EE225E/BIOE265 Spring 2014 Miki Lustig
Principles of MRI

**Assignment 1**

Due Friday Jan 31st, 2014, Self Grading Due Monday Feb 3rd, 2014

1. Read Nishimura Ch. 1 and Ch. 2.

2. Consider an old-fashioned LP gramophone phonograph. These were the preferred media for recording music from about 1910 to the late 1980's. Wikipedia has an interesting entry on this media: `http://en.wikipedia.org/wiki/Vinyl_record`. Below you can see a close in shot of 8 grooves on a 78 RPM recording, obtained from an LBNL Report 51983, 26-March-2003 Vitaliy Fadeyev and Carl Haber, (publication on the class website). The transverse undulations are transduced into music.

   (a) What is the spatial frequency of a 2 kHz tone at the outside radius (4.75 inches)?

   (b) What is the spatial frequency of a 2 kHz tone at the inside radius (1.875 inches)?

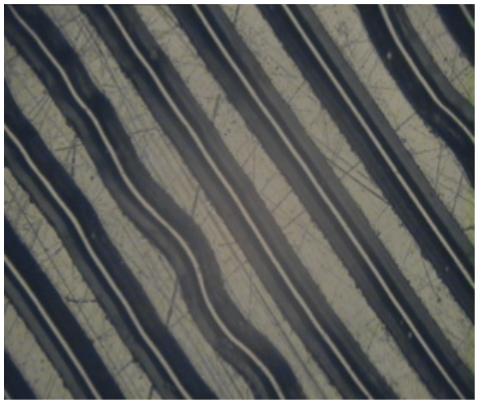


Figure 1: Micro-photograph of grooves on a 78 r.p.m recording. Illumination is coaxial. Image size is approximately $700 \times 540$ microns.

3. **Fourier Series and Gibb's Ringing**. See a nice applet at `http://www.jhu.edu/~signals/fourier2/index.html`

   (a) Which waveform on the right is represented best with a limited number of coefficients? Why?

   (b) Increase the number of harmonics to represent a rectangular pulse. Notice the Gibb's ringing at the edges. Does the amplitude change much as you vary the number of coefficients?

   (c) The approximation is the inverse FT of a windowed version of the FT of the rectangular pulse. More coefficients means a broader window in k-space. Explain the Gibb's ringing in terms of the FT of this window.

4. Proofs:

   (a) Prove the stretch or scaling theorem for 2D Fourier Transforms. That is, show that

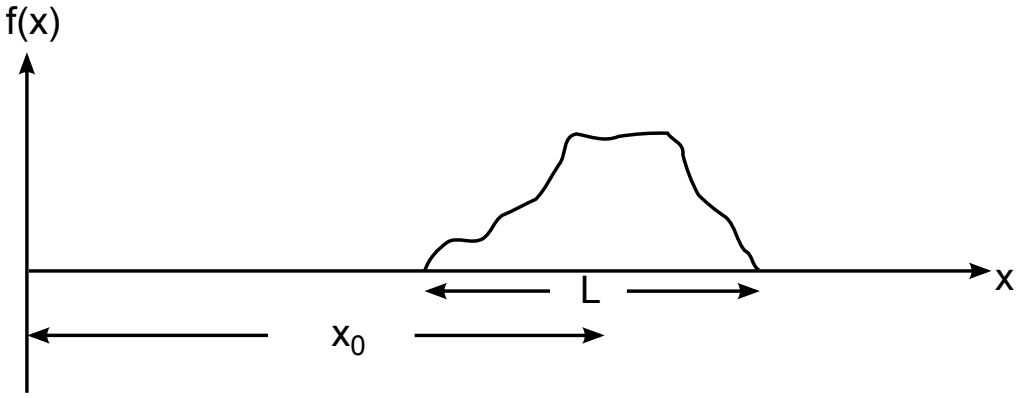
   $$\mathcal{F}\{f(x/a, y/b)\} = |ab|F(ak_x, bk_y).$$

   (b) Derive the shift theorem for 2D FTs.

   (c) Derive the convolution theorem for 2D FTs.

   (d) Derive the derivative theorem for 1D FTs. That is, find a simple expression for the FT of $\frac{df(x)}{dx}$ in terms of $F(k)$.

5. **Bandpass Sampling in k-space:** Suppose you have an image $f(x)$ in 1D x-space that has "finite support" over a region of length L. Finite support simply means that the function is zero outside the region of length L. Here we want to consider the effect of an origin shift. Suppose the support region is shifted away from the origin by a distance $x_0$, as shown in the diagram below.




   We are going to sample $F(k_x)$, the functions' FT in $k_x$ space. Suppose you are asked to specify the minimum sampling requirements in 1D Fourier space (*e.g.,* $k$-space).

   (a) Use the shah function in k-space from section 2.4 of Nishimura to show that the sampling requirement is independent of $x_0$, and only depends on L.

   (b) Find the minimum sampling rate in k-space.

   (c) Would your interpolation filter require knowledge of $x_0$ to get back $F(k_x)$ (and thus $f(x)$) from its samples ?

6. Fourier Transforms and signals:

   (a) Consider the Rect function

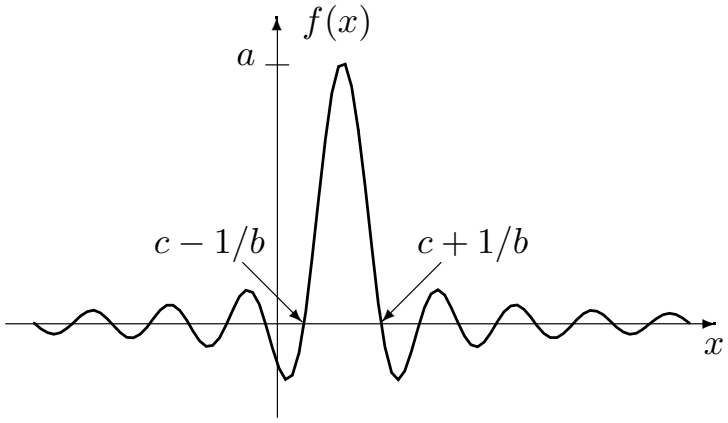
$$\sqcap(x) = \begin{cases} 1 & |x| \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

   and the triangle function

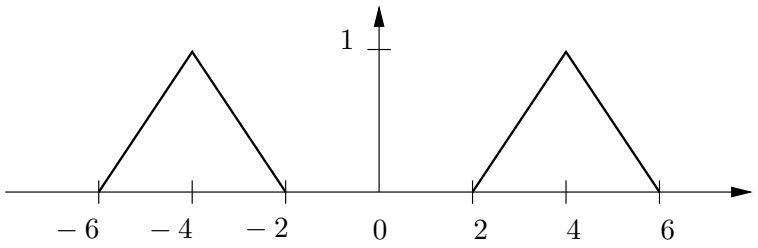
$$\Lambda(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   what is the Fourier transform of $\sqcap(x/a) * \sqcap(x/a)$?

   (b) Consider the sinc function sinc $= \frac{\sin(\pi x)}{\pi x}$. What is the transform of the following shifted scaled sinc function?

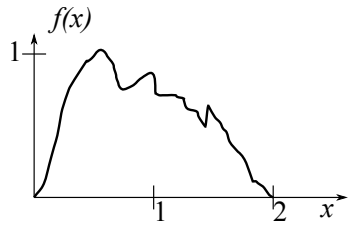



   (c) Find the Fourier transform of the following function:

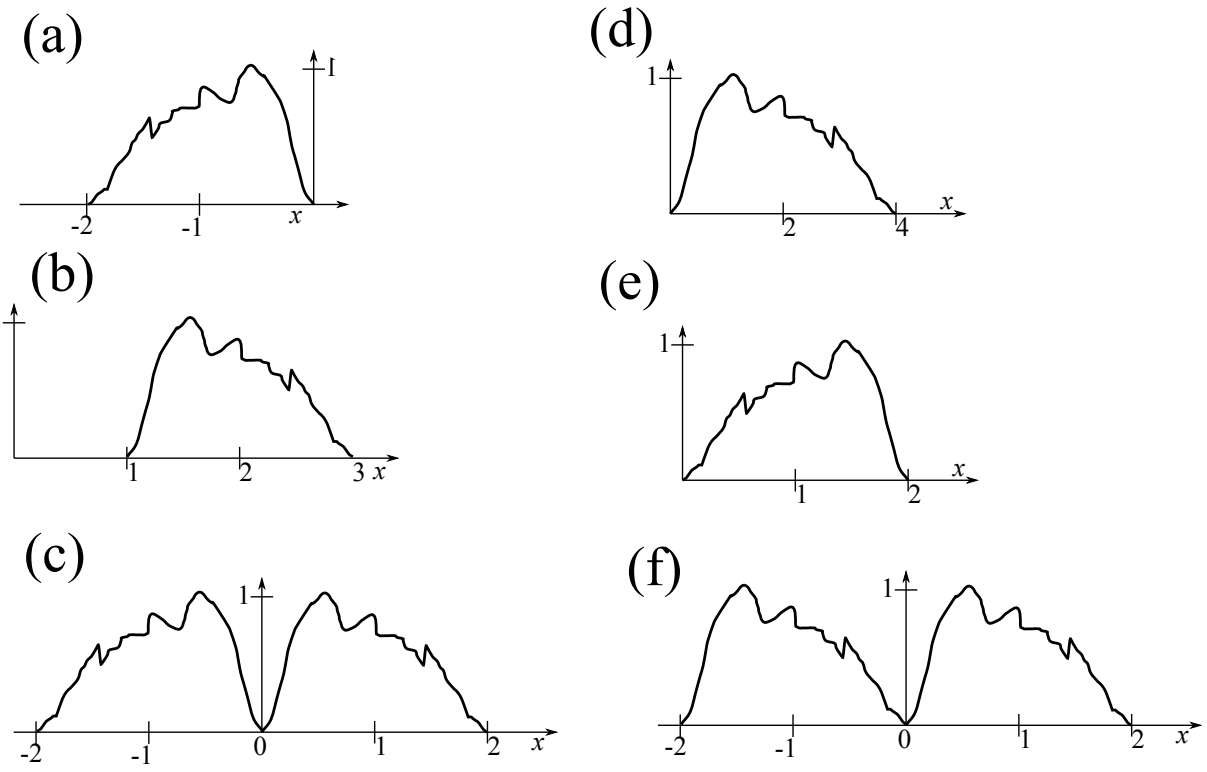



   (d) The signal $x[n] = sin(2\pi fTn)$ is a sampled sinusoid. What is the sequence $x[n]$ that has the highest discrete frequency and what is the ratio $f \cdot T$ that produces it?

7. The figures below show a signal $f(x)$ and six other signals derived from it.




Suppose $F(k_x)$ is the Fourier transform of $f(x)$. Express the Fourier transform of the other six signals in terms of $F(k_x)$.



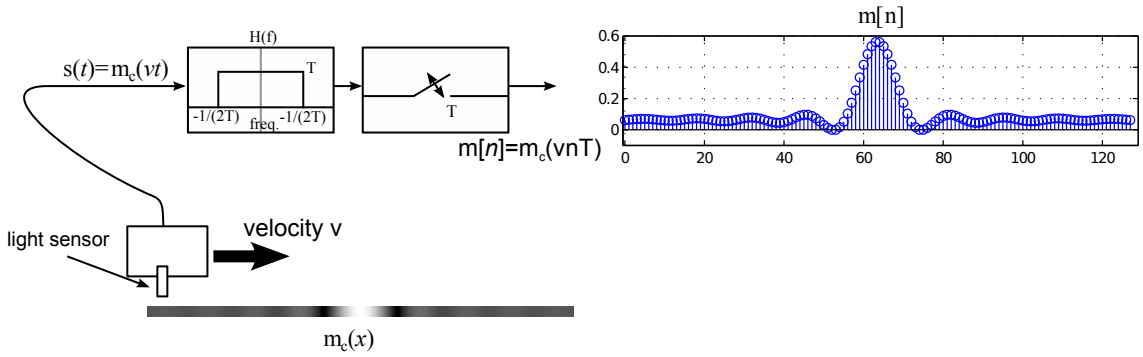

8. 2D Fourier transforms:

   (a) Find the Fourier transform $F(k_x, k_y)$ of $f(x, y) = \text{sinc}(x)\text{sinc}(y)$.

   (b) For $f(x, y) = \text{sinc}(x)\text{sinc}(y)$. Find the expression for $g(x, y) = f(x, y) * *f(x, y)$.

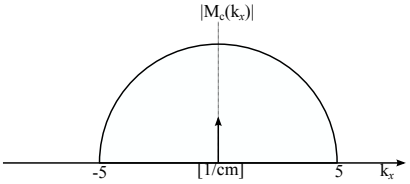
   (c) Draw the function $f(x, y) = \sqcap(x, y) * * \sqcap(x, y)$

9. *Image Scanner*
Consider the following optical scanning system:




The scanner head is equipped with a light sensor that measures the light intensity (0 to 1). The scanner head moves at a velocity $v$ while scanning a 1D object $m_c(x)$. The output of the light sensor is $s(t) = m_c(vt)$. The measure signal is filtered by an ideal low-pass anti-aliasing filter and then sampled at $T$ to produce $m[n] = s(nT)$. The antialiasing filter has a cutoff frequency $\frac{1}{2T}$ and is adjusted to the sampling interval.

(a) Given that the magnitude spectrum of $m_c$ is

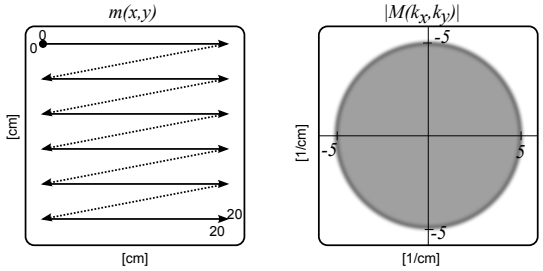



and that $T = 1$ ms, what is the maximum velocity , $v$ cm/s , such that $M_c(k_x)$ can be reconstructed from $m[n]$?

(b) Draw the magnitude FT (one period) of $m[n]$ in the interval for $v = 50$ cm/s. Does aliasing occur?

(c) Draw the magnitude FT (one period) of $m[n]$ for $v = 200$ cm/s. Does aliasing occur?

(d) We would like to use the system to scan a $20 \times 20$ cm page using raster scanning as shown below. The page contains an image, $m_c(x, y)$ with the following magnitude spectrum $|M_c(k_x, k_y)|$:




What is the maximum velocity, $v$ and the maximum distance between lines $\Delta y$ that is needed such that $m_c(x, y)$ can be reconstructed from $m[n_x, n_y]$? How many lines do you need?

(e) We would like to scan faster by increasing both the velocity $v$ and $\Delta y$ by a factor of 1.5. Draw the resulting 2D FT of $m[n_x, n_y]$ for this case. Does aliasing occur?