HW 6 (due 5/7/2009)
EE 230

1. (a) Starting from the expression for momentum relaxation due to non polar acoustic phonons:

$$\frac{1}{<\tau_m>} = \sum_{k'} \frac{k - k'\cos\theta}{k} S^a (1 - f_{k'}) + \sum_{k'} \frac{k - k'\cos\theta}{k} S^e (1 - f_{k'})$$

show that the momentum relaxation time is given by:

$$<\tau_m> = \frac{\pi\hbar^4 \rho c_s^2}{\left(2m^* k_B T\right)^{3/2} D^2} \sqrt{\frac{k_B T}{E}}$$

where θ is the angle between, k and k' and c_s is the velocity of sound.

Using the following parameters for GaAs calculate a numerical value for $<\tau_m>$ at an energy $E \sim k_B T$:

$c_s = 5240$ m/sec; $\rho = 5360$ kg/m^3; $D = 7$ eV; m$^* = 0.07 m_o$

1. (b)Starting from the expression for energy relaxation due to non polar acoustic phonons:

$$\frac{1}{<\tau_E>} = \sum_{k'} \frac{\hbar\omega}{E_k} S^a (1 - f_{k'}) + \sum_{k'} \frac{\hbar\omega}{E_k} S^e (1 - f_{k'})$$

find out the energy relaxation time for the same parameters as in (a).

1. (c) At this moment there is a lot of interest for using materials like GaAs rather than Si for transistors. Given that the typical gate delay in a nano-scale MOSFET is desired to be around 0.1 ps, do you think that interaction with non-polar acoustic phonons could be a possible bottleneck for GaAs based transistors?

2. (a) Starting from the momentum balance equation and assuming $\frac{\partial W_{zz}}{\partial z} \approx 0$ show that the following rate equation for current can be obtained:

$$\tilde{L}\frac{\partial I}{\partial t} + \tilde{R}I = -\frac{\partial V}{\partial z} \quad \text{---(1)}$$

where

$\frac{1}{\tilde{L}} = \frac{e^2 nA}{<\tau_m>}$ is an inductance per unit length; A = area

$\tilde{R} = \frac{\tilde{L}}{<\tau_m>}$ is a resistance per unit length

Show that Eq. (1) can be modified to:

$$V = RI + L\frac{\partial I}{\partial t} \qquad \text{----} \qquad (2)$$

where V is the applied potential, L is the lumped inductance and R is the lumped resistance. Eq. 2 is the well-known expression for a series R-L circuit.

Note that this inductance has no dependence on magnetic field and rather depends inversely on the number of electrons. This is often called the 'kinetic inductance' of a material.

(b) Following the approach shown in the paper, TED. 52, 8, pg. 1734 (available on the class website), find out the equations for a transmission line (Eq. 22 and 23) for a 1D conductor. Clearly state the assumptions made. Draw the equivalent circuit of the transmission line.

(c) What is the difference between the first equation found in 2. (a) (Eq. (1)), and the one derived in 2. (b) (Eq. 22 in the paper). Hint: think about what causes the current.

(d) Starting from the equation for modes, M, shown in Eq. (25) in the paper, show that

$$\frac{1}{L_k} = \frac{4e^2}{h} M \frac{<v^2>}{<v>} \text{ where } v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

Note that this expression for M accounts for one spin and one branch only.

(e) Calculate the kinetic inductance in a metallic carbon nanotube assuming that there is only one subband $E = \pm h v_F k$. How does this compare with typical magnetic inductance of nanotubes~0.5 pH/μ m?

(f) From the above analysis, how does confinement affect the speed of signal propagation?

(g) Do you think you can make an efficient on-chip antenna using the kinetic inductance? Explain.