Lab #3

Localization from range measurements. The objective of the localization problem is to infer, from a set of range measurements, the position of an object in three-dimensional space, as accurately as possible. The problem would arise in the context of trying to estimate the location of a cell phone user based on the measurements of strength of a signal emitted from the cell phone to a number of base stations (access points).

We are given anchor positions $x_i \in \mathbf{R}^3$, and associated distances from these anchor points to an unknown object, R_i, $i = 1, \ldots, m$. The problem is to estimate the position of the object, and associated measure of uncertainty around the estimated point. Geometrically, the measurements imply that the object is located at the intersection of the m spheres S_i of centers x_i and radiuses R_i, $i = 1, \ldots, m$. The main problem is to provide one point in the intersection located at some kind of center, and also a measure of the size of the intersection.

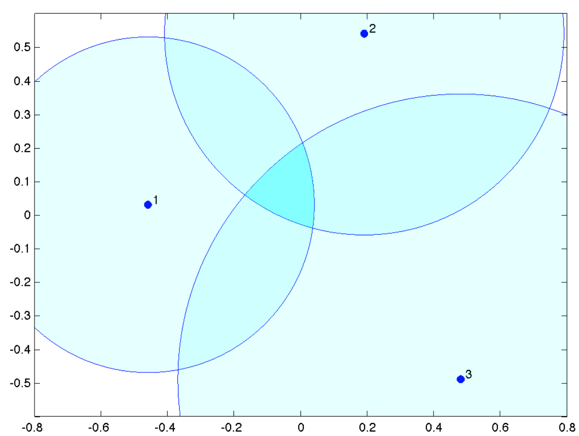

Figure 1: A localization problem with three data points in two dimensions.

In this lab we consider the localization problem with three anchor points in two dimensions, which we collect in matrix $X = (x_1, x_2, x_3)$, and associated radii measurements R_1, R_2, R_3:

$$X = (x_1, x_2, x_3) = \begin{pmatrix} -0.46 & 0.19 & 0.48 \\ 0.03 & 0.54 & -0.49 \end{pmatrix}, \quad R^T = (R_1, R_2, R_3) = \begin{pmatrix} 0.5 & 0.6 & 0.85 \end{pmatrix}.$$

Except for the last question, the answer to the following questions should be in the form of a CVX code that solves the problem, and an associated visualization of the result (you may find the file `loc_pb.m` handy).

1. We say that the data is consistent if indeed the intersection of the spheres is not empty. How would you check that the measurements are consistent?

2. Consider the case when one measurement is added:

$$x_4 = \begin{pmatrix} 0.6 \\ -0.2 \end{pmatrix}, \quad R_4 = 0.1.$$

   Is the new data set consistent?

3. Inconsistent data may arise due to faulty sensors. In that case, we would like to adjust the radius measurements so that the corrected data set is consistent, and identify the faulty sensors. Assuming that there is only a few errors, we'd like to minimize the number of adjustments that are needed. This may be hard in general. Compare two heuristics: in one, we minimize the Euclidean norm of the vector of adjustments; in the other, we use the l_1-norm. Which approach best identifies the fourth measurement above as being faulty?

4. Assume now that the data is consistent, and that we ignore the fourth data point given above. Show how to compute an inner spherical approximation to the intersection of the spheres S_i, $i = 1, \ldots, 3$. Provide the answer for the data provided, and visualize the result.

5. Do the same with a inner box approximation; that is, find a square of largest size inside the intersection.

6. Find the best outer box approximation; that is, find a square of smallest size outside the intersection.

7. How would you solve the outer spherical approximation problem? Discuss.