University of California, Berkeley
EE230 - Solid State Electronics *Prof. J. Bokor*

PROBLEM SET 2

(Due Thurs., Feb. 28, 2008)

1. [Kittel, 4.6] Consider point ions of mass M and charge q immersed in a uniform sea of conduction electrons. The ions are imagined to be in stable equilibrium when at regular lattice points. If one ion is displaced a small distance r from its equilibrium position, the restoring force is largely due to the electric charge within the sphere of radius r centered at the equilibrium position. Take the number density of ions (or of conduction electrons) as $3/4\pi R^3$, which defines R.

   a) Show that the frequency of a single ion set into oscillation is $\omega = (e^2/MR^3)^{1/2}$.

   b) Estimate the value of this frequency for sodium metal. Calculate the corresponding wavelength of light that will excite this oscillation.

   c) From the above and some simple reasoning, estimate the velocity of sound in sodium. Find a literature value for the sound velocity in sodium for comparison.

2. Consider a crystal of GaAs in which the sound velocity is 4.7×10^5 cm/sec and the optical phonon energy is 36 meV and is assumed to have no dispersion. Using the Debye model for the acoustic phonon energy and the Einstein model for the optical phonons, calculate the lattice vibration energy per cm^3 at 77K, 300K, and 1000K. If the Ga-As bond energy is 1 eV, compare the phonon energy with the crystal binding energy per cm^3.

3. A 100 fs duration laser pulse irradiates a gold film of 1000Å in thickness. This heats the electrons instantaneously. On a time scale of a few ps, the electrons cool by transferring energy to the lattice.

   a) Assuming a laser pulse energy of 1 mJ is absorbed uniformly through the thickness of the film, and a uniform laser spot size of 1 cm^2, calculate the peak electron temperature in the spot using the free-electron heat capacity.

   b) Using a Debye model for the phonons, calculate the peak lattice temperature rise in the spot after the electrons equilibrate with the phonons. Assume there is no lateral heat diffusion. (Obtain values for the sound velocity and atomic density of gold from suitable reference sources. Give the citations.)

4. Fill in the steps skipped over in the class discussion of the harmonic oscillator to show:

   a) $[a, a^\dagger] = 1$

   b)
   $$\begin{aligned} H &= \frac{1}{2}\hbar\omega(a^\dagger a + a a^\dagger) \\ &= \hbar\omega\left(N + \frac{1}{2}\right) \end{aligned}$$

c) $\langle n'|n\rangle = \delta_{n'n}$

d) $Na^{\dagger}|n\rangle = (n+1)a^{\dagger}|n\rangle$

e) $a^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$

5. Calculate the density of states in lower dimensionalities.

a) For non-interacting particles in a 2-dimensional box, derive the density of states, D(E). This is the form used for quantum well heterostructures.

b) Repeat for 1-dimensional (quantum wire) case.

Assume a density of states mass, m^* for GaAs of $0.067m_0$.

c) Plot the density of electron states in GaAs for the 3D, 2D, and 1D cases. Give your results in units of $eV^{-1}cm^{-D}$, where D is the dimensionality.

d) What is the value of the Fermi level (at 0 K) for electron concentrations of 10^{18} cm^{-3} (3D case), 10^{12} cm^{-2} (2D case), 10^{6} cm^{-1} (1D case)?

e) Plot the function f(E)D(E) [where f(E) is the Fermi-Dirac function] as a function of energy, E, at T = 300K for each of the 3D, 2D and 1D cases. This plot shows the motivation for going towards lower dimensional systems for semiconductor lasers.

6. [Kittel, 7.5, see figure 12 in the book] For energies within the band-gap at the boundary (k=G/2) of the first Brillouin zone, we can have states if we allow k to become complex. Use the nearly free electron model with $V(x) = -2U\cos(2\pi x/a)$. Show that the result for small Im(k) is:

$$[Im(k)]^2 = \left(2\frac{m}{\hbar^2}\right)^2\frac{U^2}{G^2}$$

Give a physical interpretation. [Hint: consider what might happen at a metal/semiconductor interface.]