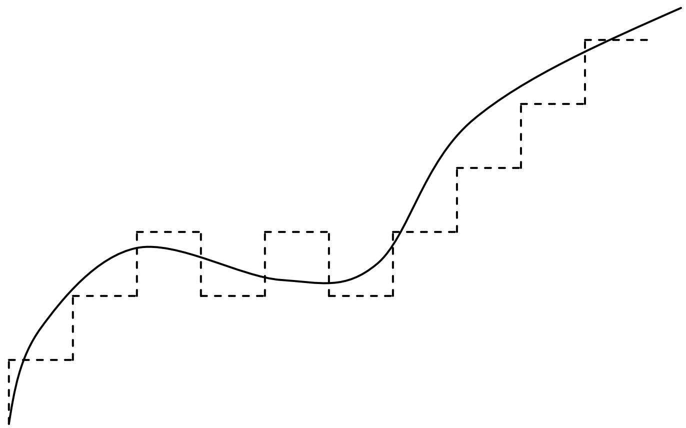
The Delta Modulator

- The simplest linear prediction.

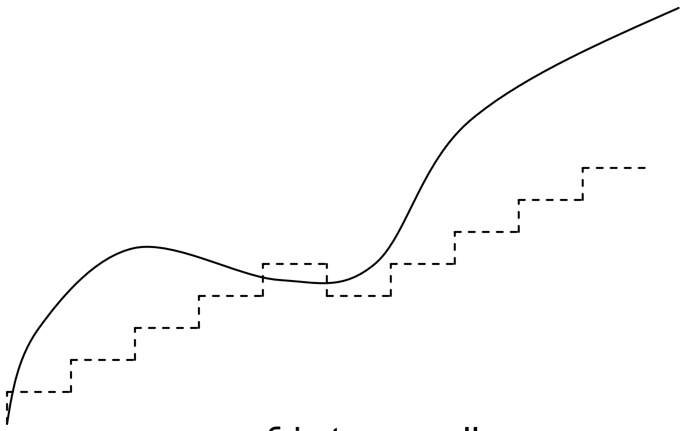
$$\hat{f}_0 = f_1$$

- The quantizer has only two levels.

$$\hat{e} = \begin{cases} \xi & e > 0 \\ -\xi & e < 0 \end{cases}$$



ξ is appropriate



ξ is too small

Error Analysis of a Lossy Predictive Coder

- Let q represent the quantization error for e, then

$$\hat{f} = f_p + \hat{e}_p = f_p + e_p - q = f - q$$

- Therefore, the error between the original and the reconstructed value e_f is exactly the same as the quantization error.
- Because the error usually has a non-uniform distribution, a non-uniform quantizer optimized for the distribution of the error signal is usually used.
- A vector quanitizer can be used for the error.

Design of the Predictor

- For lossy predictive coders, the optimal predictor should be designed to minimize the MSE between the original f_0 and the predicted values $\hat{f}_0$

$$\sigma_p^2 = E\left\{ | f_0 - \sum_k a_k \hat{f}_k |^2 \right\}$$

- Since $\hat{f}_k$ is related to a_k and the quantizer in a complicated relation, the precise minimization of the error is difficult.
- In practice, one simply assumes that the quantization error is negligible.

Estimation of the Correlation Coefficients

- One can use spatial sample averaging to approximate statistical ensemble mean.
- To estimate the correlation between a pixel f(m, n) and its neighbor at f(m-k, n-l), denoted by $R_f(k, l)$, one can use

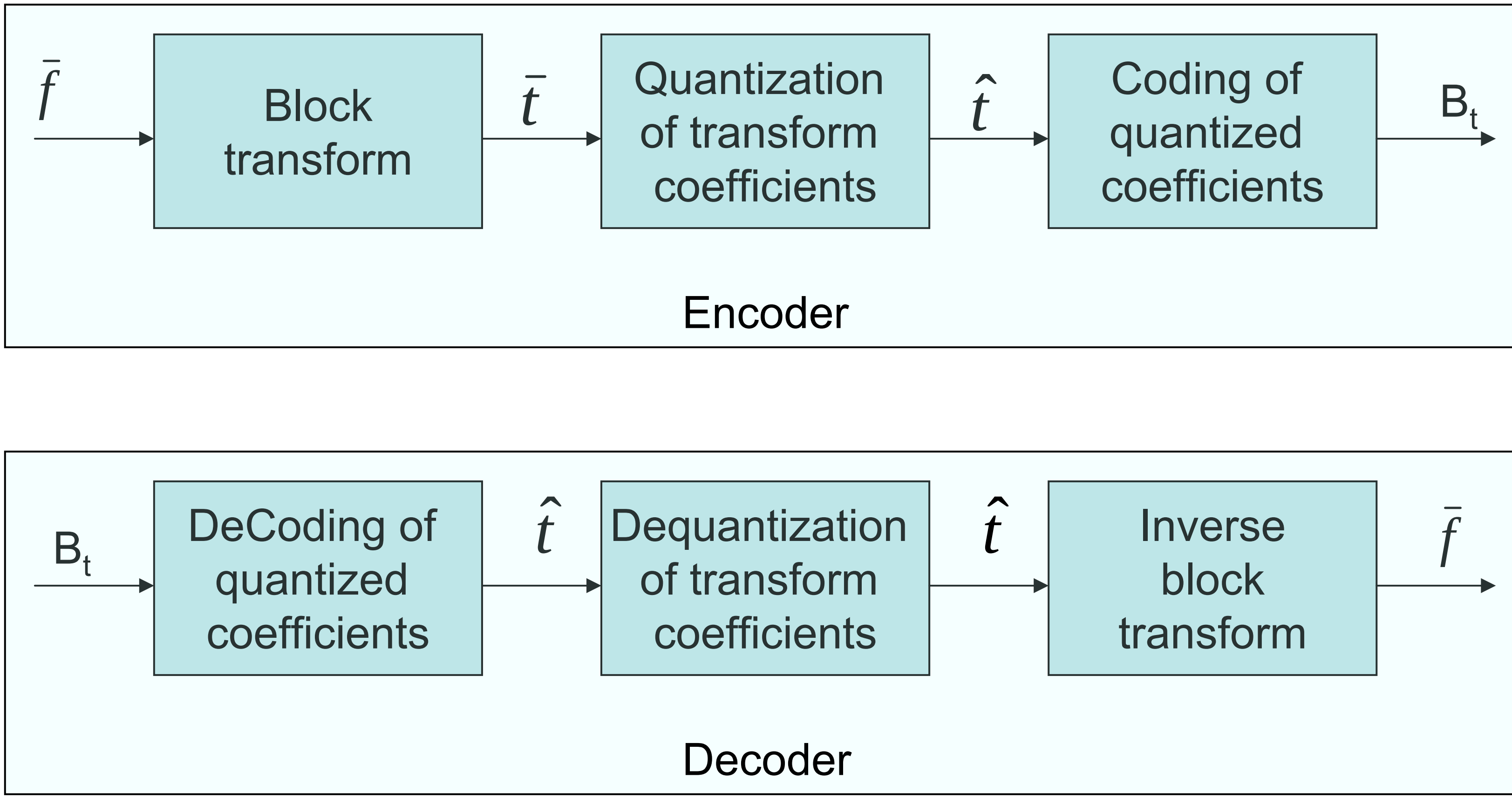
$$R_f(k,l) = \frac{1}{N_s} \sum_{m,n} f(m,n) f(m-k, n-l)$$

where N_s is the number of pairs of f(m, n) and f(m-k, n-l) that are included in the summation.

Non-Adaptive v.s. Adaptive Predictive Coding

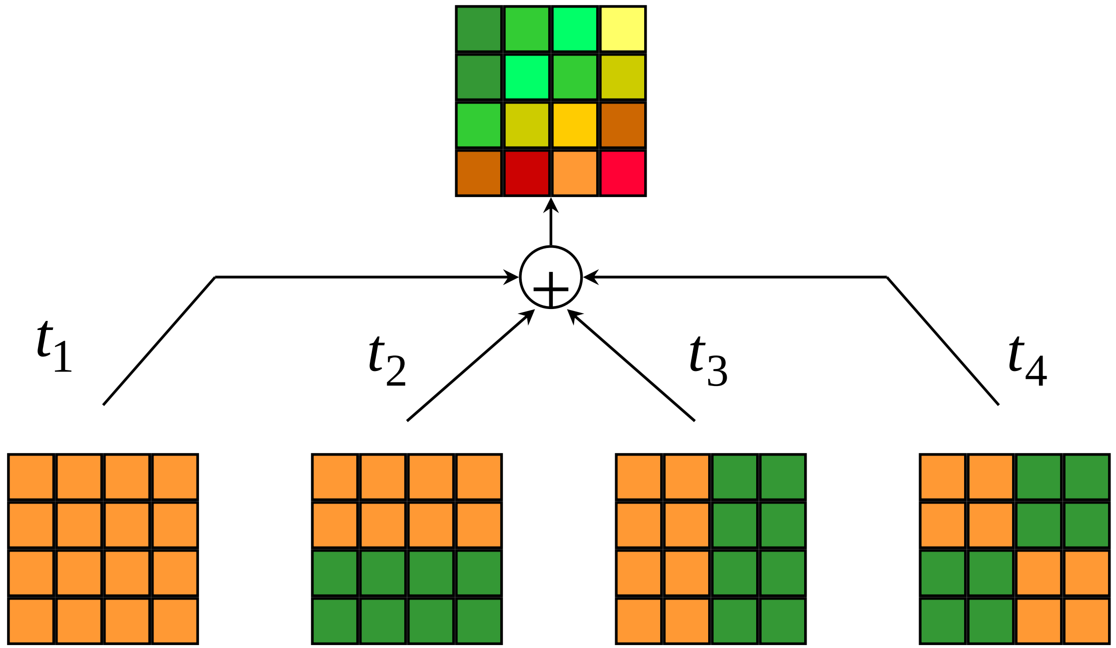
- In non-adaptive coders, a fixed set of prediction coefficients are used across the entire image.
- In adaptive coders, one updates the correlation coefficients R(k, l) and hence the prediction coefficient a_k based on local samples.
  - In *forward adaptive* predictive coder, for each block of pixels, the correlation coefficients are calculated for this block and the optimal coefficients are used.
  - In *backward adaptive* predictive coder, the correlation coefficients and consequently the prediction coefficients are updated based on the past reconstructed samples, in both the encoder and decoder.

Diagrams for Transform Coding System



Transform Coding

- Represent an image as the linear combination of some basis images and specify the linear coefficients.



Transform Basis Design

- Optimality Criteria:
  - *Energy compaction:* a few basis images are sufficient to represent a typical image.
  - *Decorrelation:* coefficients for separated basis images are uncorrelated.
- Karhunen Loeve Transform (KLT) is the Optimal transform for a given covariance matrix of the underlying signal.
- Discrete Cosine Transform (DCT) is close to KLT for images that can be modeled by a first order Markov process (*i.e.,* a pixel only depends on its previous pixel).

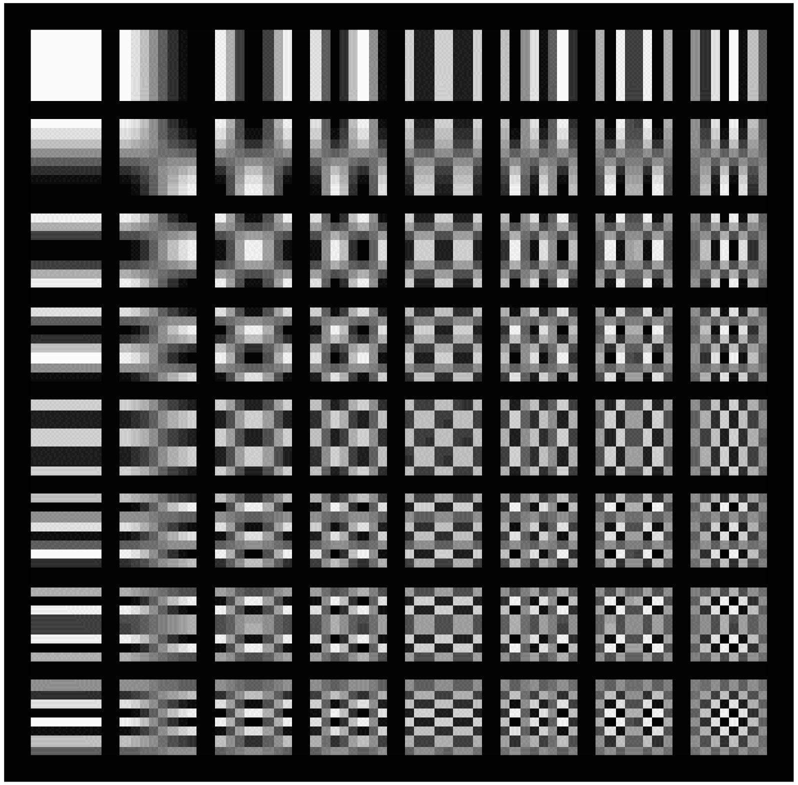
Basis Images of DCT

$$h(m,n,u,v) = \alpha(u)\alpha(v)\cos\left[\frac{(2m+1)u\pi}{2N}\right]\cos\left[\frac{(2n+1)v\pi}{2N}\right]$$

$$\text{where } \alpha(u) = \begin{cases} \sqrt{1/N} & u = 0 \\ \sqrt{2/N} & u = 1,\ldots,N-1 \end{cases}$$

$$T(u,v) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n)h(m,n,u,v)$$

$$f(m,n) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} T(u,v)h(m,n,u,v)$$



DCT on a Real Image Block

```
>>imblock = lena256(128:135,128:135)-128
imblock=
 54   68   71   73   75   73   71   45
 47   52   48   14   20   24   20   -8
 20  -10   -5  -13  -14  -21  -20  -21
-13  -18  -18  -16  -23  -19  -27  -28
-24  -22  -22  -26  -24  -33  -30  -23
-29  -13    3  -24  -10  -42  -41    5
-16   26   26  -21   12  -31  -40   23
 17   30   50   -5    4   12   10    5
```

```
>>dctblock =dct2(imblock)
dctblock=
 31.0000   51.7034    1.1673  -24.5837  -12.0000  -25.7508   11.9640   23.2873
113.5766    6.9743  -13.9045   43.2054   -6.0959   35.5931  -13.3692  -13.0005
195.5804   10.1395   -8.6657   -2.9380  -28.9833   -7.9396    0.8750    9.5585
 35.8733  -24.3038  -15.5776  -20.7924   11.6485  -19.1072   -8.5366    0.5125
 40.7500  -20.5573  -13.6629   17.0615  -14.2500   22.3828   -4.8940  -11.3606
  7.1918  -13.5722   -7.5971  -11.9452   18.2597  -16.2618   -1.4197   -3.5087
 -1.4562  -13.3225   -0.8750    1.3248   10.3817   16.0762    4.4157    1.1041
 -6.7720   -2.8384    4.1187    1.1118   10.5527   -2.7348   -3.2327    1.5799
```

In JPEG, “imblock-128” is done before DCT to shift the mean to zero

Quantization of DCT Coefficients

- Use uniform quantizer on each coefficient
- Different coefficient is quantized with different step-size (Q):
  - Human eye is more sensitive to low frequency components
  - Low frequency coefficients with a smaller Q
  - High frequency coefficients with a larger Q
  - Specified in a normalization matrix
  - Normalization matrix can then be scaled by a scale factor

Default Normalization Matrix in JPEG

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Actual step size for C(i,j): Q(i,j) = QP*M(i,j)

Example: Quantized Indices

```
>>dctblock =dct2(imblock)
dctblock=
 31.0000   51.7034    1.1673  -24.5837  -12.0000  -25.7508   11.9640    23.2873
 113.5766    6.9743  -13.9045   43.2054   -6.0959   35.5931  -13.3692  -13.0005
 195.5804   10.1395   -8.6657   -2.9380  -28.9833   -7.9396    0.8750    9.5585
  35.8733  -24.3038  -15.5776  -20.7924   11.6485  -19.1072   -8.5366  0.5125
  40.7500  -20.5573  -13.6629   17.0615  -14.2500   22.3828   -4.8940  -11.3606
   7.1918  -13.5722   -7.5971  -11.9452   18.2597  -16.2618   -1.4197    -3.5087
  -1.4562  -13.3225   -0.8750    1.3248   10.3817   16.0762    4.4157    1.1041
  -6.7720   -2.8384    4.1187    1.1118   10.5527   -2.7348   -3.2327    1.5799
```

```
>>QP=1;
>>QM=Qmatrix*QP;
>>qdct=floor((dctblock+QM/2)./(QM))
qdct =
    2    5    0   -2    0   -1    0    0
    9    1   -1    2    0    1    0    0
   14    1   -1    0   -1    0    0    0
    3   -1   -1   -1    0    0    0    0
    2   -1    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0
```

Only 19 coefficients are retained out of 64

Example: Quantized Coefficients

```
%dequantized DCT block
>>  iqdct=qdct.*QM
iqdct=
```

32	55	0	-32	0	-40	0	0
108	12	-14	38	0	58	0	0
196	13	-16	0	-40	0	0	0
42	-17	-22	-29	0	0	0	0
36	-22	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Original DCT block

dctblock=

31.0000	51.7034	1.1673	-24.5837	-12.0000	-25.7508	11.9640	23.2873
113.5766	6.9743	-13.9045	43.2054	-6.0959	35.5931	-13.3692	-13.0005
195.5804	10.1395	-8.6657	-2.9380	-28.9833	-7.9396	0.8750	9.5585
35.8733	-24.3038	-15.5776	-20.7924	11.6485	-19.1072	-8.5366	0.5125
40.7500	-20.5573	-13.6629	17.0615	-14.2500	22.3828	-4.8940	-11.360
7.1918	-13.5722	-7.5971	-11.9452	18.2597	-16.2618	-1.4197	-3.5087
-1.4562	-13.3225	-0.8750	1.3248	10.3817	16.0762	4.4157	1.1041
-6.7720	-2.8384	4.1187	1.1118	10.5527	-2.7348	-3.2327	1.5799

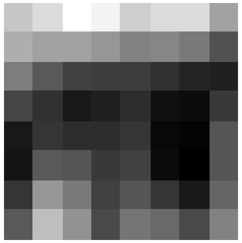
Example: Reconstructed Image

%reconstructed image block

```
>>  qimblock=round(idct2(iqdct))
```

qimblock=

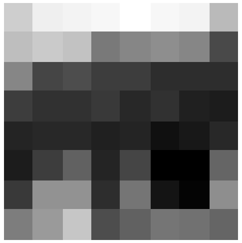
58	68	85	79	61	68	67	38
45	38	39	33	22	24	19	-2
21	2	-11	-12	-13	-19	-24	-27
-8	-19	-31	-26	-20	-35	-37	-15
-31	-17	-21	-20	-16	-39	-41	0
-33	3	-1	-14	-11	-37	-44	1
-16	32	18	-10	1	-16	-30	8
3	54	30	-6	16	11	-7	23



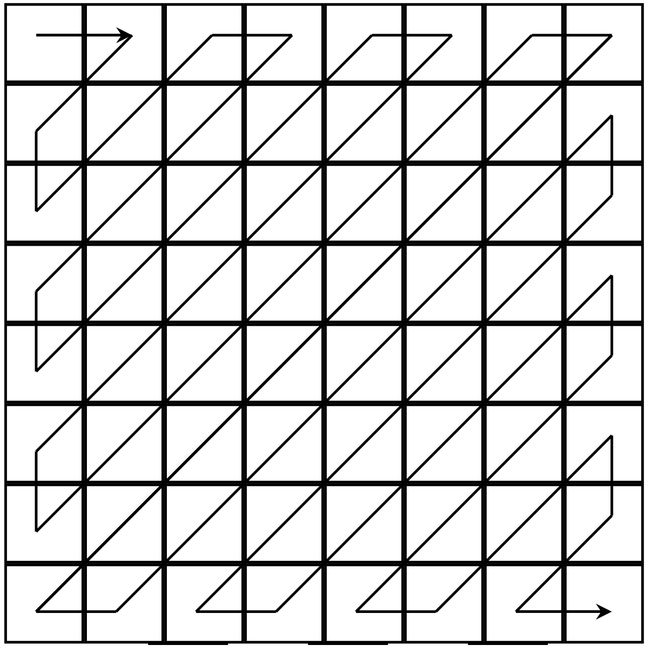
Original image block

imblock=

54	68	71	73	75	73	71	45
47	52	48	14	20	24	20	-8
20	-10	-5	-13	-14	-21	-20	-21
-13	-18	-18	-16	-23	-19	-27	-28
-24	-22	-22	-26	-24	-33	-30	-23
-29	-13	3	-24	-10	-42	-41	5
-16	26	26	-21	12	-31	-40	23
17	30	50	-5	4	12	10	5



Zig-Zag Ordering of DCT Coefficients



0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

Zig-Zag ordering: converting a 2D matrix into a 1D array, so that the frequency (horizontal+vertical) increases in this order, and the coefficient variance decreases in this order.

DCT Coefficient Distribution

Example

qdct =

2	5	0	-2	0	-1	0	0
9	1	-1	2	0	1	0	0
14	1	-1	0	-1	0	0	0
3	-1	-1	-1	0	0	0	0
2	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

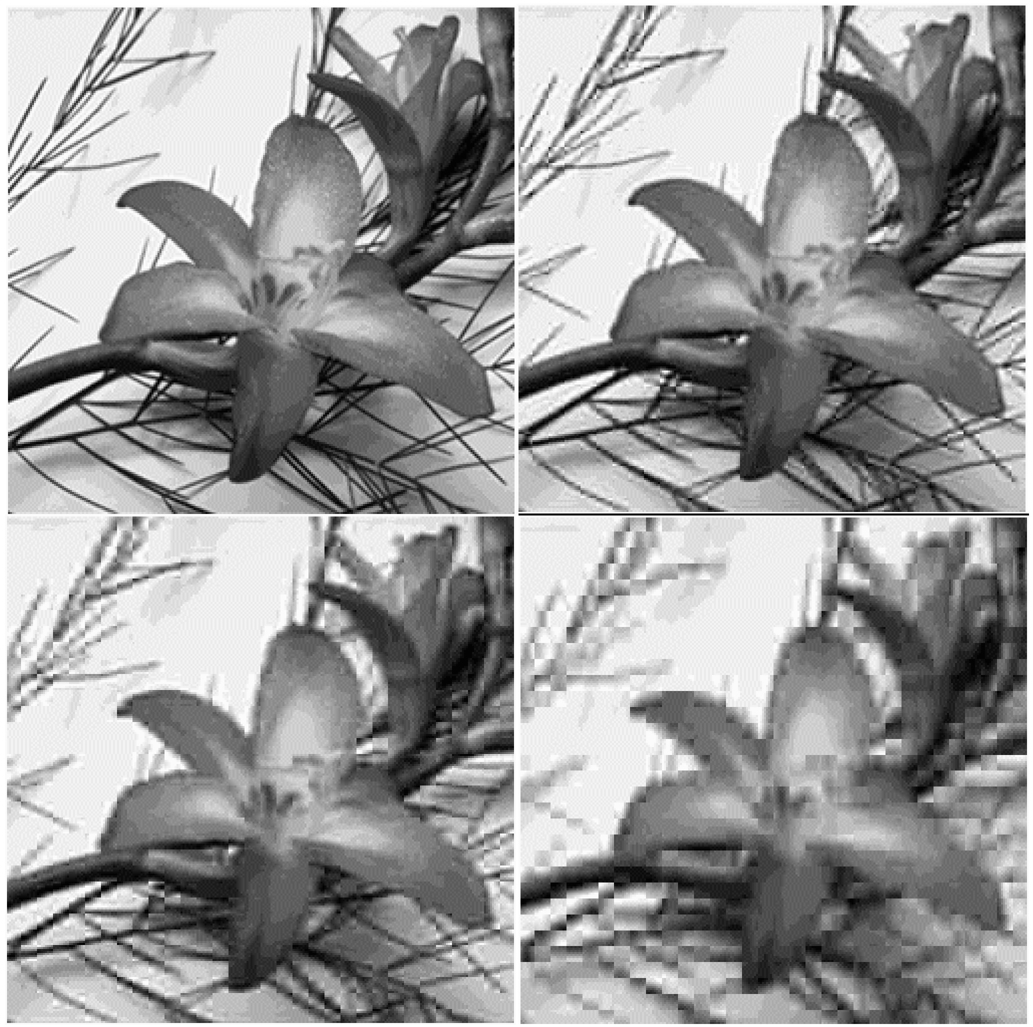
Run-length symbol representation:
{2,(0,5),(0,9),(0,14),(0,1),(1,-2),(0,-1),(0,1),(0,3),(0,2),(0,-1),(0,-1),(0,2),(1,-1),(2,-1), (0,-1), (4,-1),(0,-1),(0,1),EOB}

EOB: End of block, one of the symbol that is assigned a short Huffman codeword

Approximation by DCT Basis

Original

With 16/64 Coefficients



With 8/64 Coefficients

With 4/64 Coefficients

Coding of DCT Coefficients

- Always code the first L coefficients or coefficients in certain locations (zonal-coding)
- Code all non-zero coefficients after quantization (run-length coding) -- Used in JPEG standard
  - Runlength coding:
    - Ordering coefficients in the zig-zag order
    - Each symbol=(length-of-zero, non-zero-value)
    - Code each symbol using Huffman coding

What is JPEG

- The **Joint Photographic Expert Group** (JPEG), under both the International Standards Organization (ISO) and the International Telecommunications Union-Telecommunication Sector (ITU-T)
  - www.jpeg.org
- Has published several standards
  - JPEG: lossy coding of continuous tone still images
    - Based on DCT
  - JPEG-LS: lossless and near lossless coding of continuous tone still images
    - Based on predictive coding and entropy coding
  - JPEG2000: scalable coding of continuous tone still images (from lossy to lossless)
    - Based on wavelet transform

The 1992 JPEG Standard

- Contains several modes:
  - Baseline system (what is commonly known as JPEG!): lossy
    - Can handle gray scale or color images (8bit)
  - Extended system: lossy
    - Can handle higher precision (12 bit) images, providing progressive streams, etc.
  - Lossless version
- Baseline version
  - Each color component is divided into 8x8 blocks
  - For each 8x8 block, three steps are involved:
    - Block DCT
    - Perceptual-based quantization
    - Variable length coding: Runlength and Huffman coding

Coding of Quantized DCT Coefficients

- DC coefficient: Predictive coding
  - The DC value of the current block is predicted from that of the previous block, and the error is coded using Huffman coding
- AC Coefficients: Runlength coding
  - Many high frequency AC coefficients are zero after first few low-frequency coefficients
  - Runlength Representation:
    - Ordering coefficients in the zig-zag order
    - Specify how many zeros before a non-zero value
    - Each symbol=(length-of-zero, non-zero-value)
  - Code all possible symbols using Huffman coding
    - More frequently appearing symbols are given shorter codewords
    - For more details on the actual coding table, see Handout (Sec.8.5.3 in [Gonzalez02]
- One can use default Huffman tables or specify its own tables.
- Instead of Huffman coding, arithmetic coding can be used to achieve higher coding efficiency at an added complexity.

Coding of DC Symbols

- Example:
  - Current quantized DC index: 2
  - Previous block DC index: 4
  - Prediction error: -2
  - The prediction error is coded in two parts:
    - Which category it belongs to (Table of JPEG Coefficient Coding Categories), and code using a Huffman code (JPEG Default DC Code)
      - DC= -2 is in category “2”, with a codeword “100”
    - Which position it is in that category, using a fixed length code, length=category number
      - “-2” is the number 1 (starting from 0) in category 2, with a fixed length code of “10”.
      - The overall codeword is “10010”

JPEG Tables for Coding DC

TABLE 8.17 JPEG coefficient coding categories.

Range	DC Difference Category	AC Category
0	0	N/A
−1, 1	1	1
−3, −2, 2, 3	2	2
−7, ..., −4, 4, ..., 7	3	3
−15, ..., −8, 8, ..., 15	4	4
−31, ..., −16, 16, ..., 31	5	5
−63, ..., −32, 32, ..., 63	6	6
−127, ..., −64, 64, ..., 127	7	7
−255, ..., −128, 128, ..., 255	8	8
−511, ..., −256, 256, ..., 511	9	9
−1023, ..., −512, 512, ..., 1023	A	A
−2047, ..., −1024, 1024, ..., 2047	B	B
−4095, ..., −2048, 2048, ..., 4095	C	C
−8191, ..., −4096, 4096, ..., 8191	D	D
−16383, ..., −8192, 8192, ..., 16383	E	E
−32767, ..., −16384, 16384, ..., 32767	F	N/A

TABLE 8.18 JPEG default DC code (luminance).

Category	Base Code	Length	Category	Base Code	Length
0	010	3	6	1110	10
1	011	4	7	11110	12
2	100	5	8	111110	14
3	00	5	9	1111110	16
4	101	7	A	11111110	18
5	110	8	B	111111110	20

Coding of AC Coefficients

- Example:
  - First symbol (0,5)
    - The value ‘5’ is represented in two parts:
    - Which category it belongs to (Table of JPEG Coefficient Coding Categories), and code the “(runlength, category)” using a Huffman code (JPEG Default AC Code)
      - AC=5 is in category “3”,
      - Symbol (0,3) has codeword “100”
    - Which position it is in that category, using a fixed length code, length=category number
      - “5” is the number 4 (starting from 0) in category 3, with a fixed length code of “011”.
      - The overall codeword for (0,5) is “100011”
  - Second symbol (0,9)
    - ‘9’ in category ‘4’, (0,4) has codeword ‘1011’,’9’ is number 9 in category 4 with codeword ‘1001’ -> overall codeword for (0,9) is ‘10111001’
  - ETC

JPEG Tables for Coding AC (Run,Category) Symbols

Run/ Category	Base Code	Length	Run/ Category	Base Code	Length
0/0	**1010 (= EOB)**	**4**			
0/1	00	3	8/1	11111010	9
0/2	01	4	8/2	111111111000000	17
0/3	100	6	8/3	1111111110110111	19
0/4	1011	8	8/4	1111111110111000	20
0/5	11010	10	8/5	1111111110111001	21
0/6	111000	12	8/6	1111111110111010	22
0/7	1111000	14	8/7	1111111110111011	23
0/8	1111110110	18	8/8	1111111110111100	24
0/9	1111111110000010	25	8/9	1111111110111101	25
0/A	1111111110000011	26	8/A	1111111110111110	26
1/1	1100	5	9/1	111111000	10
1/2	111001	8	9/2	1111111110111111	18
1/3	1111001	10	9/3	1111111111000000	19
1/4	111110110	13	9/4	1111111111000001	20
1/5	11111110110	16	9/5	1111111111000010	21
1/6	1111111110000100	22	9/6	1111111111000011	22
1/7	1111111110000101	23	9/7	1111111111000100	23
1/8	1111111110000110	24	9/8	1111111111000101	24
1/9	1111111110000111	25	9/9	1111111111000110	25
1/A	1111111110001000	26	9/A	1111111111000111	26
2/1	11011	6	A/1	111111001	10
2/2	11111000	10	A/2	1111111111001000	18
2/3	1111110111	13	A/3	1111111111001001	19
2/4	1111111110001001	20	A/4	1111111111001010	20
2/5	1111111110001010	21	A/5	1111111111001011	21
2/6	1111111110001011	22	A/6	1111111111001100	22
2/7	1111111110001100	23	A/7	1111111111001101	23

TABLE 8.19
JPEG default AC code (luminance) *(continues on next page)*.

JPEG Performance for B/W images

65536 Bytes
8 bpp

4839 Bytes
0.59 bpp
CR=13.6

3037 Bytes
0.37 bpp
CR=21.6

1818 Bytes
0.22 bpp
CR=36.4

JPEG for Color Images

- Color images are typically stored in (R,G,B) format
- JPEG standard can be applied to each component separately
  - Does not make use of the correlation between color components
  - Does not make use of the lower sensitivity of the human eye to chrominance samples
- Alternate approach
  - Convent (R,G,B) representation to a YCbCr representation
    - Y: luminance, Cb, Cr: chrominance
  - Down-sample the two chrominance components
    - Because the peak response of the eye to the luminance component occurs at a higher frequency (3-10 cpd) than to the chrominance components (0.1-0.5 cpd). (Note: cpd is cycles/degree)
- JPEG standard is designed to handle an image consists of many (up to 100) components

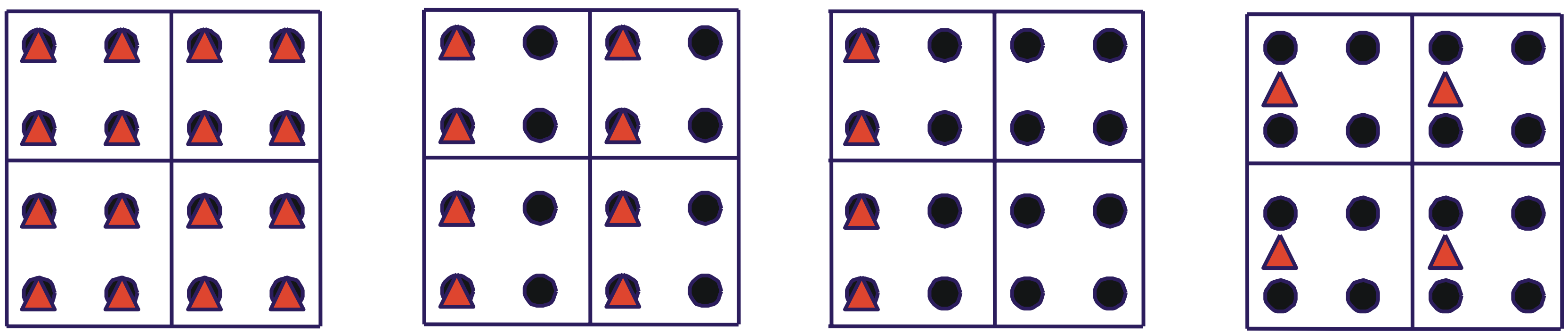
RGB <-> YCbCr Conversion

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ 128 \\ 128 \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.000 & -0.001 & 1.402 \\ 1.000 & -0.344 & -0.714 \\ 1.000 & 1.772 & 0.001 \end{bmatrix} \begin{bmatrix} Y \\ C_b - 128 \\ C_r - 128 \end{bmatrix}$$

Note: Cb ~ Y-B, Cr ~ Y-R, are known as color difference signals.

Chrominance Subsampling



4:4:4
For every 2x2 Y Pixels
4 Cb & 4 Cr Pixel
(No subsampling)

4:2:2
For every 2x2 Y Pixels
2 Cb & 2 Cr Pixel
(Subsampling by 2:1
horizontally only)

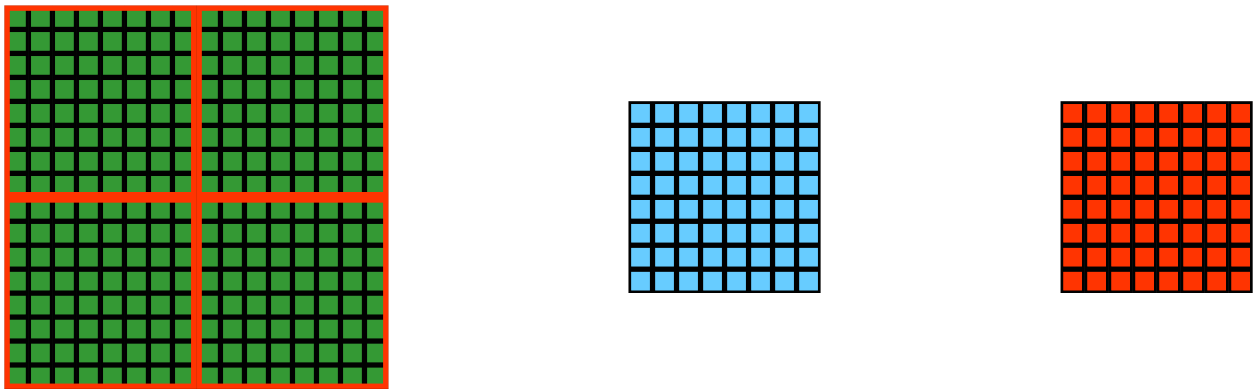
4:1:1
For every 4x1 Y Pixels
1 Cb & 1 Cr Pixel
(Subsampling by 4:1
horizontally only)

4:2:0
For every 2x2 Y Pixels
1 Cb & 1 Cr Pixel
(Subsampling by 2:1 both
horizontally and vertically)

● Y Pixel ▲ Cb and Cr Pixel

4:2:0 is the most common format

Coding Unit in JPEG



4 8x8 Y blocks   1 8x8 Cb blocks   1 8x8 Cr blocks

Each basic coding unit (called a data unit) is a 8x8 block in any color component. In the interleaved mode, 4 Y blocks and 1 Cb and 1 Cr blocks are processed as a group (called a minimum coding unit or MCU) for a 4:2:0 image.

Default Quantization Table

For luminance

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

For chrominance

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

The encoder can specify the quantization tables different from the default ones as part of the header information

Performance of JPEG

- For color images at 24 bits/pixel (bpp)
  - 0.25-0.5 bpp: moderate to good
  - 0.5-0.75 bpp: good to very good
  - 0.75-1.5 bpp: excellent, sufficient for most applications
  - 1.5-2 bpp: indistinguishable from original
  - From: G. K. Wallace: The JPEG Still picture compression standard, *Communications of ACM*, April 1991.
- For grayscale images at 8 bpp
  - 0.5 bpp: excellent quality

JPEG Performance

487x414 pixels,
Uncompressed, 600471 Bytes,24 bpp
85502 Bytes, 3.39 bpp, CR=7

487x414 pixels
41174 Bytes, 1.63 bpp, CR=14.7

JPEG Pros and Cons

- Pros
  - Low complexity
  - Memory efficient
  - Reasonable coding efficiency

- Cons
  - Single resolution
  - Single quality
  - No target bit rate
  - Blocking artifacts at low bit rate
  - No lossless capability
  - Poor error resilience
  - No tiling
  - No regions of interest

JPEG2000 Features

- Improved coding efficiency
- Full quality scalability
  - From lossless to lossy at different bit rate
- Spatial scalability
- Improved error resilience
- Tiling
- Region of interests
- More demanding in memory and computation time

What is Scalability?

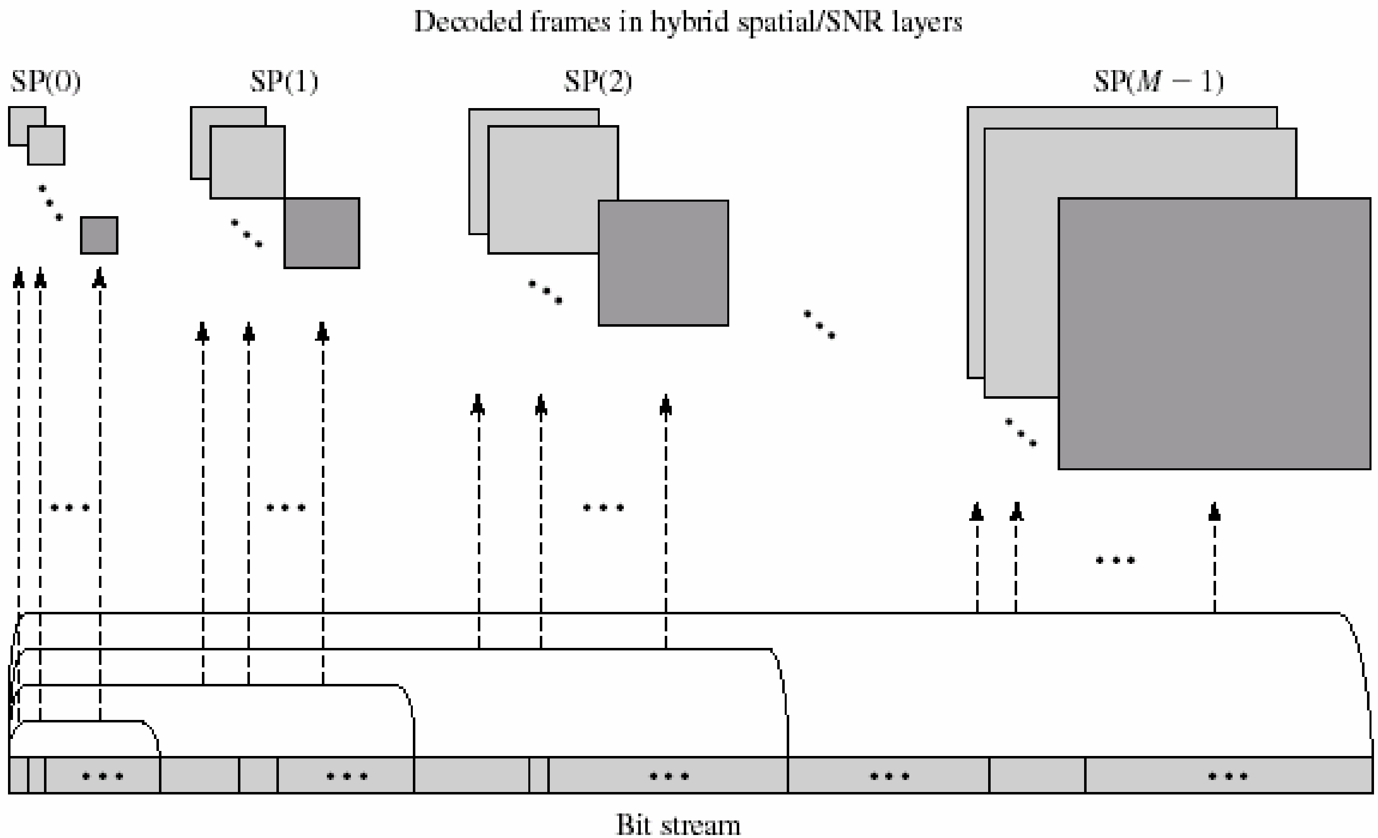


Figure 11.7 $N \times M$ layers of combined spatial/quality scalability. Reprinted from I. Sodagar, H.-J. Lee, P. Hatrack, and Y.-Q. Zhang, Scalable wavelet coding for synthetic/natural hybrid images, *IEEE Trans. Circuits Syst. for Video Technology* (March 1999), 9:244–54. Copyright 1999 IEEE.

Quality Scalability of JPEG2000

(a) (b) (c)

▲ 17. Example of SNR scalability. Part of the decompressed image "bike" at (a) 0.125 b/p, (b) 0.25 b/p, and (c) 0.5 b/p.

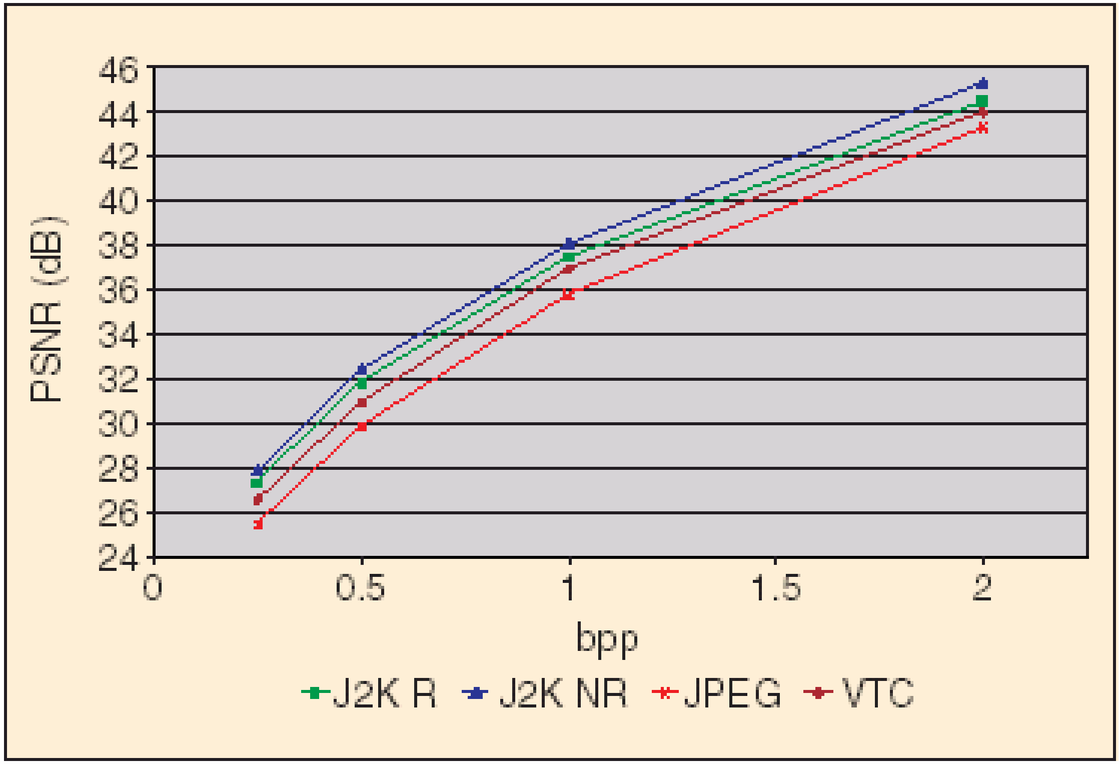
Figures in this slide are extracted from: A. Skodras, C. Christopoulos, T. Ebrahimi, The JPEG2000 Still Image Compression Standard, IEEE Signal Processing Magazine, Sept. 2001.

Spatial Scalability of JPEG2000

▲ 18. Example of the progressive-by-resolution decoding for the color image "bike."

From [skodras01]

JPEG2000 vs. JPEG: Coding Efficiency

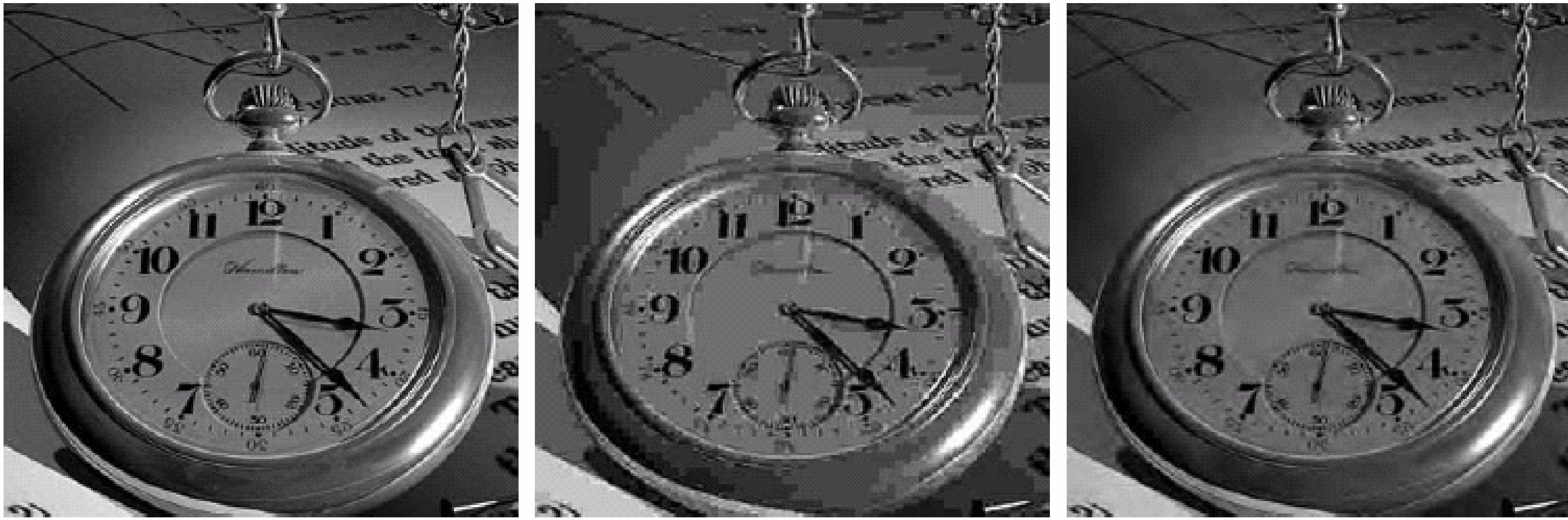


▲ *19. PSNR results for the lossy compression of a natural image by means of different compression standards.*

From [skodras01]

J2K R: Using reversible wavelet filters; J2K NR: Using non-reversible filter; VTC: Visual texture coding for MPEG-4 video

Example Image

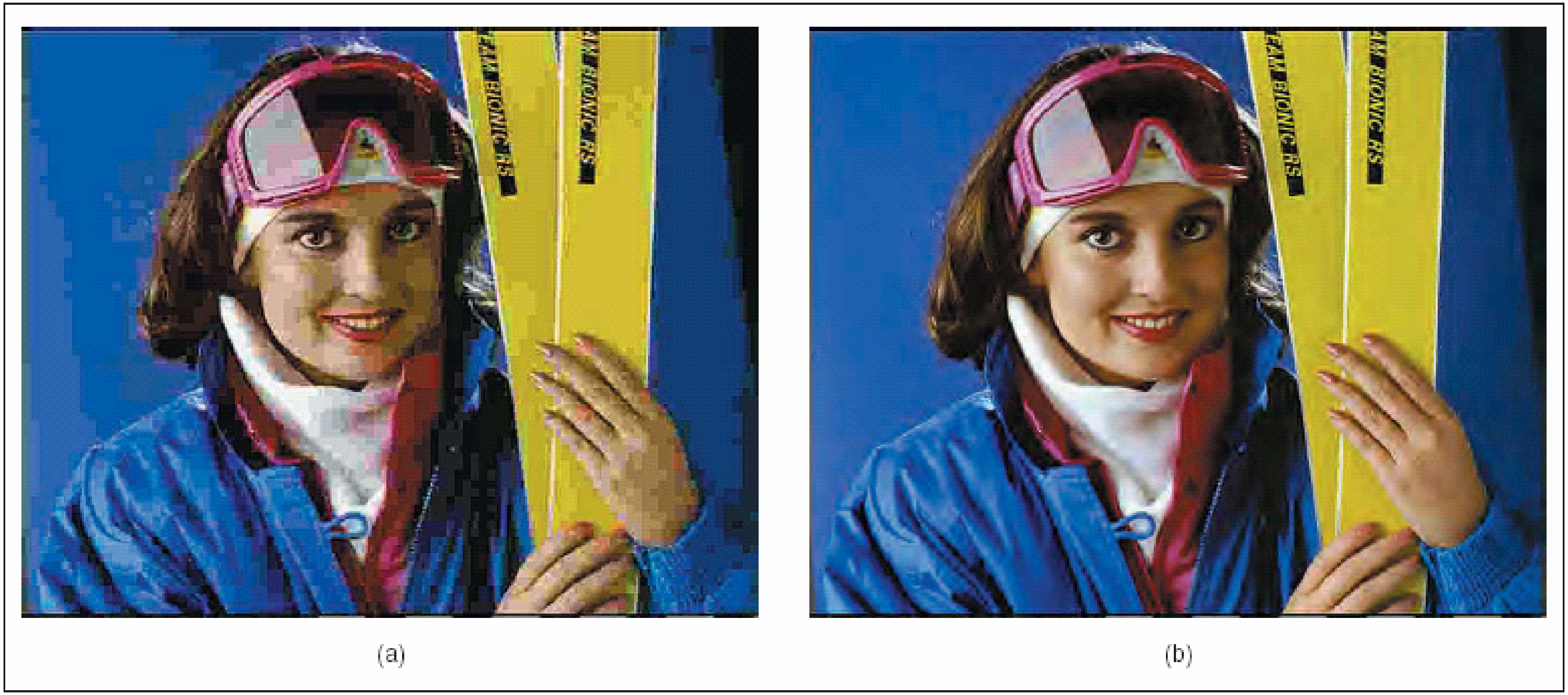


(a) (b) (c)

▲ 20. Image "watch" of size 512 × 512 (courtesy of Kevin Odhner): (a) original, and reconstructed after compression at 0.2 b/p by means of (b) JPEG and (c) JPEG 2000.

From [skodras01]

Another Example



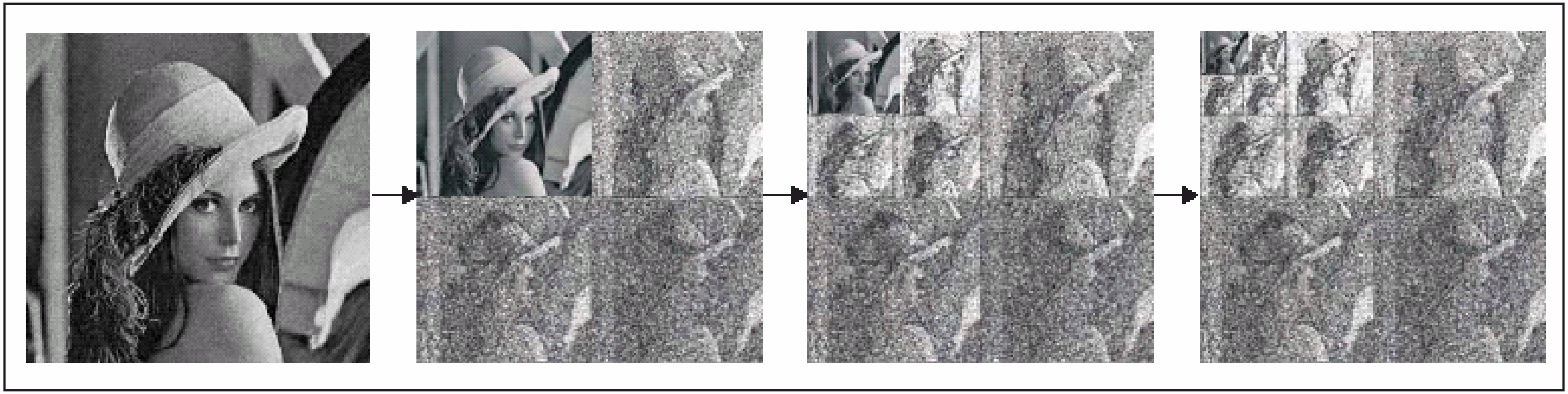
▲ 21. Reconstructed image "ski" after compression at 0.25 b/p by means of (a) JPEG and (b) JPEG 2000.

From [skodras01]

How J2K Achieves Scalability?

- Core: Wavelet transform
  - Yields a multi-resolution representation of an original image
- Still a transform coder
  - Block DCT is replaced by a full frame wavelet transform
    - Also known as subband or wavelet coder
  - Wavelet coefficients are coded bit plane by bit plane
  - Spatial scalability can be achieved by reconstructing from only low resolution wavelet coefficients
  - Quality scalability can be achieved by decoding only partial bit planes

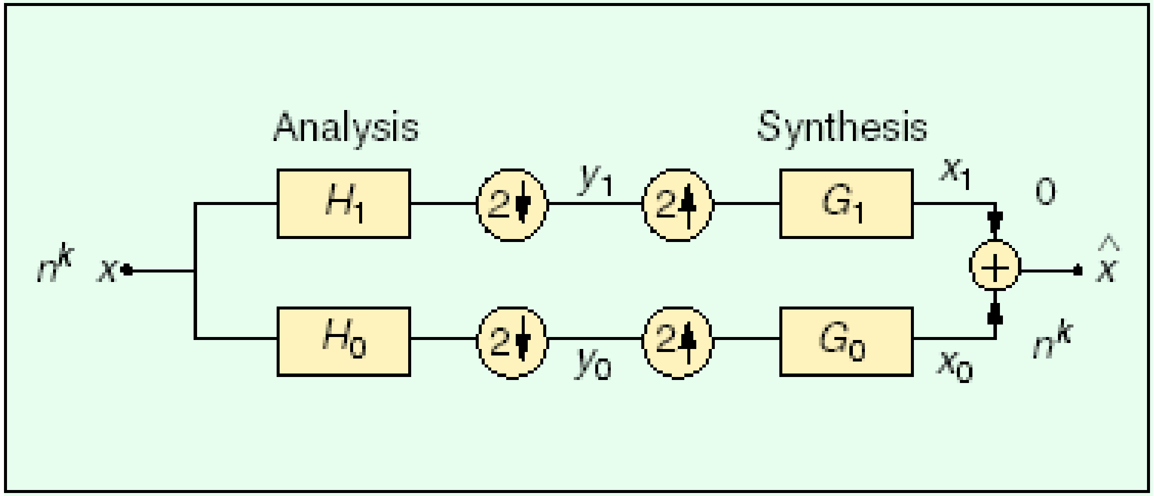
Wavelet Decomposition



▲ 6. *Three-level dyadic wavelet decomposition of the image "Lena."*

From [skodras01]

Two Band Subband Decomposition

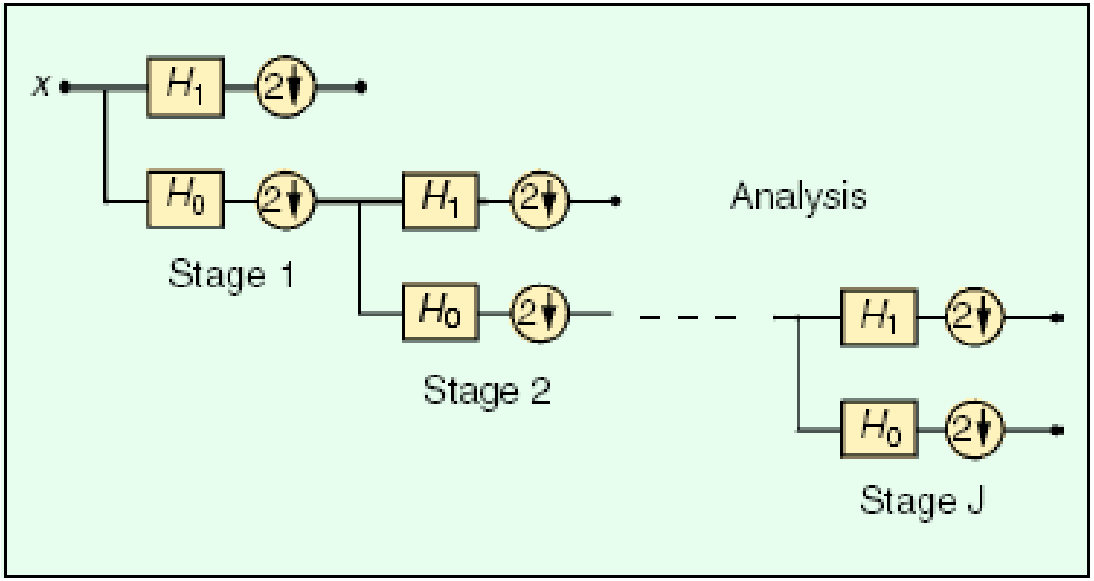


▲ 1. Multirate filter bank. Two channel analysis followed by synthesis.

From [Vetterli01]

H0: Lowpass filter, y0: a blurred version of x
H1: Highpass filter, y1: edges in x

Wavelet Transform = Subband Tree

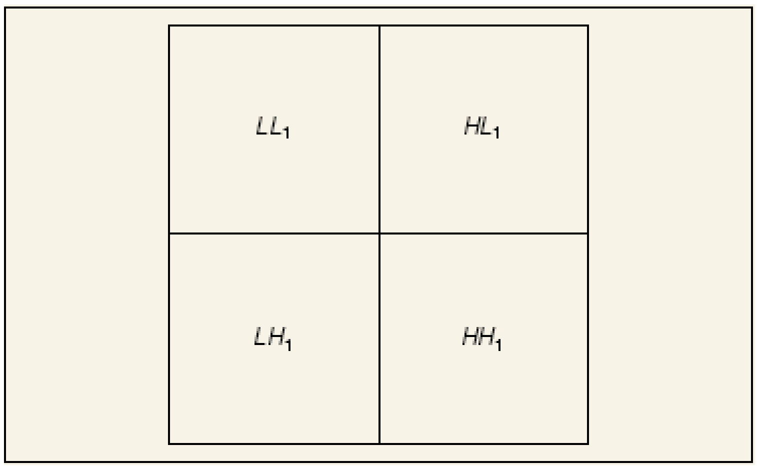


▲ 3. Iterated filter bank. The lowpass branch gets split repeatedly to get a discrete-time wavelet transform.

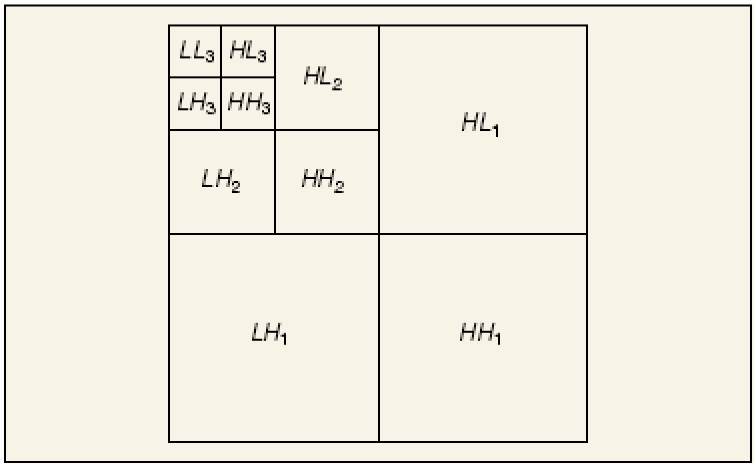
From [Vetterli01]

The above structure can be applied to rows of an image first, and then columns, forming 2D wavelet decomposition

Wavelet Transform for Images



▲ *4. The subband labeling scheme for a one-level, 2-D wavelet transform.*

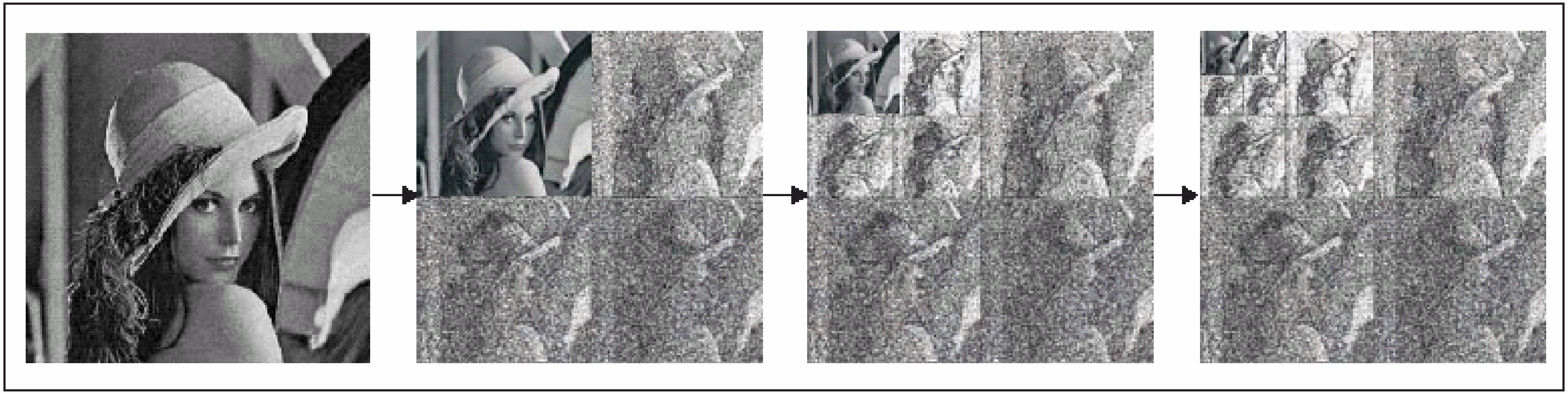


▲ *6. The subband labeling scheme for a three-level, 2-D wavelet transform.*

From [Usevitch01]

2D wavelet transform is accomplished by applying the 1D decomposition along rows of an image first, and then columns.

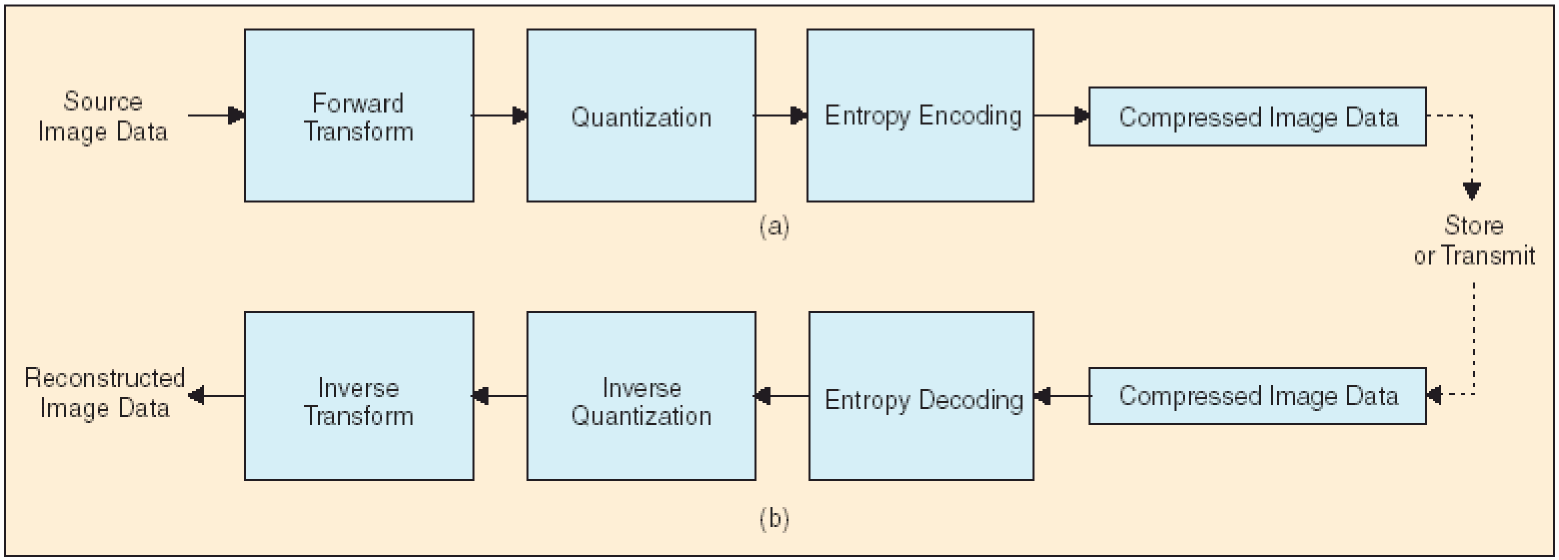
Wavelet Decomposition



▲ 6. *Three-level dyadic wavelet decomposition of the image "Lena."*

From [skodras01]

JPEG2000 Codec Block Diagram



▲ 2. *General block diagram of the JPEG 2000 (a) encoder and (b) decoder.*

- Quantization: Each subband may use a different step-size. Quantization can be skipped to achieve lossless coding
- Entropy coding: Bit plane coding is used, the most significant bit plane is coded first.
- Quality scalability is achieved by decoding only partial bit planes, starting from the MSB. Skipping one bit plane while decoding = Increasing quantization stepsize by a factor of 2.

Homework

1.Consider an image predictor: $f(m, n) = \alpha.f(m, n\text{-}1)+\beta f(m\text{-}1, n)+\gamma f(m\text{-}1, n\text{-}1)$, where m and n correspond to row and column indices, respectively. Assume that the image values have been scaled and shifted so that they have zero mean and unit variance. Suppose the correlation coefficients are ρ for two horizontally or vertically adjacent pixels, and ρ^2 for two diagonally or anti–diagonally adjacent pixels. Find the optimal values for α, β, and γ that will minimize the mean square error of prediction, and determine the corresponding minimal error.

2.Consider the following image compression scheme. The pixels in an image is scanned from left to the right and from the top to the bottom. Each new pixel is predicted by the average of the pixel above and the one to the left. Let f and $\hat{f}$ represent the original and $\hat{f}$ the predicted values, and $e = f - \hat{f}$ the prediction error. The prediction error is quantized to "0", "B", or "-B" according to:

$$\hat{e} = \begin{cases} -B & e < -T \\ 0 & -T \le e \le T \\ B & e > T \end{cases}$$

Fig. 2

0	0	1	5	6
0	0	1	5	6
2	2	4	7	8
3	3	7	4	2
6	6	5	1	0

All the possible quantized symbols ("0", "B" and "-B") are assigned binary codewords using Huffman coding.

For the image given in Fig. 2:

(a) Determine the predicted image. For simplicity, ignore the pixels in the first row and column. (i.e. Assuming those pixels are coded directly and losslessly). Also, for simplicity, always use the original values in the previous positions for prediction (i.e. using open loop predictive coding)

(b) Determine the prediction error image.

(c) Determine the quantized error image, assuming B = 3, T = 2.

(d) Determine the reconstructed image.

(f) Design a Huffman codebook for all the symbols based on their occurrence frequencies in this example. Calculate the average bit rate (bits/pixel) using this method.

(g) Note that in practice, one cannot assume that the original values for the previous pixels are available for prediction. Determine the reconstructed image without assuming the original values are available for the reference pixels used for prediction. That is, put the reconstructed value (the predicted value plus the quantized error values) for each pixel back to its position immediately, and when predicting a new pixel, use the reconstructed values for its past neighbors (This is known as closed-loop predictive coding).

Reading

- Prof. Yao Wang's Lecture Notes, Section 9.4.2 – 9.5
- R. Gonzalez, "Digital Image Processing," Section 8.5 - 8.6
- A.K. Jain, "Fundamentals of Digital Image Processing," Section 11.4 – 11.5