

## HW #5

Due Nov. 4 (Tuesday) in class

	Unit	GaAs	$\text{Al}_x\text{Ga}_{1-x}\text{As}$ , $0 < x < 0.45$
<b>Bandgap Energy</b>	eV	1.424	$1.424 + 1.247x$
<b>Electron Effective Mass</b>	$m_0$	0.067	$0.067 + 0.083x$
<b>Hole Effective Mass</b>	$m_0$	0.5	$0.5 + 0.29x$
<b>Dielectric Constant</b>	$\epsilon_0$	13.1	$13.1 - 3x$
<b>Conduction Band Discontinuity</b>	%	-	$\Delta E_C \sim 67\% \Delta E_g$
<b>Valence Band Discontinuity</b>	%	-	$\Delta E_V \sim 33\% \Delta E_g$

- Consider a double heterostructure (DH) laser with  $\text{Al}_{0.4}\text{Ga}_{0.6}\text{As}$  cladding layers and GaAs active (gain) layer. Use the Table above for their refractive indices. The active layer is  $0.1 \mu\text{m}$  thick. The laser is  $200 \mu\text{m}$  long and  $1 \mu\text{m}$  wide. Both mirrors are cleaved facets with a power reflectivity of 30%. The residue loss is  $20 \text{ cm}^{-1}$ . Assume the carrier lifetime is 1 ns (a constant value, which is an approximation). Use linear gain approximation  $g = a \cdot (N - N_{tr})$  where  $a = 10^{-16} \text{ cm}^2$  and  $N_{tr} = 10^{18} \text{ cm}^{-3}$ 
  - Solve the fundamental TE mode as outlined in class. Calculate and plot the optical field profile.
  - Numerically calculate the optical confinement factor using the field profile in Part a).
  - Find the “V parameter” of the DH waveguide. How many modes does the waveguide support?
  - Calculate the approximate confinement factor of the DH waveguide using V parameter. Compare that with the numerical calculation in Part b).
  - Find the threshold gain and threshold current of the laser
  - What is the external quantum efficiency of the laser? Express your answers in both percentage (%) and in W/A.
  - Ignore the material dispersion, find the mode spacing of this laser (in wavelength, with unit of nm).
- In this problem, you will show that the laser output is basically amplified spontaneous emission. When we include the spontaneous emission term in the rate equation, the carrier concentration is very close to, but never reaches, the threshold value.

The rate equations we discussed in class are:

$$\frac{dN}{dt} = \eta_i \frac{I}{qV_{active}} - \frac{N}{\tau} - v_g g(N) \cdot S$$

$$\frac{dS}{dt} = \Gamma v_g g(N) \cdot S - \frac{S}{\tau_p} + \Gamma \beta \cdot R_{sp}$$

where  $N$  is carrier concentration,  $S$  is the photon density;  $V_{active}$  is the volume of the active layer (i.e., laser width x length x thickness);  $\tau$  is the carrier lifetime,  $\frac{N}{\tau} = AN + BN^2$  (assume Auger recombination is negligible),  $v_g = c/n_g$  is the speed of light in semiconductor;  $g(N) = a \cdot (N - N_{tr})$  is gain coefficient. Here, we use linear gain approximation.  $\Gamma$  is the optical confinement factor;  $\tau_p = \frac{1}{v_g(\alpha_m + \alpha_i)}$  is the photon lifetime;  $\beta$  is the spontaneous emission factor, i.e., the fraction of the spontaneous emission coupled to the lasing mode; and  $R_{sp} = BN^2$  is the spontaneous rate.

- In steady state, show that  $S(N) = \frac{\Gamma\beta \cdot R_{sp}(N)}{1/\tau_p - \Gamma v_g g(N)}$  for both below and above threshold.
- In steady state, show that  $I(N) = \frac{qV_{active}}{\eta_i} \left( \frac{N}{\tau} + v_g g(N) \cdot S(N) \right)$  for both below and above threshold.
- Show that the output power  $P(N) = \hbar\omega \frac{V_{active}}{\Gamma} \alpha_m v_g S(N)$ .
- Since both output power  $P(N)$  and current  $I(N)$  are both functions of  $N$ , we can plot the light-versus-current (L-I) curve using  $N$  as a parameter. Use the following parameters for the plot:

$$R_1 = R_2 = 30\%, \quad N_{tr} = 10^{18} \text{ cm}^{-3}, \quad a = 10^{-16} \text{ cm}^2, \quad \Gamma = 50\%$$

$$\alpha_i = 10 \text{ cm}^{-1}, \quad L = 300 \text{ }\mu\text{m}, \quad w = 1 \text{ }\mu\text{m}, \quad t = 0.1 \text{ }\mu\text{m}, \quad \tau = 1 \text{ nsec}$$

$$\eta_i = 100\%, \quad \eta_r = 90\%$$

$$\beta = 10^{-2}$$

$$A = 2 \times 10^8 \text{ sec}^{-1}, \quad B = 10^{-9} \text{ sec}^{-1} \text{ cm}^{-3}$$

$$\lambda = 1550 \text{ nm}, \quad n_r = 3.5 \text{ (effective refractive index)}$$

Calculate and plot the L-I curve of the laser (i.e.,  $P(N)$  versus  $I(N)$ ) in both linear and logarithmic scales for the vertical axis. Choose the range of  $N$  such that the output power goes from 0 to ~ 10 mW.

**Please note that the  $N < N_{th}$  even above threshold, though it is getting closer and closer to  $N_{th}$  above threshold. To show clearly the behavior around threshold, you need to use a very fine increment for  $N$ . Alternatively, you can use non-uniform intervals with much denser points near  $N_{th}$ .**

- Plot the carrier concentration  $N$  as a function of current  $I$  for the same range of  $I$  as in part d. Again, use both linear and logarithmic scale for the vertical axis.
- To show effect of  $\beta$  on the L-I characteristics, plot L-I curves for  $\beta = 0.001, 0.01, \text{ and } 0.1$  in the same graph. Choose the ranges of  $N$  such that the output power reaches 10 mW for all  $\beta$ 's. Likewise, plot N-I curve for the three  $\beta$  values. Show your plots in log-linear and linear-linear plots for both families of curves.