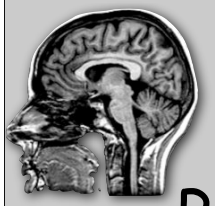
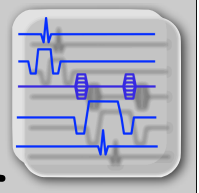




Principles of MRI

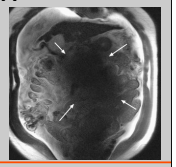
EE225E / BIO265

Lecture 06

Instructor: Miki Lustig
UC Berkeley, EECS

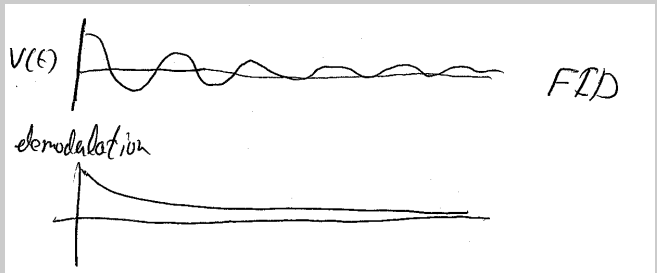
Last time

- B_0
  - Polarization → M_0
  - Resonance $\omega = -\gamma B_0$
- B1 - Transmit RF field
  - Magnetization precesses around rotating field and is tipped away
  - Very Small Field! (usually < 0.35 G)
  - Resonance is essential - easy to describe in rotating frame
  - $\Delta B_1 \sim \pm 10\% \ @ \ 1.5T$
    - Leads to different flips in space



Last Time

- Relaxation
  - Longitudinal: T_1
  - Transverse: T_2
- B1 - Reception RF field
  - Precession causes EMF in coil



Last Time

- Gradient Fields
  - Encode position onto frequency

$$\vec{G} = \left[\frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial z} \right]$$

  - Small concomitant fields Bx, By are also created. These do not contribute much to precession - fields are NOT oscillating at Larmor freq.

Imaging

(1) place sample in B_0

M_z Develops $\sim 5T_1$

(2) Excite using $B_1(t)$

creates transverse magn.

(3) Instantaniouse precession of M_{xy}

Induces EMF in Coil

(4) Encode Position in Freq. using gradients

1-D "Projection"

Some Limitations

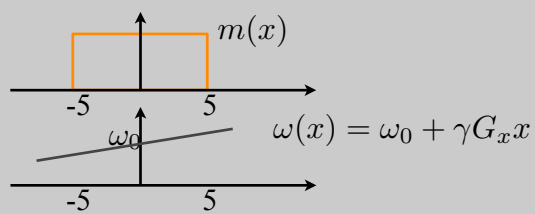
Gradient strength
Gradient duration } Resolution

⇓

signal decay (T_2)
Field Inhomogeneity (T_2^*)
DIFFUSION

TYPICAL RESOLUTION: $\leq 1mm$
(~20μm in small animal scanners)

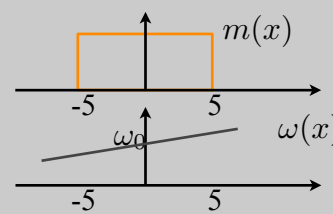
Example: Square RECT



$$\omega(x) = \omega_0 + \gamma G_x x$$

- What is the signal after excitation?
  - Signal proportional to m_{xy}
  - Comes from the entire volume

Example: Square RECT



$$\omega(x) = \omega_0 + \gamma G_x x$$

- Magnetization at position $x=x_0$

$$m_{xy}(x) = m(x)e^{-i(\omega_0+\gamma G_x x)t}$$

- Signal from everything

$$s(t) = \int_x m(x)e^{-i(\omega_0+\gamma G_x x)t}dx$$

Example: Square RECT

$$
\begin{aligned}
s(t) &= \int_x m(x) e^{-i(\omega_0 + \gamma G_x x)t} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i\gamma G_x x t} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi(\frac{\gamma}{2\pi} G_x t)x} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi k_x x} dx \\
&= e^{-i\omega_0 t} \mathcal{F}\left\{m(x)\right\}\big|_{k_x = \frac{\gamma}{2\pi} G_x t}
\end{aligned}
$$

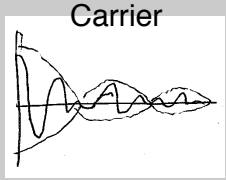
$$k_x = \frac{\gamma}{2\pi} G_x t$$

Example: Square RECT

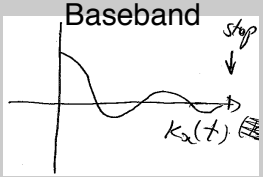
$$s(t) = e^{-i\omega_0 t} \mathcal{F}\left\{m(x)\right\}\big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

$$m(x) = \mathrm{rect}(\cdot) \quad \Rightarrow \quad \mathcal{F}\{m(x)\} \sim \mathrm{sinc}(\cdot)$$

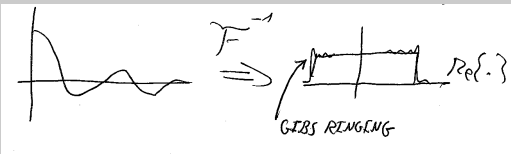
Carrier



Baseband



Inverse FT of baseband signal yields 1D projection (Almost..)



Imaging

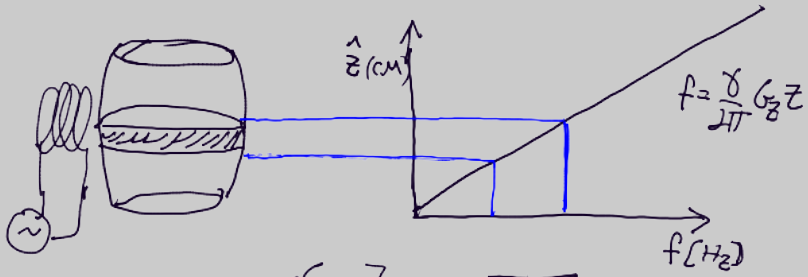
Q. How do we get an image?

several key components:

1. Selective excitation (Dimension reduction)
2. Spatial encoding (Freq/phase) ← more later
3. Signal decay, M_0 recovery
4. Repeat N Times
5. Image Reconstruction

Selective Excitation

want to excite a slice



$\mathcal{F}\{B_1(t)\} =$

Gradient (G_z) maps position to bandwidth

Bandlimited $B_1(t) \Longrightarrow$ Excites a slab in space

Excitation Field

$$\underbrace{e^{-i\omega_0 t}}_{\text{carrier}} \quad \underbrace{B_1(t)}_{\text{envelope}}$$

(center of slice) (shape of slice)

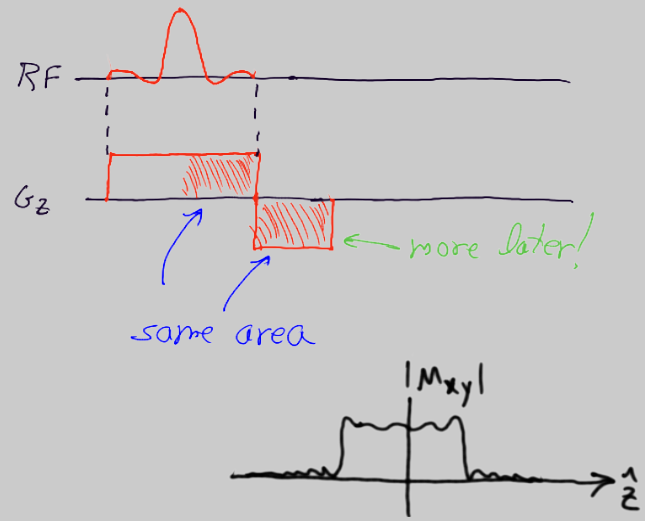
choose $B_1(t)$

$$\mathcal{F}\{B_1(t)\} \simeq \mathrm{rect}\left(\frac{f}{A}\right)$$

$$B_1(t) \simeq A\,\mathrm{sinc}(At)$$

finite in time
ripple in slice

we prow RF excitation as a pulse seqence:



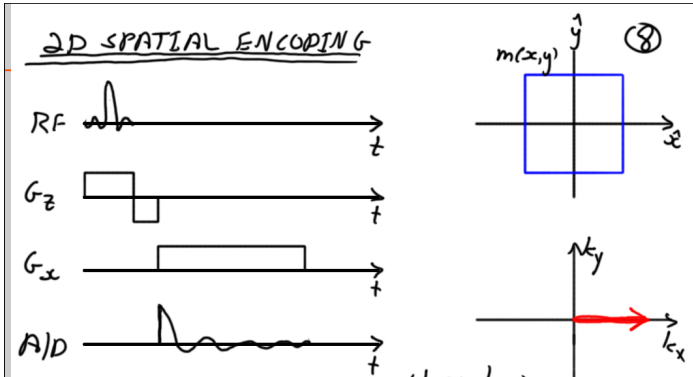
Quiz

$$B_1(t) = e^{-i\omega_0 t}\mathrm{sinc}(5000t)$$

$$G_z = 1\ G/cm$$

What is the excited slice thickness?

2D SPATIAL ENCODING



$$s(t) = \iint m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\, dx\, dy$$

$$k_x = \frac{\gamma}{2\pi} G_x t \qquad k_y = 0$$

Need k_x-k_y coverage:

Try: $G_x = G\cos\theta_n$ $\quad \theta_n = \frac{\pi}{N} n \quad n = [0, N-1]$

$G_y = G\sin\theta_n$

The Central Section Theorem

(9)

(Projection slice)

R. Bracewell

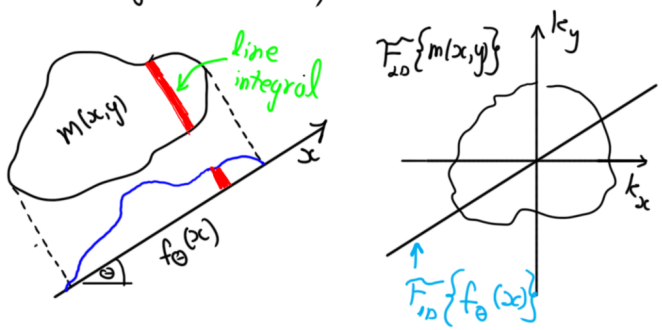


Image Space — k-space

Transform (1D) of projection of an object

$\Longrightarrow$ Diameter through transform(2D) of object

Very cool!... and useful...

Projection slice $\Rightarrow$ basis for CT

(10)

Proof (for $\theta=0$)

$$p(x)=\int_{-\infty}^{\infty} m(x,y)\,dy$$

$$M(k_x,0)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\,e^{-i2\pi(k_x x+k_y y)}\,dx\,dy=$$

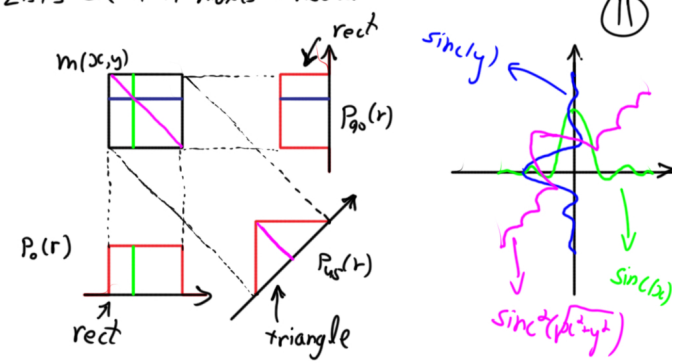
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\,e^{-i2\pi k_x x}\,dx\,dy=$$

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} m(x,y)\,dy\right]e^{-i2\pi k_x x}\,dx=$$

$$=\int_{-\infty}^{\infty} p(x)\,e^{-i2\pi k_x x}\,dx=\mathcal{F}\{p(x)\}$$

Let's see if it makes sense...

(11)

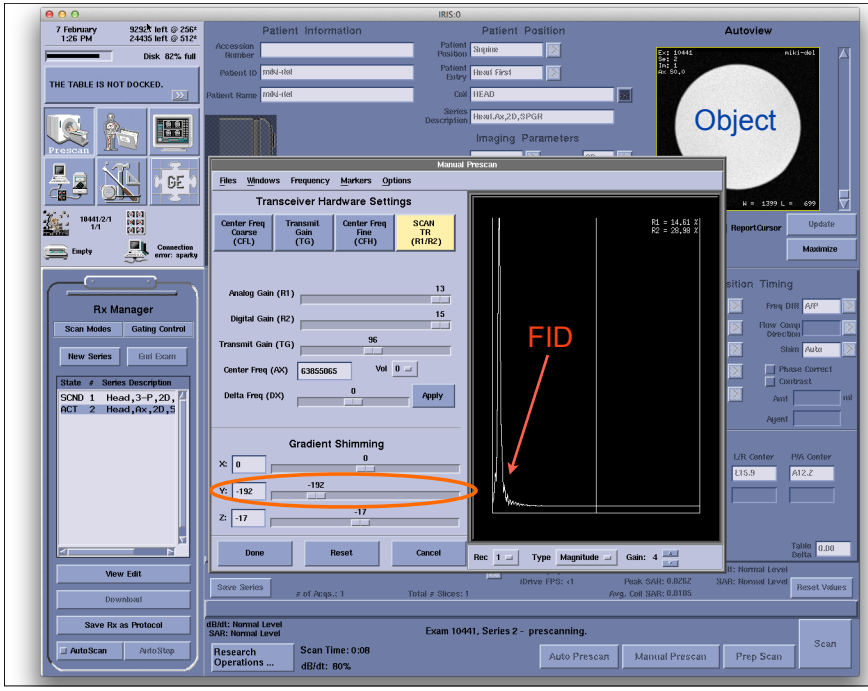


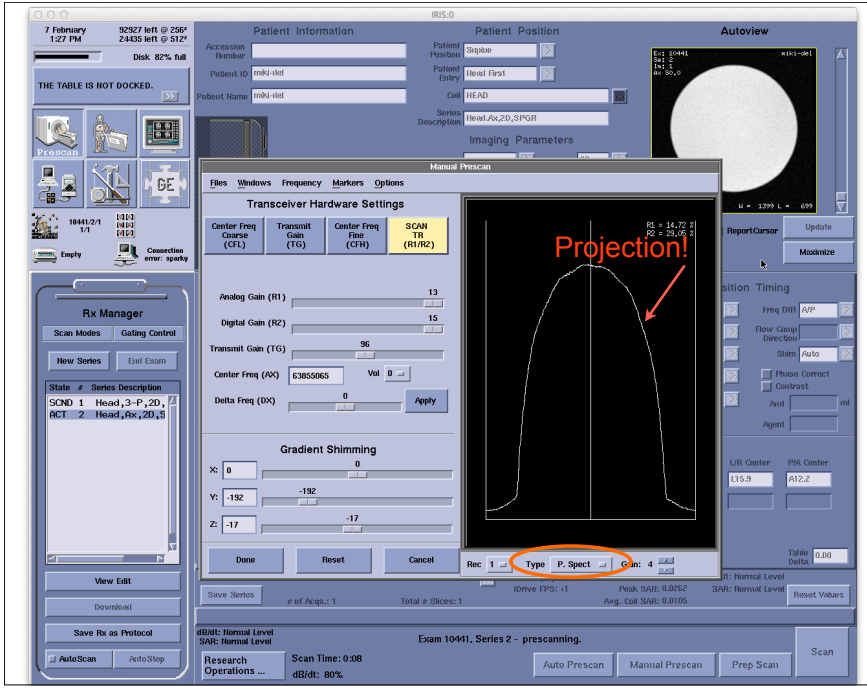
How to reconstruct?

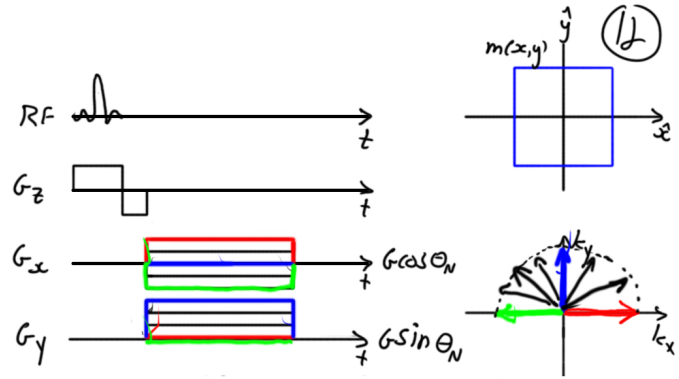
(-) filtered back-projection

(-) Interpolation (in k-space)

Good class project





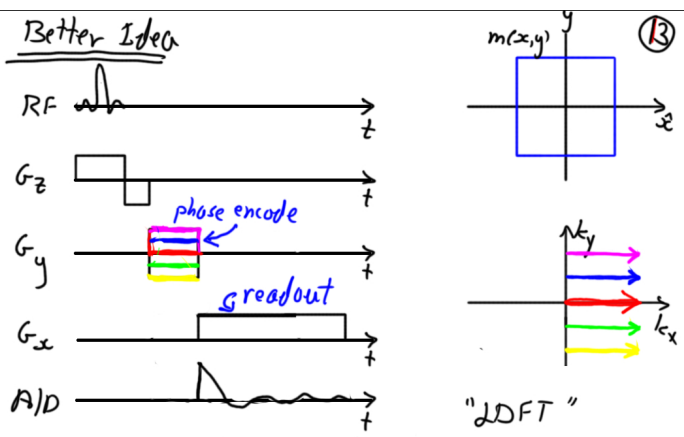


Projection Reconstruction (Lauterber)

$\mathcal{F}_{1D}^{-1}\{\text{piameter}\} \Rightarrow$ Projection

However... not quite diameter yet...

Better Idea



"2DFT"

"SPIN-WARP"

(still not perfect)

(-) Make "horizontal projections"

(-) Encode phase in y prior to readout in x

Other approaches to Imaging

14

(-) Localization is the key!

(1) B_0 localization

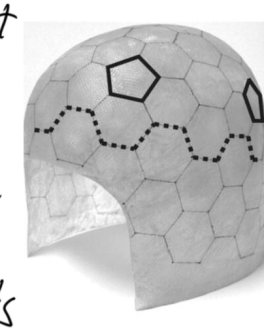
(-) saddle point

(-) local magnet

(2) RF

(-) Local coils

(-) Local Excitation



(3) Non linear gradients

Complex value & Demodulation

(15)

Physical signal:

$$s_p(t) = A(t)\cos(\omega_0 t + \phi(t)) =$$

real $\Longrightarrow$ trigo

$$\underbrace{A(t)\cos(\phi(t))}_{I(t)}\cos(\omega_0 t) + \underbrace{\left[-A(t)\sin(\phi(t))\right]}_{Q(t)}\sin(\omega_0 t)$$

$I(t)$ in phase

$Q(t)$ quadrature

Analysis signal (convinient)

$$s_r = A(t)e^{-i\omega_0 t + \phi(t)}$$

complex

real

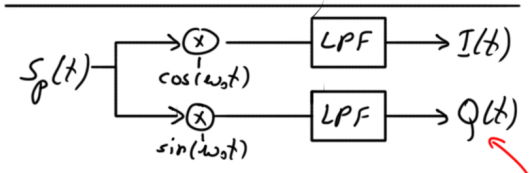
$s_p = \mathrm{Re}\{s_r\}$

Baseband signal

$$s(t) = s_r(t)e^{+i\omega_0 t} = A(t)e^{-i\phi(t)} = I(t) + iQ(t)$$

Quadrature Phase-Sensitive Detection:

(16)



often used in communication

$$s(t) = I(t) + iQ(t)$$

$I(t) \rightarrow M_x$ in rotating frame

$Q(t) \rightarrow M_y$ in rotating frame