

## FINAL PROBLEM SET

(Due 4:00PM, Monday, May 19, 2008)

**Turn in at my office (508 Cory) on or before 4PM sharp on Monday, May 19, 2008. Late papers will not be graded.**

Take home, open book. You may consult any sources you wish, however, only results from class notes, the two textbooks (*Introduction to Solid State Physics, 8th edition*, by Charles Kittel, and *Fundamentals of Carrier Transport*, by M. Lundstrom, either edition), or standard references (integral tables, physical constants, etc.) may be cited without derivation. **You must work strictly individually. No collaboration with anyone else, either in or out of this class is permitted. Violation of this rule will be considered to be an incident of cheating on an exam. See the Academic Dishonesty link on the class web page.**

If there are any important corrections or other announcements necessary during the period of the exam, I will post them on the class web site. Please check it periodically. I will do my best to respond in a timely fashion to questions about the exam that are emailed to me.

1. Consider electron impact ionization in a semiconductor, where a single high-energy electron loses energy by creating an electron-hole pair. If the initial kinetic energy of the electron is  $E_g$  (the band-gap energy), then the final state in an impact ionization process would have two electrons and one hole, all at rest.
  - a) Show that, in this case, momentum is not conserved.
  - b) Show further that, in order to simultaneously conserve both energy and momentum, the initial energy of the electron must exceed a threshold value considerably larger than  $E_g$ .
  - c) Find this threshold energy. Assume isotropic parabolic bands for electrons and holes and further assume a direct gap semiconductor. Assume unequal electron and hole effective mass.
2. We will make a simplified energy balance model of electron transport to obtain an analytical solution that displays velocity saturation and overshoot. Assume a uniform slab of length  $L_{\text{eff}}$ . Further assume a uniform electric field,  $E_x$ , with a step-like behavior at  $x = 0$  (the source). Finally, assume that charge density remains uniform throughout the slab. (This assumption is not valid if there is significant velocity variation along the slab, since current must be continuous. We will make a patch at the end to try to account for this.)
  - a) Write down the steady-state energy balance equation for this situation. Neglect the drift term in the energy density and the electronic heat diffusion term in the energy flux, as well as in the current. Further assume that the momentum relaxation time takes the form

$$\tau_m = \tau_{m0} \frac{T_0}{T_e}, \text{ and that the energy relaxation time, } \tau_w \text{ is a constant.}$$

b) Show that the electron temperature along the channel is given by:

$$T_e(x) = \frac{T_0}{2} [1 + A \tanh(Bx + c)]$$

Take  $T_e(x=0) = T_0 = 300K$  as a boundary condition, and find A, B, and C.

c) Use  $\mu_0 = 600 \text{ cm}^2/\text{Vsec}$  for the low field mobility and an energy relaxation time,  $\tau_w$ , of 0.4 psec. Plot  $T_e(x)$  for  $L_{eff} = 0.3 \text{ }\mu\text{m}$  and for  $E_x = 10^4 \text{ V/cm}$ ;  $5 \times 10^4 \text{ V/cm}$ ; and  $10^5 \text{ V/cm}$ .

d) Since  $T_e$  is a function of  $x$ , and  $\mu$  is a function of  $T_e$ , clearly  $v_d$  is *non-uniform* along  $x$ , violating current continuity. Find the *average* drift velocity along the device as

$$v_{av} = \frac{L_{eff}}{t_{transit}}$$

where

$$t_{transit} = \int_0^{L_{eff}} \frac{1}{v_d} dx$$

Plot  $v_{av}$  as a function of  $L_{eff}$  over the range  $10 \text{ nm} \leq L_{eff} \leq 1.0 \text{ }\mu\text{m}$  and for  $E_x = 10^3 \text{ V/cm}$ ,  $10^4 \text{ V/cm}$ , and  $10^5 \text{ V/cm}$ .

3. [20 points] Use the data contained in the figure below to deduce the energy relaxation time for electrons in GaAs quantum wells.

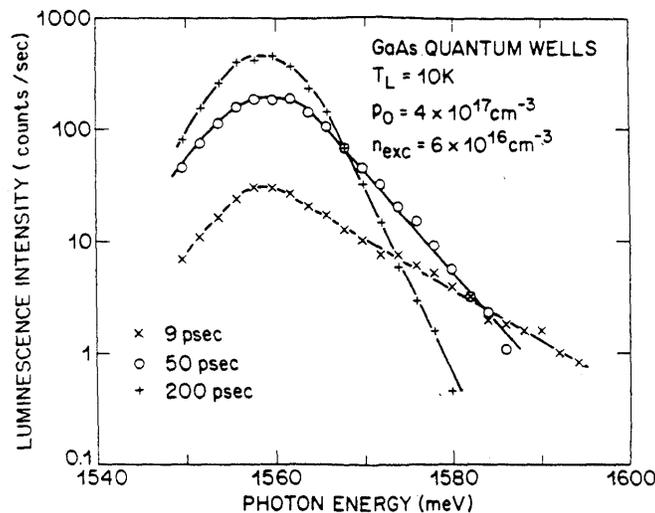


FIGURE 5. Time-resolved luminescence spectra of  $p$ -modulation-doped GaAs-AlGaAs quantum wells obtained using the luminescence upconversion technique. Note the large dynamic range, which allows a good determination of the carrier temperature from the spectra. (From [64].)

4. In problem set 4, you showed that, even for the complex conduction band structure of silicon, the net carrier acceleration is always along the direction of the force, for arbitrary direction of the force relative to the crystalline axes. In other words, the current is parallel to the applied field. Will this still be true for high fields (high enough to cause appreciable carrier heating)? Give a qualitative explanation for your answer, but be as specific and detailed as possible.
  
5. The Si(111) 2x1 reconstructed surface consists of  $\pi$ -bonded chains which can be treated as 1D semiconducting quantum wires. Two electronic surface bands are formed, a  $\pi$  "bonding" band, and a  $\pi^*$  "anti-bonding" band. A direct band-gap of 0.45 eV is formed between the two bands, and both bands have approximately equal effective mass,  $m^* = 0.16m_e$ . The separation between the chains is 6.65 Å. The atomic structure and a calculated electronic band structure is shown in Fig. 1. Note that the bandstructure calculation does not accurately match the measured value of the band-gap.

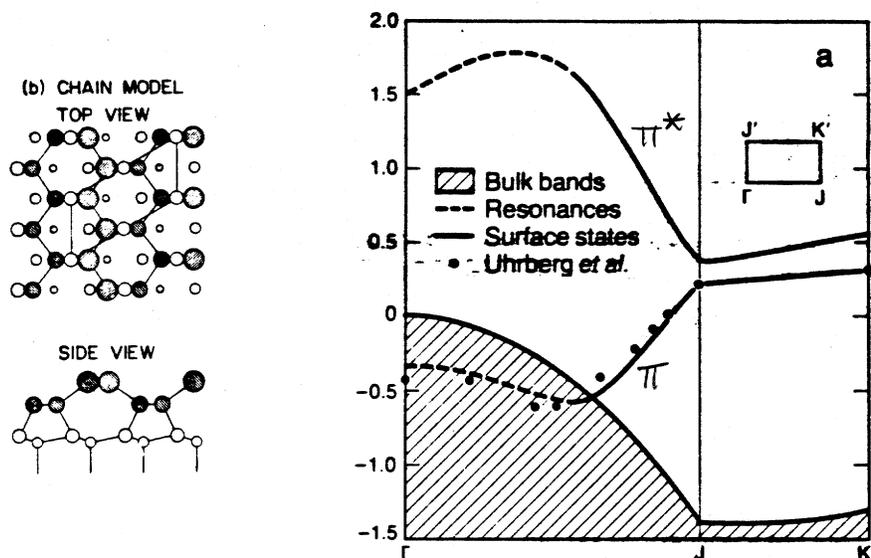


Figure 1

An optical absorption experiment shows a strong absorption due to  $\pi - \pi^*$  interband transitions. The peak absorption is found to be 2% at 0.47 eV. The absorption spectrum is roughly Gaussian in shape with a full width at half maximum of 0.1 eV. The results of this experiment are shown in Fig. 2. Since optical absorption occurs, this means that radiative recombination across this surface band-gap must also occur. Your task is to estimate the radiative recombination coefficient given the above data.

The recombination rate can be written:

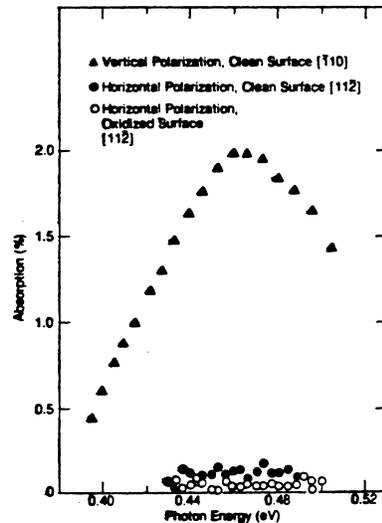


FIG. 2. Si(111)2x1 surface-state absorption spectrum. Saturation oxidation was obtained after ~1 h at  $10^{-7}$  Torr.

Figure 2

$$R = R_o \frac{np}{n_i}$$

$R$  is the recombination rate per unit surface area,  $n$  is the areal density of electrons in the  $\pi^*$  band,  $p$  is the areal density of holes in the  $\pi$  band, and  $n_i$  is the intrinsic areal density specific to this bandstructure. (Since the chains are 1-D, the areal density is equal to the linear density on individual chains divided by the inter-chain spacing.)

a) Give a physical interpretation for the fact that the band dispersions are strongly dispersive in the  $\Gamma$ - $J$  direction but nearly flat in the  $J$ - $K'$  direction.

b) Adapt the van Roosbroeck-Shockley analysis to this situation to calculate  $R_o$ . Use whatever means necessary to reach a numerical result. A numerical integration by computer or programmable calculator is perfectly acceptable. As an alternative, you may make whatever approximations necessary to at least get reasonable estimate, and then discuss the accuracy of the estimate given the approximations.

c) Derive a general expression for  $n_i^2$  and evaluate for  $T = 300\text{K}$ .

d) Assume the Fermi-level is pinned at the bottom of the  $\pi^*$  band such that  $n_o$  is given by the minimum "degenerate" density. What is the lifetime of excess holes in the  $\pi$  state?