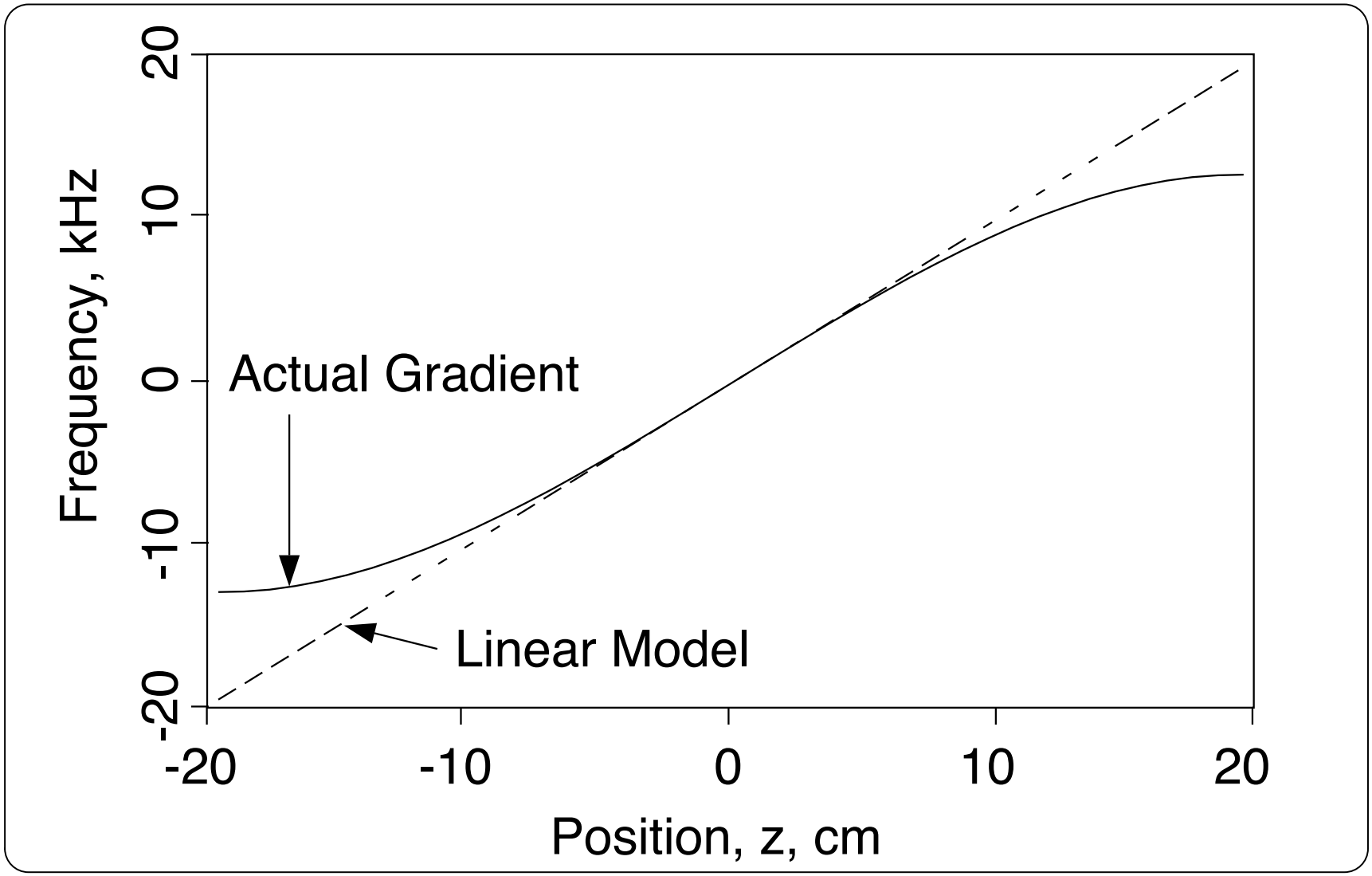
Followup on Homework

- Gradient non-Linearities

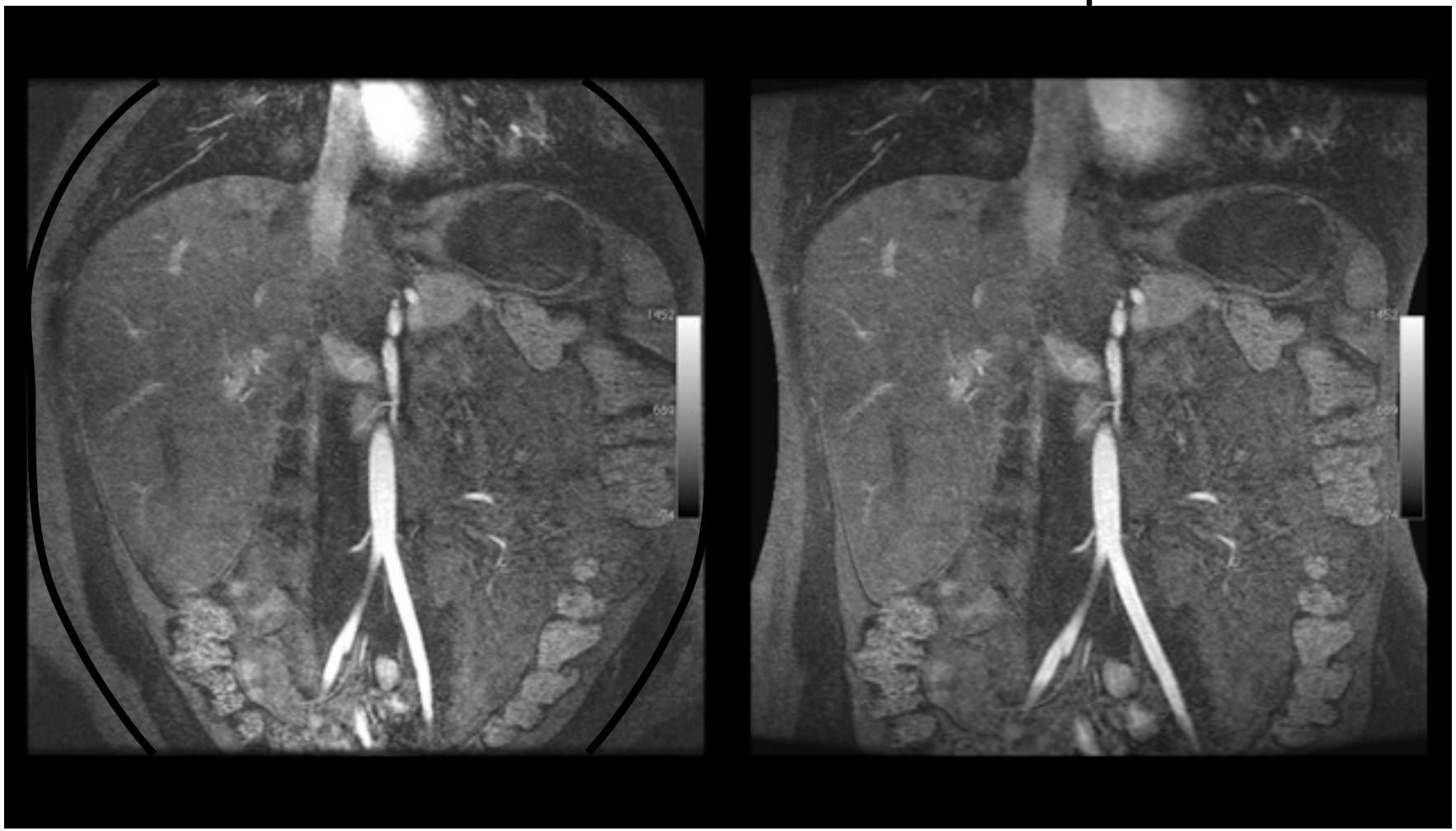


Followup on Homework

- Gradient non-Linearities

Before correction

Grad-warp correction



Followup on Last time

- Magnetization:

$$M_0 = \frac{N\gamma^2\hbar^2 I_z(I_z+1)B_0}{3kT}$$

linear

quadratic

linear

Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

Jan H. Ardenkjær-Larsen*, Björn Fridlund, Andreas Gram, Georg Hansson, Lennart Hansson, Mathilde H. Lerche, Rolf Servin, Mikkel Thaning, and Klaes Golman

Amersham Health Research and Development AB, Medeon, SE-205 12 Malmö, Sweden

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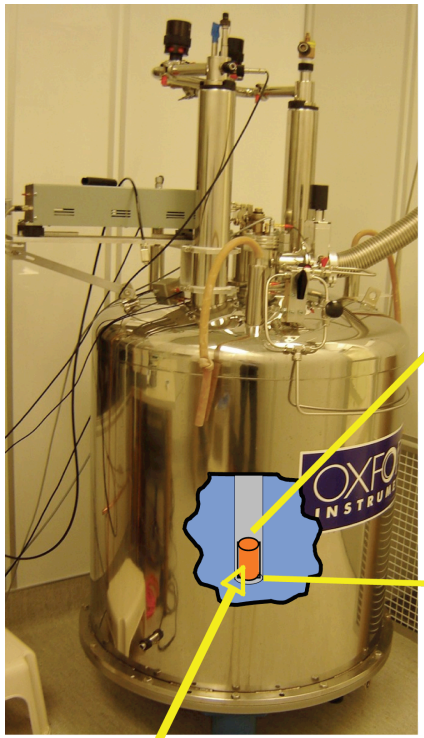
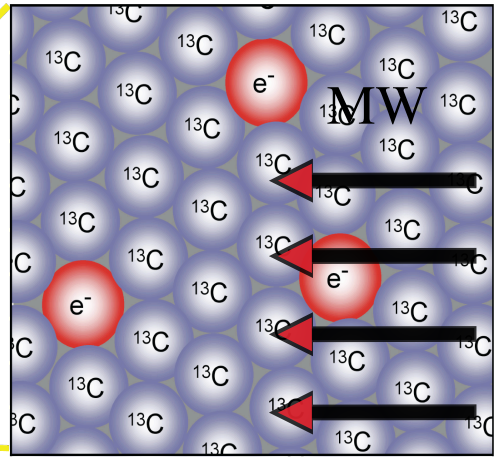


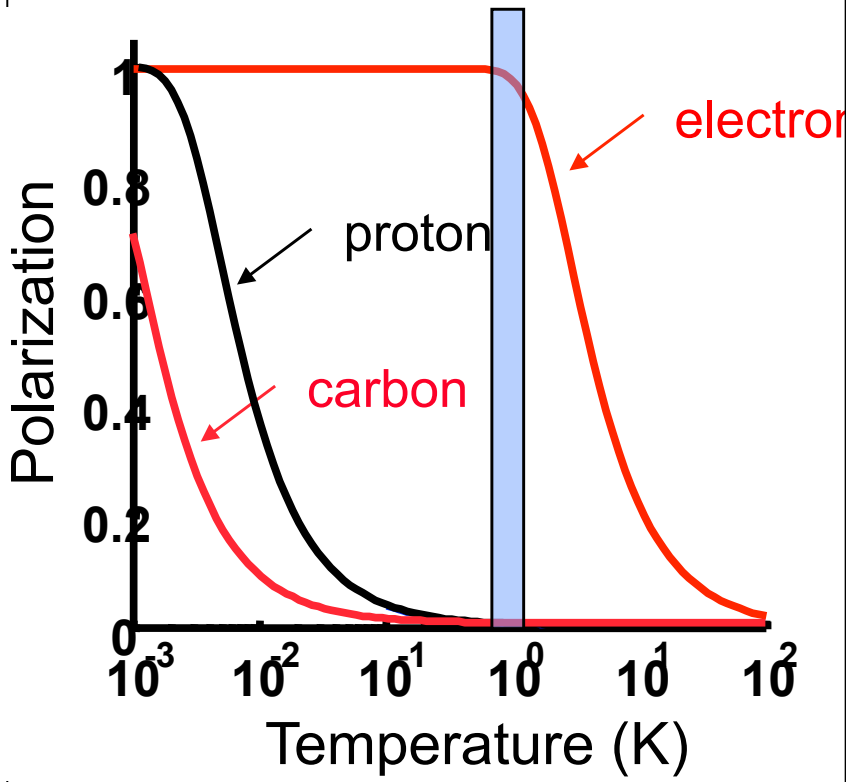
3.35T, ~1.2K

Solid material doped with unpaired electrons



P_e= 94% and P_C= 0.086%

Microwaves transfer the polarization from electrons to nuclei



Slide: Simon Hu, UCSF

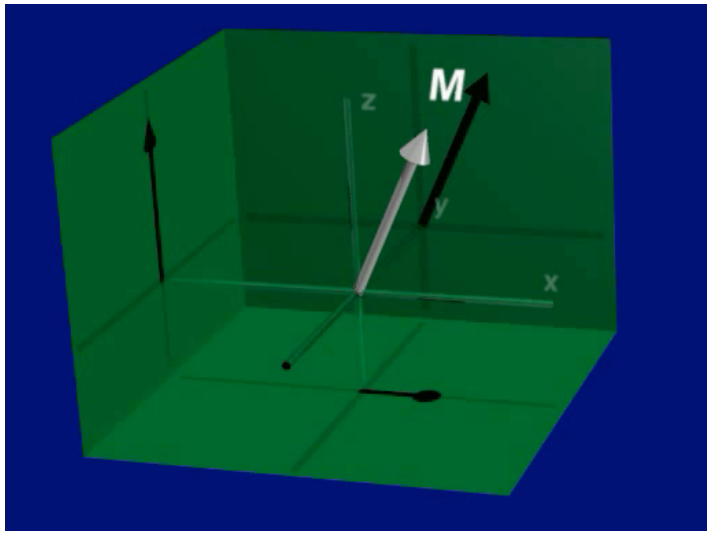
Precession

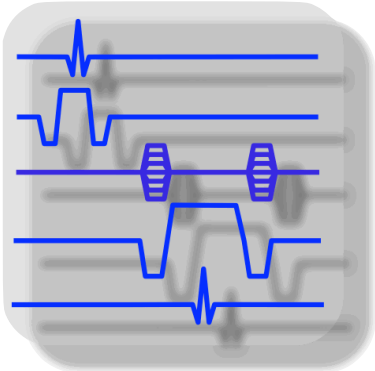
- Recall from Last time:

$$\vec{S} + \vec{\mu} + \vec{B} \Rightarrow \text{Precession}$$

Solution: $\omega = \gamma |B|$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$





Principles of MRI

EE225E / BIO265

Lecture 09

Instructor: Miki Lustig
UC Berkeley, EECS

Mathematical Description of MRI

- Three Elements:
  - Precession about $\vec{B}$ (all fields)
  - Transverse decay
  - Longitudinal recovery

Mathematical Description of MRI

- Plan:

  1) Derive Math for each element

  2) Put together : e.g., the BLOCH equation

  3) Solve the Bloch eqn. for special cases

     a) Excitation CH. 6 (later)

     b) Reception CH. 5 (first)

        i) Derive k-space (AGAIN!!!)

        ii) Pulse sequence

        iii)Sampling

Precession

- We apply fields: B_0, B_1, G

Precession

Magnetization is:

$$\vec{M} = [M_x, M_y, M_z]^T$$

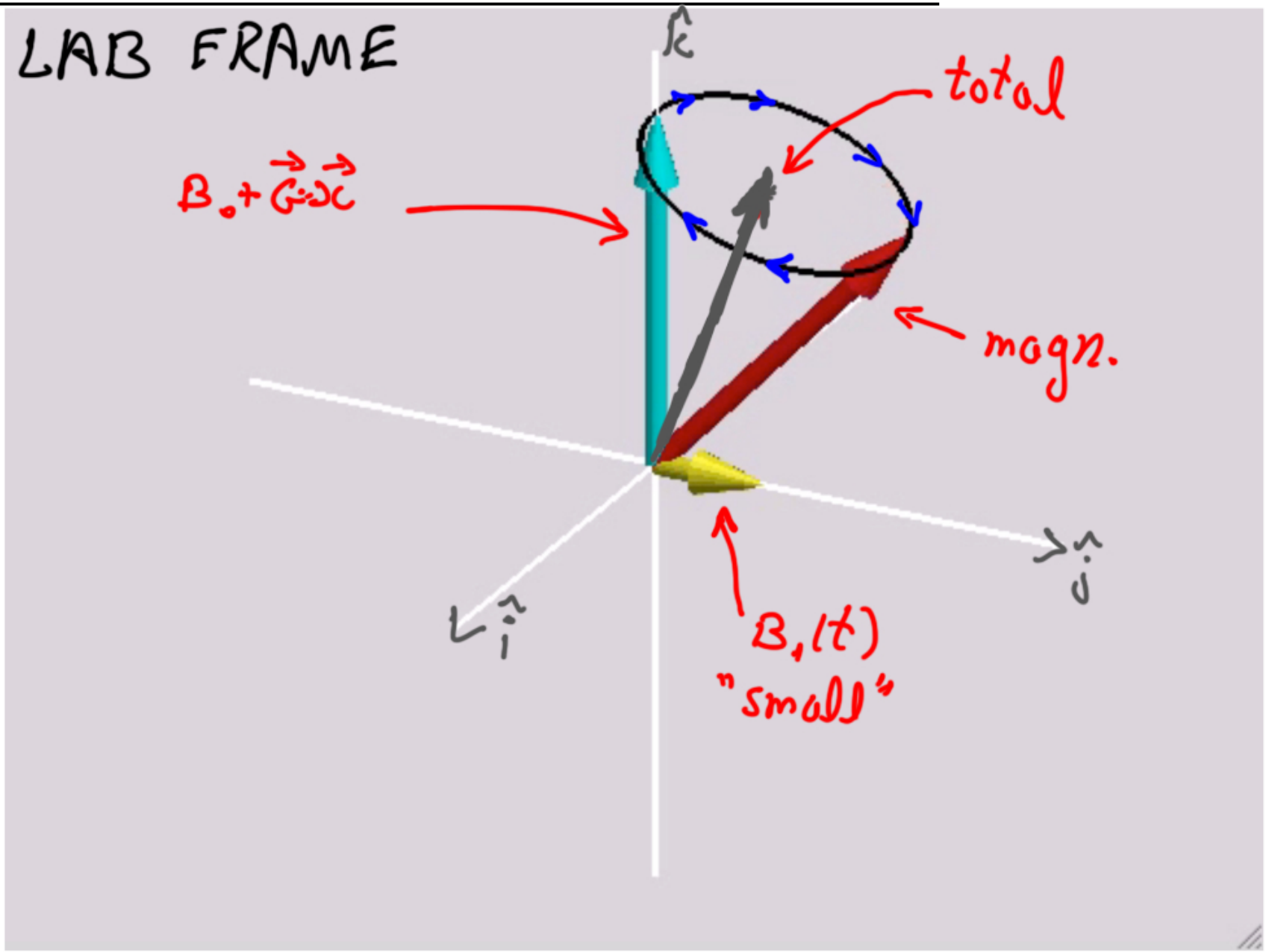
- $\vec{M}$ Precesses around $\vec{B}$
- Frequency of rotation is

$$\omega = \gamma |\vec{B}|$$

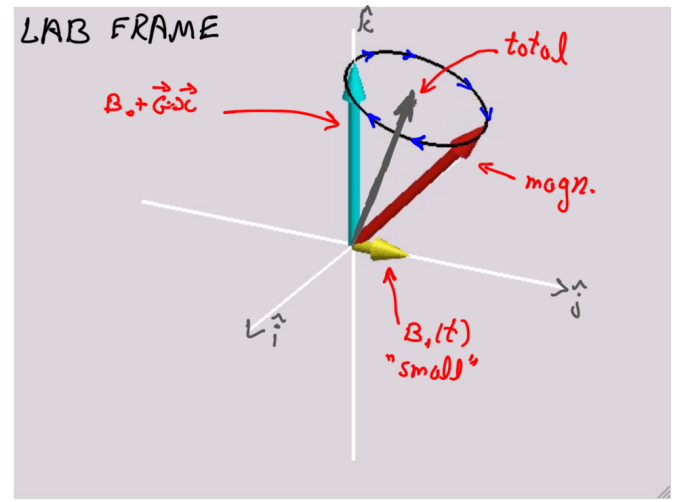
normal

- Axis of rotation is $\vec{n} \frac{\vec{B}}{|B|}$

Precession


LAB FRAME
$\hat{k}$
total
$B_0 + \vec{G} \cdot \vec{x}$
magn.
$\hat{j}$
$\hat{i}$
$B_1(t)$
"small"

Precession



- Described by cross product

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M}$$

- "-" Due to negative gyromagnetic ratio of protons

  or:

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

- B_0 Dominates! Hard to see other terms

Rotating Frame

- Change coordinates:

$$[\hat{i}_r,\ \hat{j}_r,\ \hat{k}_r]^T = [\hat{i}\cos\omega_0 t,\ \hat{j}\sin\omega_0 t,\ \hat{k}]^T$$

- In the rotating frame at w_0:

Rotating Frame

- For $\omega_0 = \gamma B_0$ MAIN FIELD GOES AWAY!

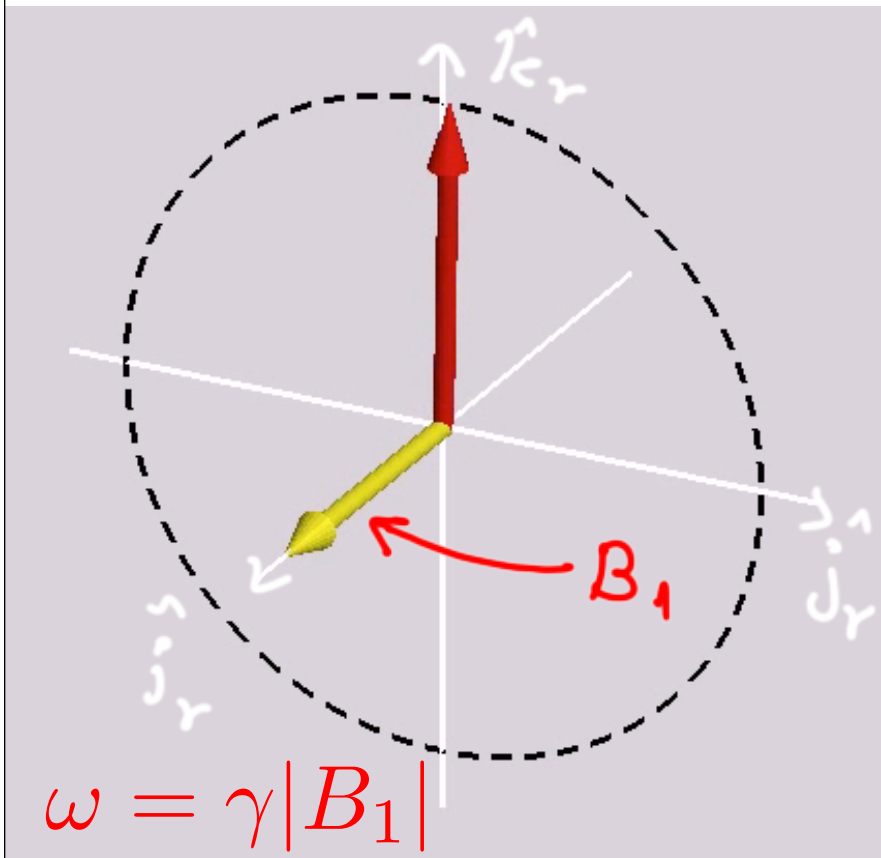
$$\vec{B}_{ROT} = \vec{G} \cdot \vec{x}\, \hat{k}_r + B_{1,x}\, \hat{i}_r + B_{1,y}\, \hat{j}_r$$

Much simpler!

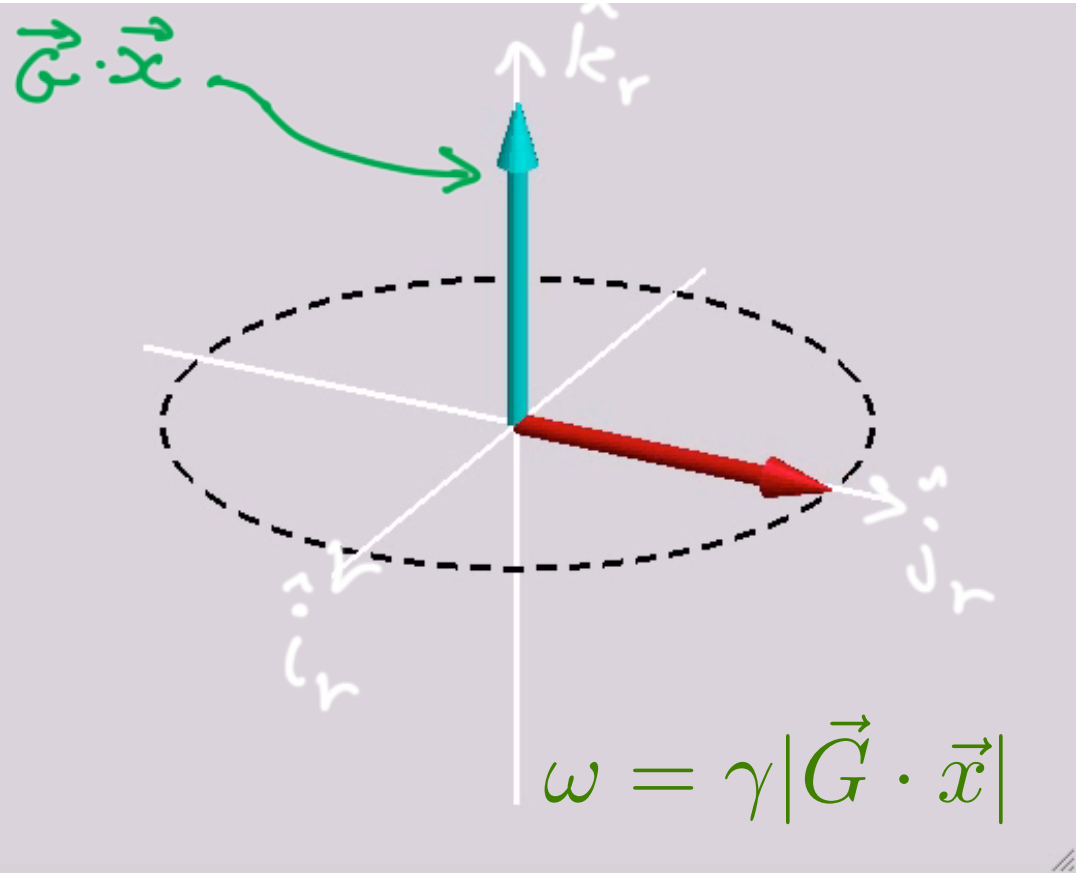
- $\vec{M}_{ROT}$ Precesses about <u>applied</u> fields $\vec{G} \cdot \hat{x}$ and B_{1x}, B_{1y}

Examples

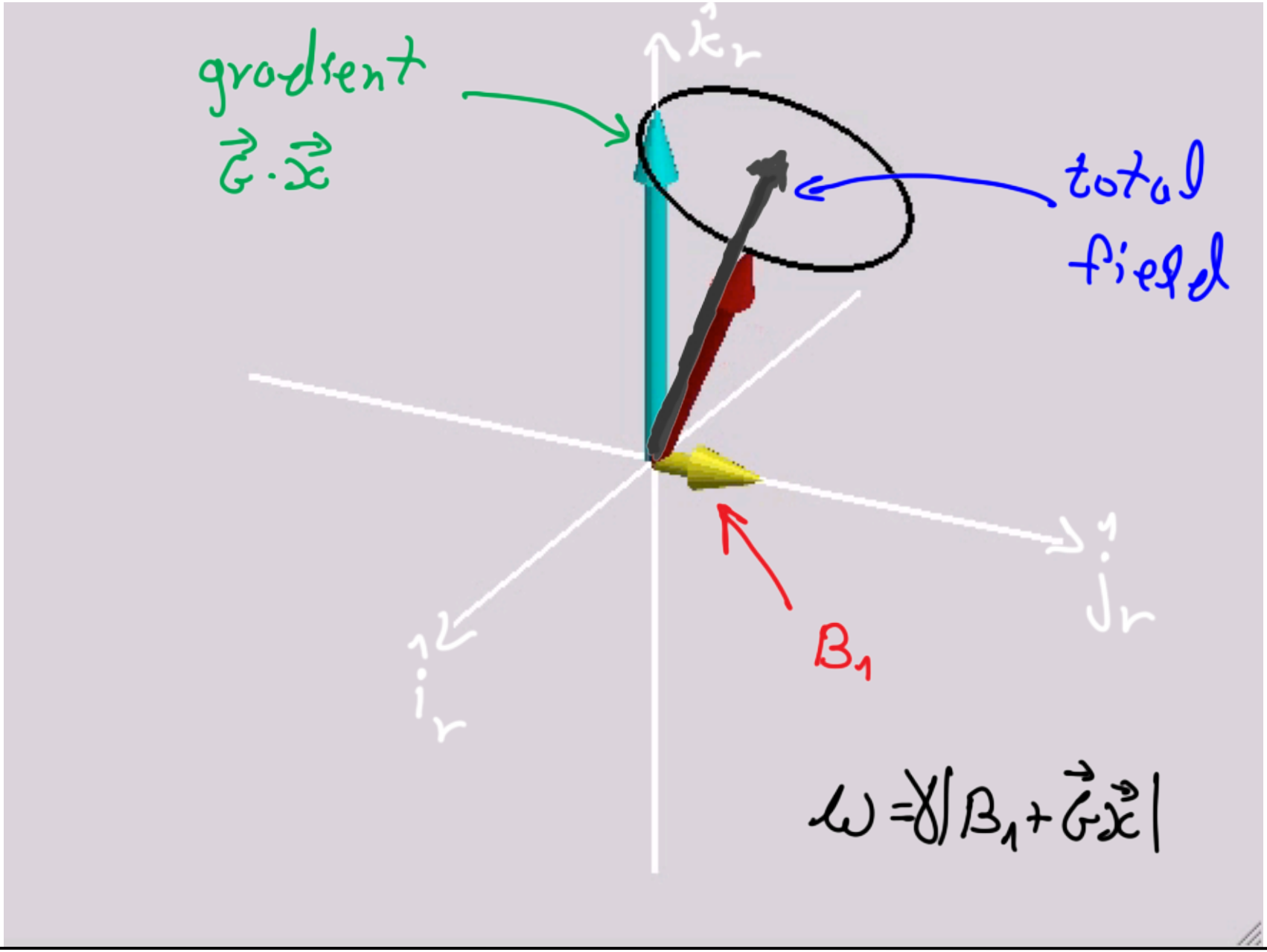
Excitation



Precession



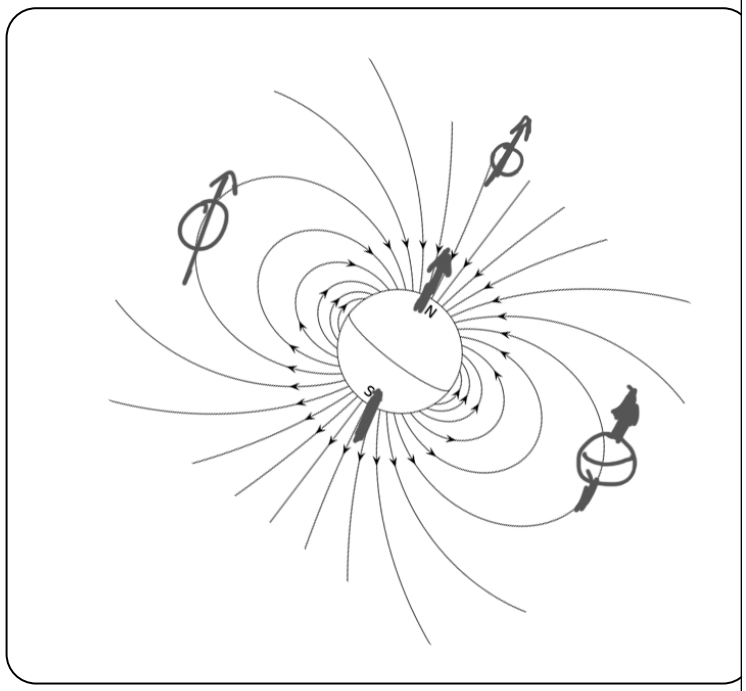
Examples


gradient
$\vec{G}\cdot\vec{x}$
total field
B_1
$\omega = \gamma |B_1 + \vec{G}\vec{x}|$

Relaxation

- <u>T2 Decay</u>
  - Transverse magnetization decays
  - Due to loss of coherence between spins
  - Also called spin-spin relaxation

  - Not a strong function of B_0
  - Dipole effect stronger in solids



Transverse Relaxation (T2)

- Let

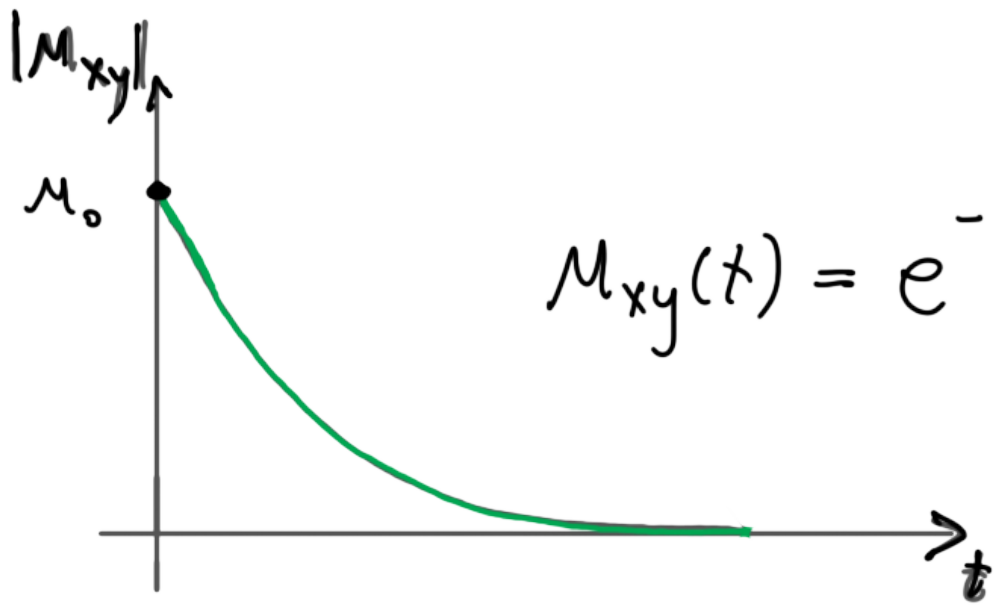
$$M_{xy} = M_x + iM_y$$

- Then

$$\frac{dM_{xy}}{dt} = -\frac{1}{T_2} M_{xy}$$

Transverse Relaxation (T2)

- Solution:



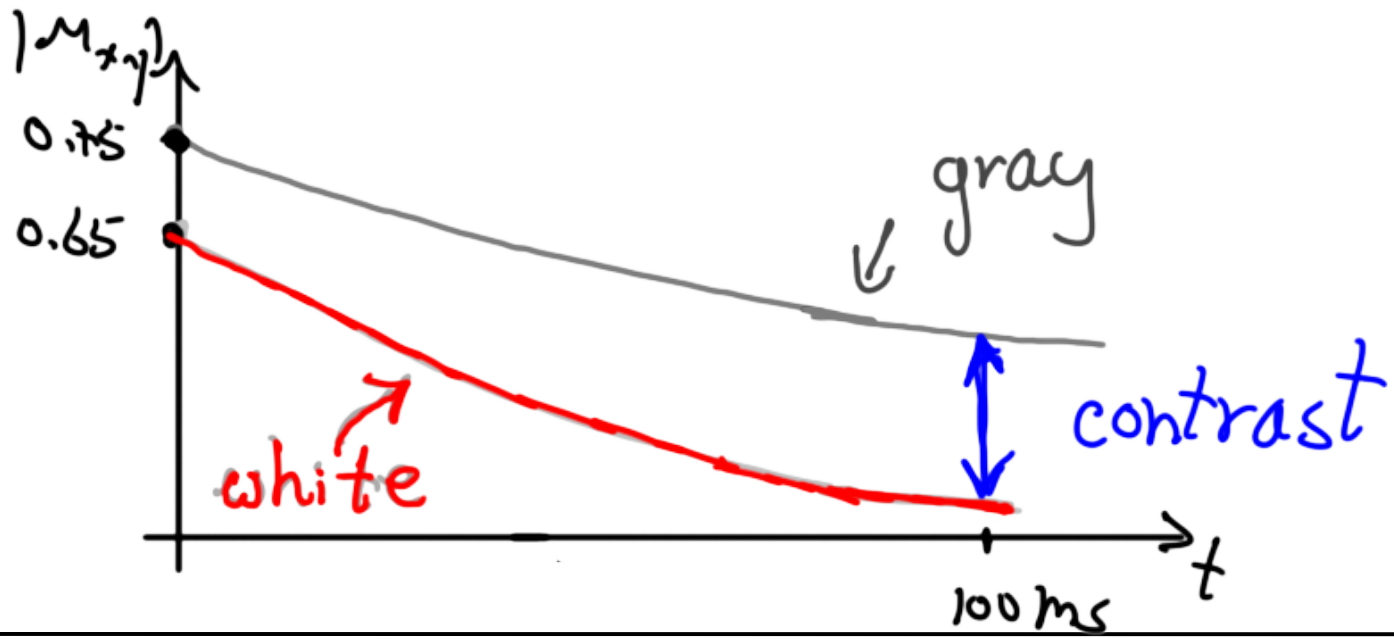
$$M_{xy}(t) = e^{-\frac{t}{T_2}} M_{xy}(0)$$

Major source of contrast

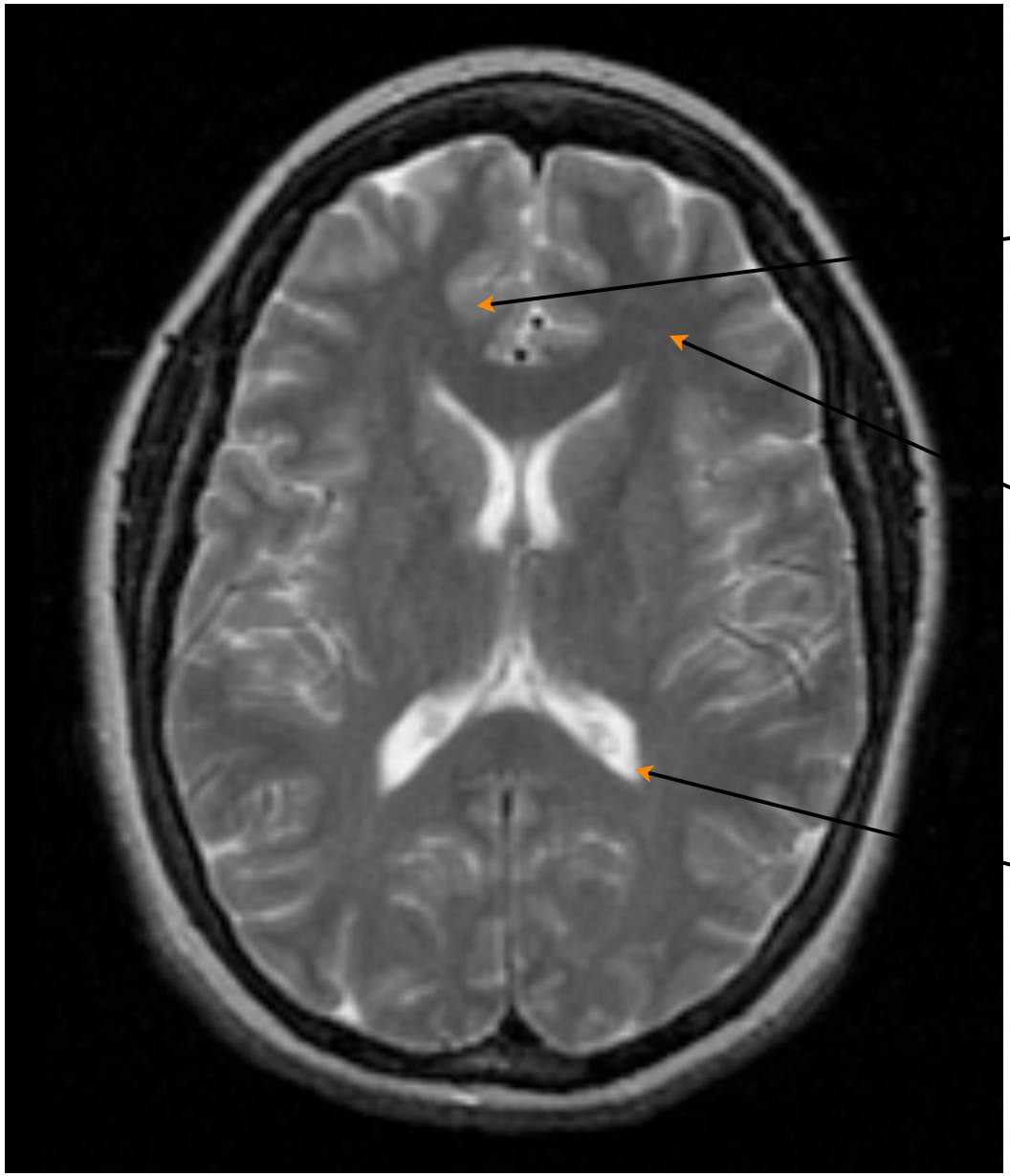
Transverse Relaxation (T2)

- Example: Brain @ 1.5T
  - white matter T2=92ms, Density=0.65
  - gray matter T2=100ms, Density = 0.75

Excite, wait 100ms, collect data



T2 Example



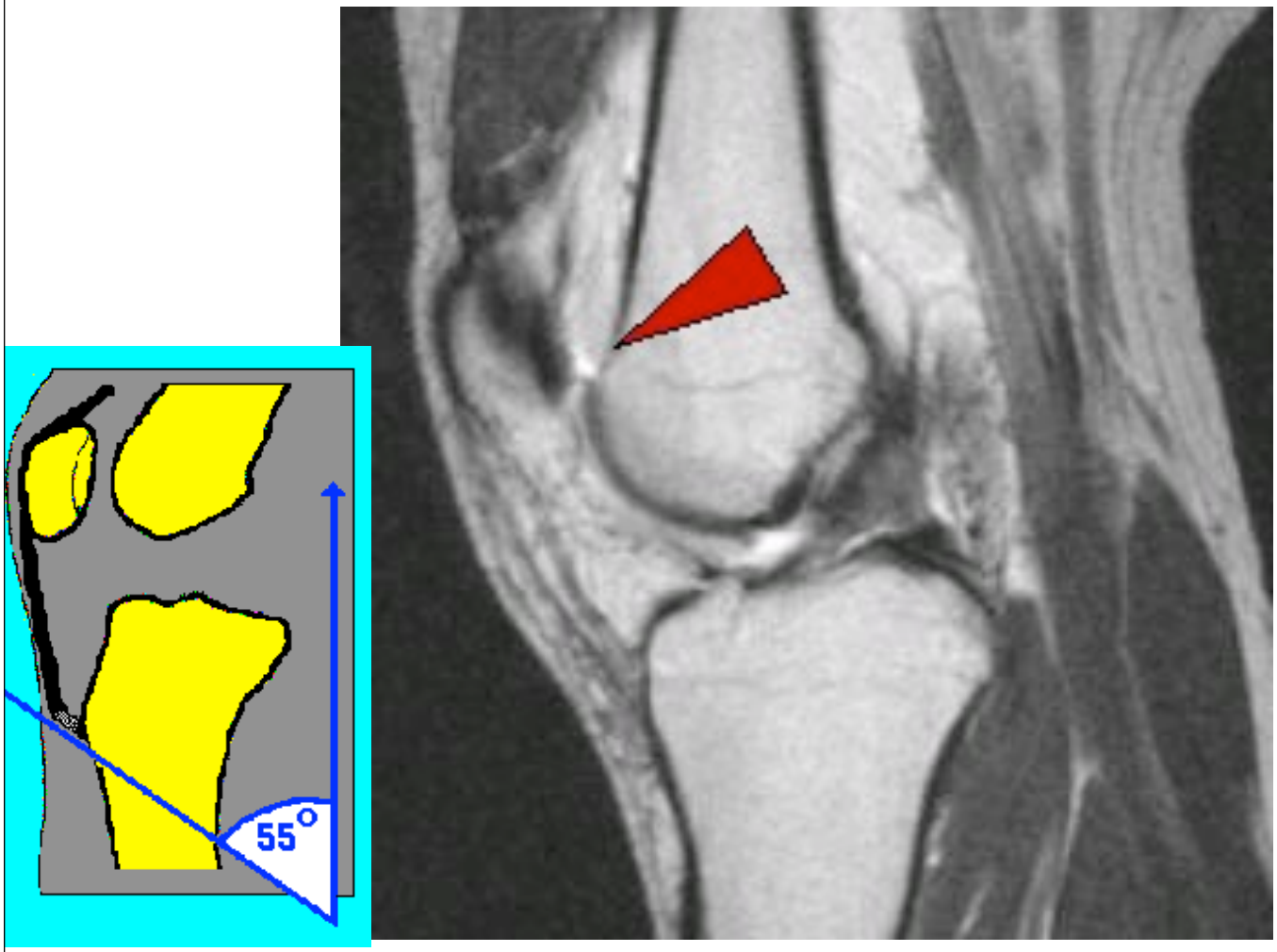
Gray matter
lighter

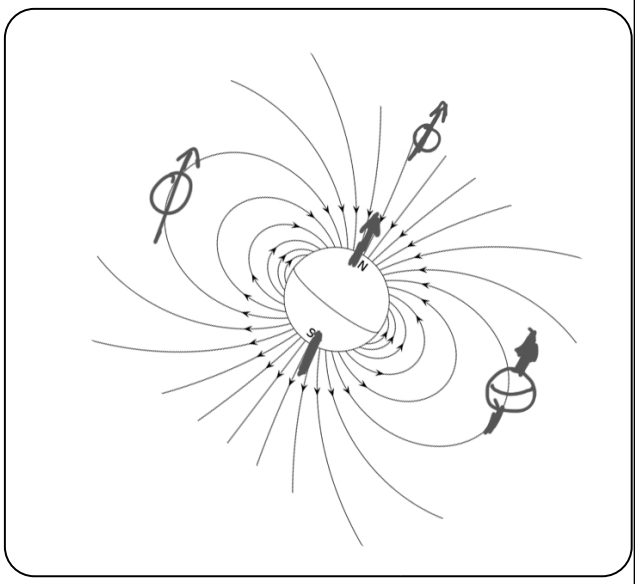
white matter
darker

CSF
Bright!

Magic Angle ~55 degrees

- Longer T2 due to dipole decoupling

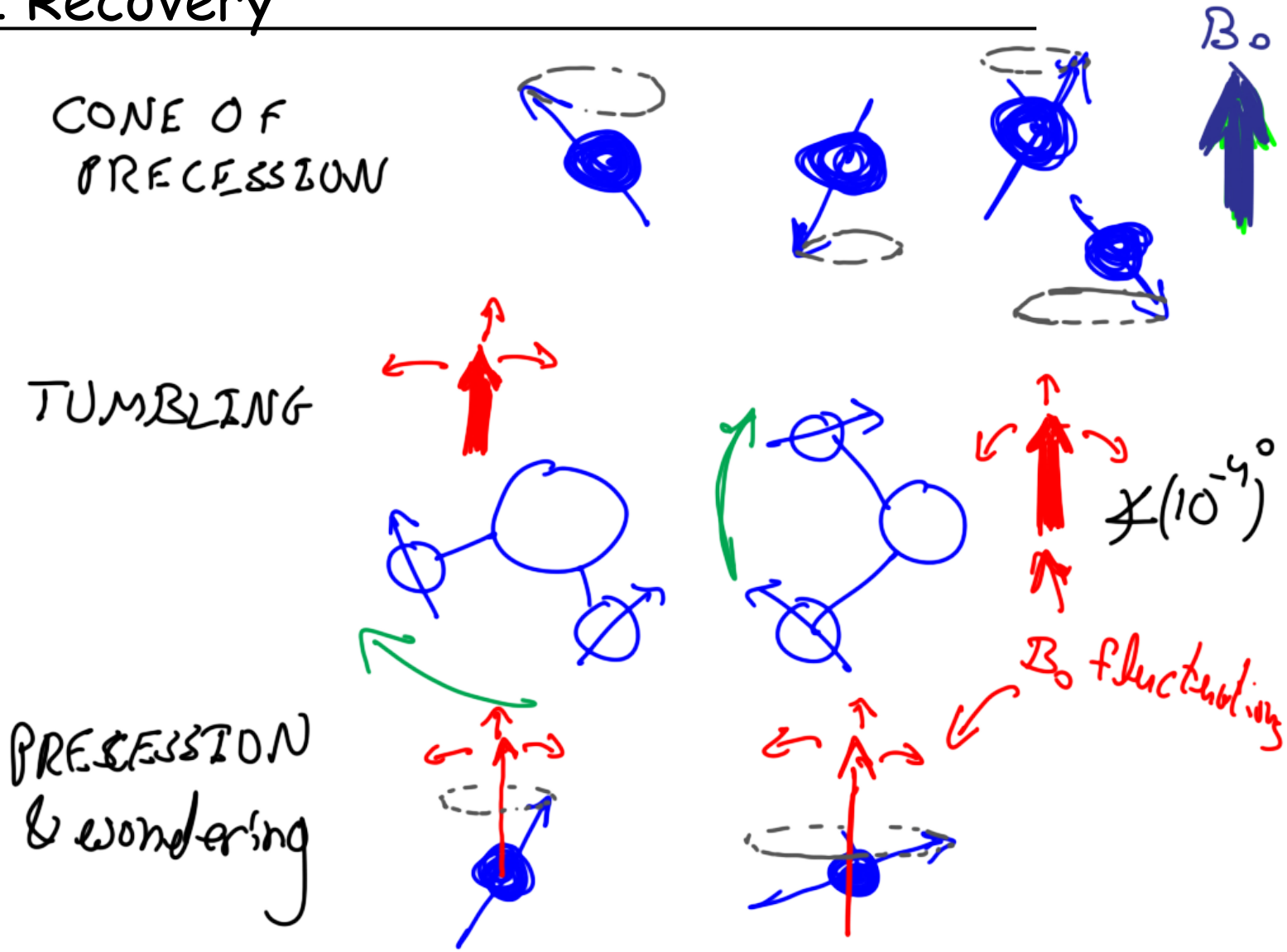




Relaxation

- <u>T1 Recovery</u>
  - Longitudinal relaxation
  - Due to Spin-Lattice interaction
  - Thermal bouncing of molecules - lose cone of precession - align with field
  - Strong dependency on B_0 , since energy level depends on B_0
  - B_0 stong - hard to transition - T1 long

T1 Recovery



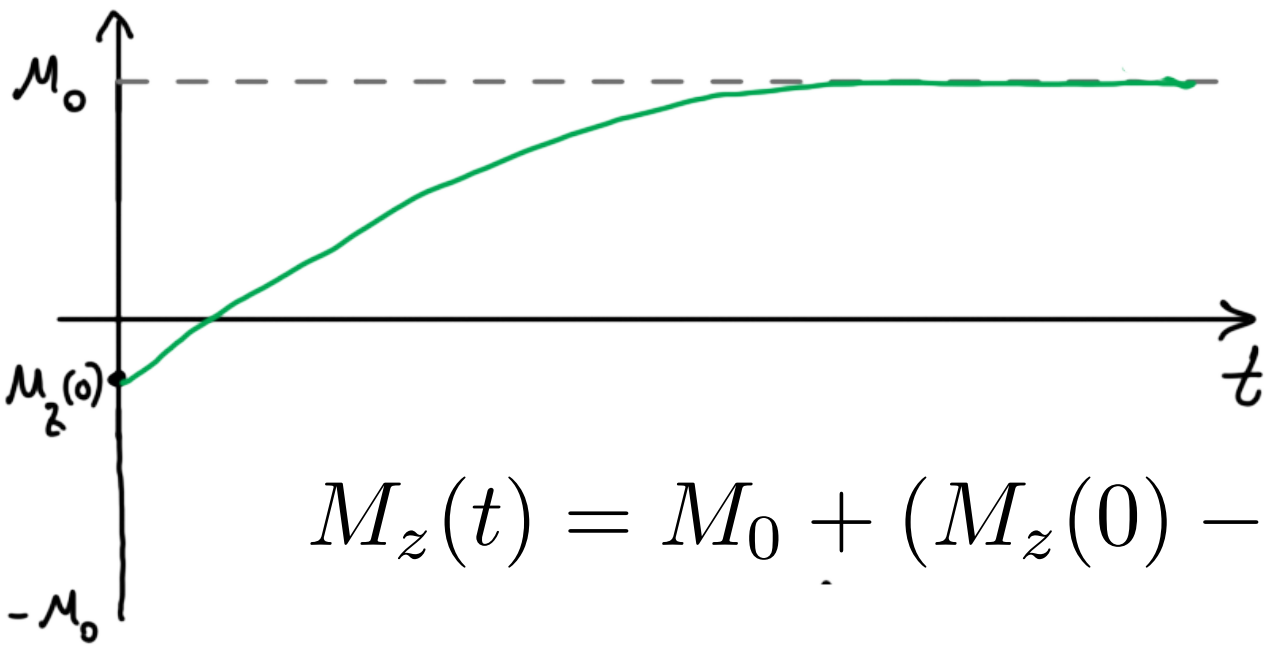
- Bias towards up - stable anisotropic dist.

T1 Recovery

- Magnetization recovers to equilibrium M_0

$$\frac{dM_z}{dt} = -\frac{M_z - M0}{T_1}$$

- Solution:



$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T1}$$

T1 Recovery

After 90° pulse, $M_z = 0$

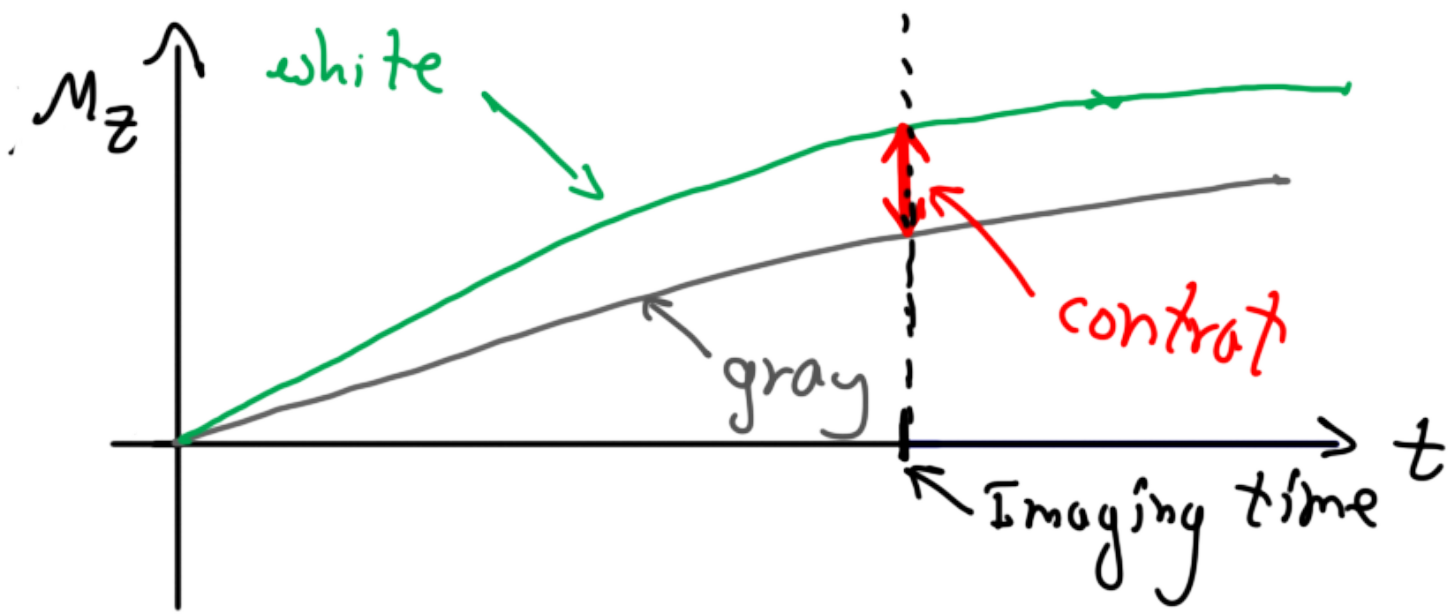
$$M_z(t) = M_0 - M_0 e^{-\frac{t}{T_1}} = M_0 (1 - e^{-\frac{t}{T_1}})$$

Major source of contrast as well!

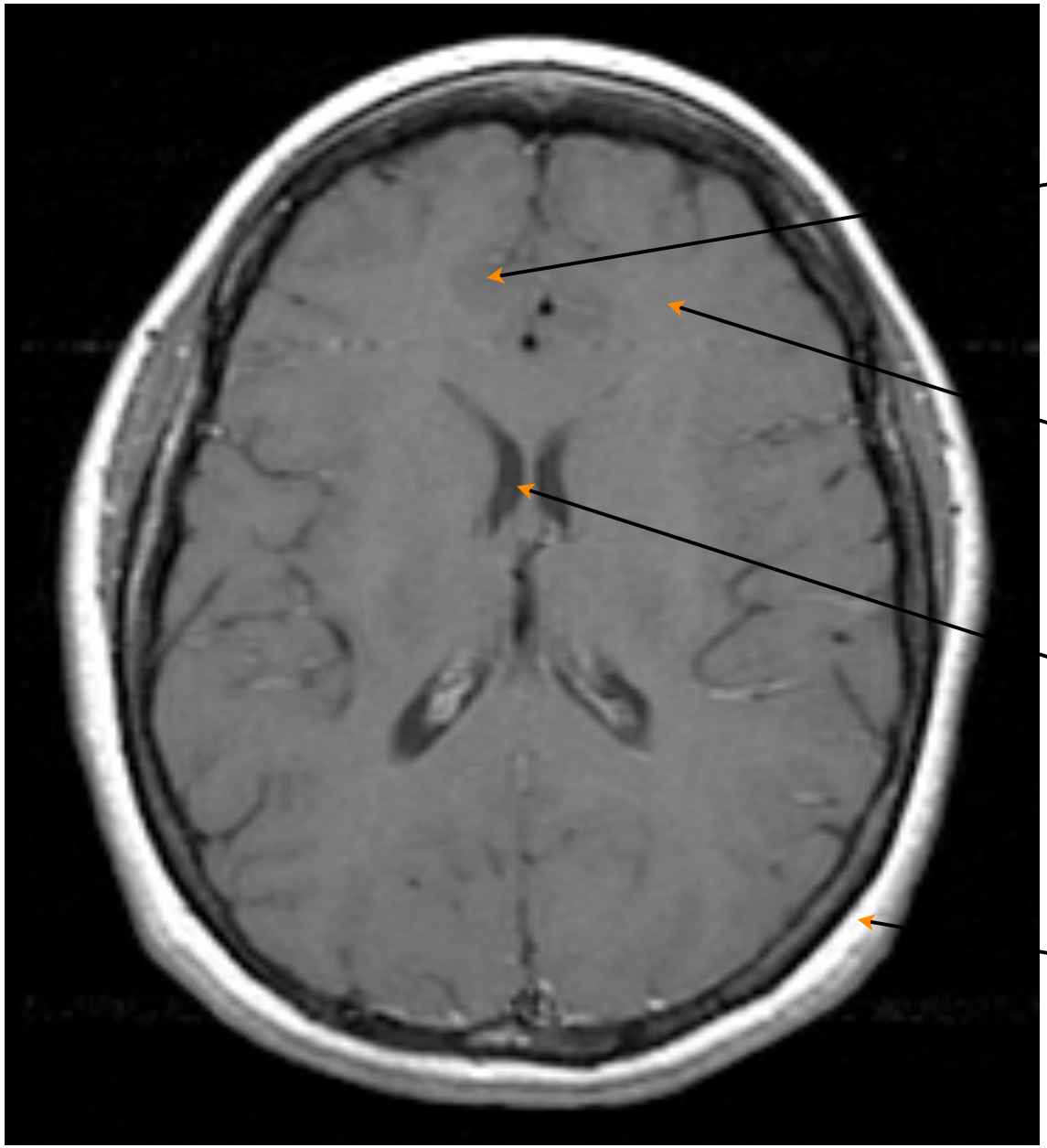
T1 Contrast

- Brain at 1.5T
  - Gray Matter T1 = 900ms
  - White Matter T1 = 800ms

Excite 90, wait, excite again, image....



T1 Contrast Example



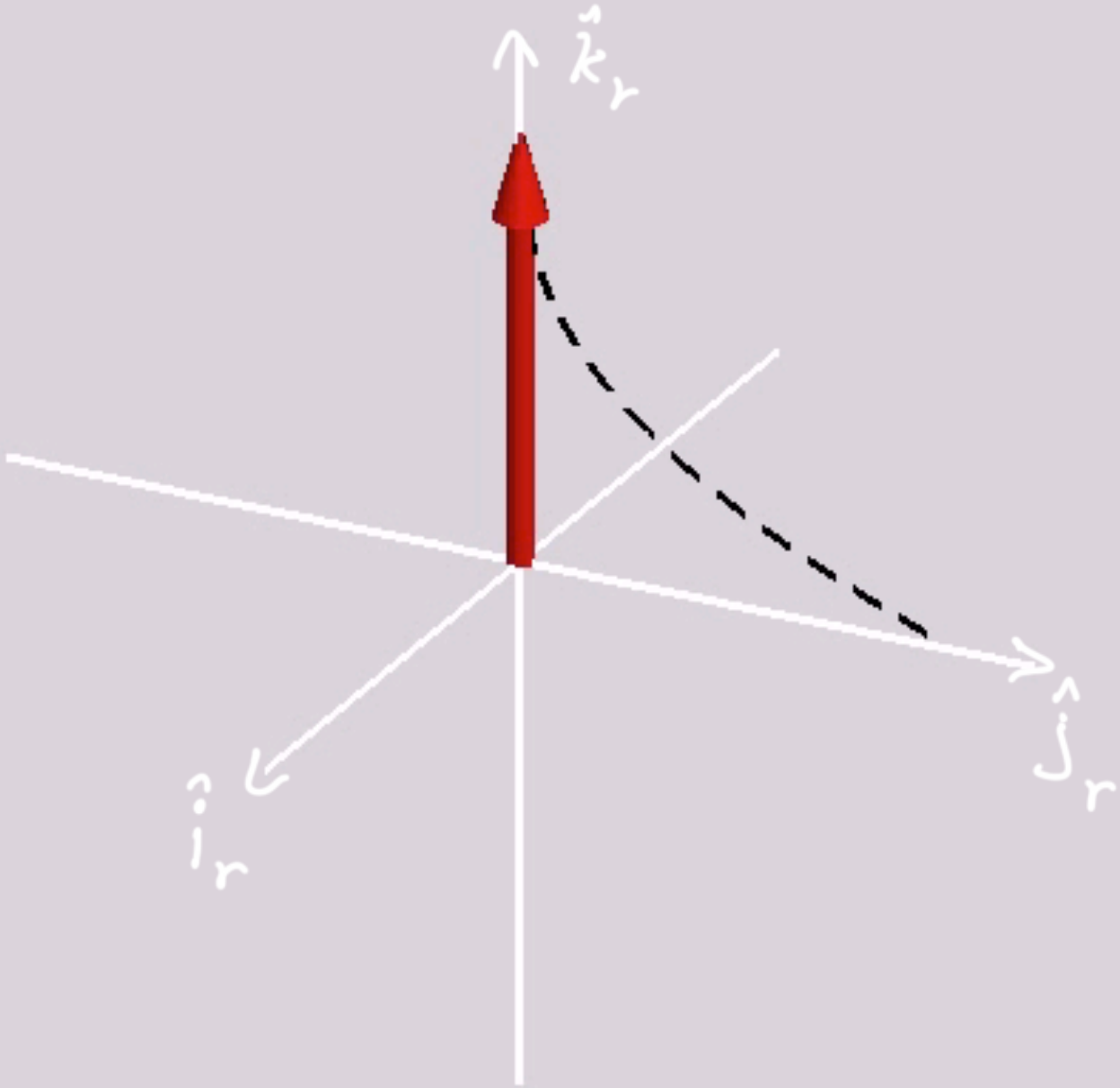
Gray matter
darker

white matter
lighter

CSF
Dark!

Fat
Bright!

Relaxation



The Bloch Equation

- Combine Precession and relaxation

$$\frac{d\vec{M}}{dt} = \underbrace{-\gamma \vec{B} \times \vec{M}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{i} + M_y \hat{j}}{T_2}}_{\text{transverse decay}} - \underbrace{\frac{M_z - M_0}{T_1} \hat{k}}_{\text{long. recovery}}$$

- Phenomenological: Fits observations
  - Describes most of MRI
  - Sometimes Fails.... J-coupling, Magn, transfer