

### Assignment 3

Due Feb 10, 2012

#### 1. Nishimura 3.3

#### 2. Precession in Tops (brain teasers)

- Explain why only *spinning* tops precess. That is, explain why static tops will not.
- Suppose you wanted to create an approximate macro version of the precessing quantum nuclear magnetic moment. Dr. Bore thinks you could do this by embedding a permanent magnetic (PM) moment within a toy top. Would you embed the PM perpendicular or parallel to the rotation axis? If you altered the applied external magnetic field, do you believe you would alter the mechanical precession frequency of the top— that is, the precession would change from that determined merely by gravity? Could you create both positive and negative gyromagnetic ratios?

#### 3. Magnetic field steps

Suppose you placed two test tubes (each 50 mL) at 1.5 T and one test tube (50 mL) at 1.6 T. Suppose both coils were visible with equal sensitivity to the RF coil. Sketch the intensity of the received signal as a function of frequency.

#### 4. RF Field

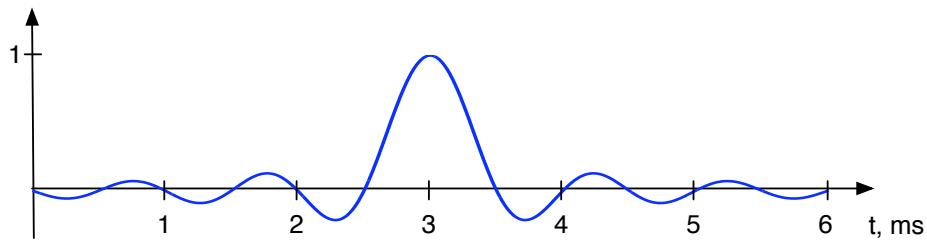
- Find the amplitude of an RF pulse that performs a 90 degree excitation in exactly 1 ms at 1.5T.
- Find the amplitude of an RF pulse that performs a 90 degree excitation in exactly 1 ms at 3T.

#### 5. Gradient field bandwidth

Sketch the spectrum of received signal on a 1.5 T MRI scanner versus frequency for an object of diameter = 25 cm with a gradient field of peak amplitude 4 G/cm.

#### 6. RF Excitation.

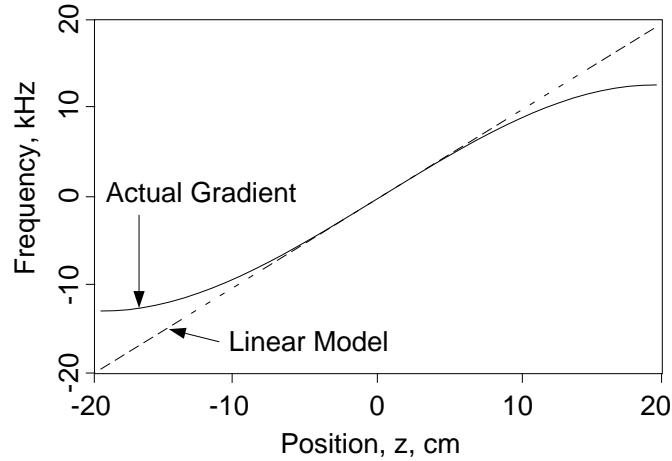
If we apply an RF waveform in addition to a gradient  $G$ , we can excite a slice through an object. Assume that the RF envelope  $B_1(t)$  is the 6 ms segment of a sinc(), as shown



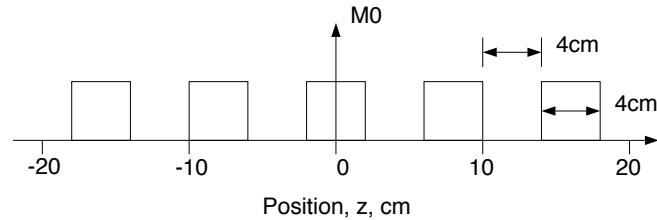
Write an expression for  $B_1(t)$ , and its Fourier transform. Assuming a gradient  $G$  of 0.94 G/cm, how wide is the excited slice?

**7. Non-Linear gradients.** One of the key elements in MRI is the use of a gradient field  $G$  which establishes a linear relationship between resonance frequency and position. While the linear model is convenient for analysis, real gradients are seldom exactly linear. In this problem we will look at some of the consequences of gradient non-linearity. This is a REAL situation in every scanner!

- (a) Consider a gradient system with the response shown by the solid line. A linear model is shown as the dashed line.



Assume the object is a sequences of rectangles of uniform intensity.



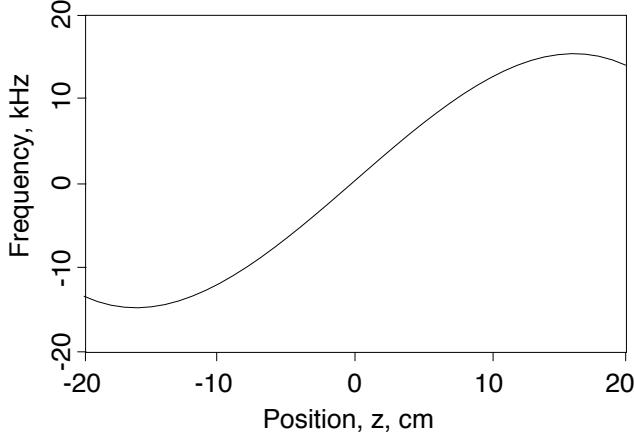
Sketch the one dimensional image we would get if we encode using the real gradient (solid line) but use the linear approximation (dashed line) when we reconstruct the data (i.e. assign spatial positions to different frequencies.) Things to look for are spatial distortion, and intensity variations.

- (b) The non-linearity in the gradient can be measured, and then used to more accurately reconstruct the data. Assume that the gradient field produces a frequency

$$\omega(z) = \gamma(G_{ideal}z + Hz^3) \quad (1)$$

where  $G_{ideal} = 0.235$  G/cm, and  $H = -1.9 \times 10^{-4}$  G/(cm<sup>3</sup>), and  $z$  ranges from  $\pm 20$  cm. If we have a data acquisition window of 10 ms, we can resolve frequencies of 100 Hz. What spatial resolution does this provide at  $z = 0$ , 10, and 20 cm?

- (c) Would this gradient profile work for spatial encoding for MRI? Why or why not? Assume that the object extends from -20 cm to 20 cm.



## 8. Design of Time-Optimal Gradient Waveforms

A key element in pulse sequence design in MRI is the design of the gradient waveforms. A very common problem is to design a gradient waveform with a certain desired area. To minimize the duration of the sequence, we often would like the waveform to be as short as possible. However, the gradient pulse has to be realizable by the system and therefore must satisfy the system constraints of maximum gradient amplitude and slew-rate.

Given that the system is limited to maximum gradient amplitude  $G_{\max} = 4 \text{ G/cm}$  and a slew-rate of  $S_{\max} = \frac{dG(t)}{dt} = 15000 \text{ G/cm/s}$

- (a) Find the shortest gradient waveform that has an area of  $\int_t G(\tau) d\tau = 8e-4 \text{ G*s/cm}$ . Draw the waveform. Point out the maximum gradient, and its duration. What is the shape of the waveform?
- (b) Find the shortest gradient waveform that has an area of  $\int_t G(\tau) d\tau = 16e-4 \text{ G*s/cm}$ . Draw the waveform. Point out the maximum gradient, and its duration. What is the shape of the waveform?
- (c) **Matlab assignment:** In this part, we will write a matlab function to design minimum-time gradient waveforms. This function will be used later in class, so make sure you get it right. Write a function that accepts the desired gradient area (in  $\text{G*s/cm}$ ), the maximum gradient amplitude (in  $\text{G/cm}$ ), the maximum slew-rate (in  $\text{G/cm/s}$ ) and sampling interval (in s). The function will return (a discrete) shortest gradient waveforms that satisfy the constraints:

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g = minTimeGradientArea(area, Gmax, Smax, dt);
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- Plot the result of the function for area=6e-4, Gmax=4, Smax = 15000, dt=4e-6;
- Plot the result of the function for area=6e-4, Gmax=1, Smax = 5000, dt=4e-6;