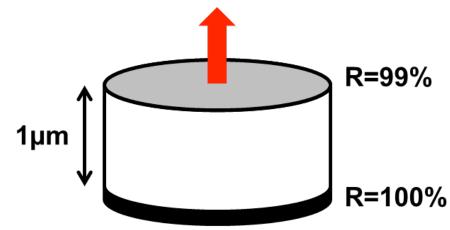
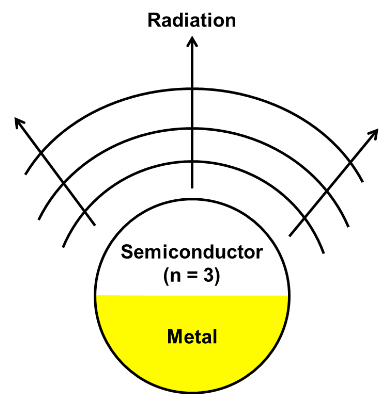
HW #1
Due Feb. 25 (Thursday) in class

1. Consider a vertical cavity surface-emitting laser with a top mirror reflectivity of 99% and a bottom mirror reflectivity of 100%, an effective cavity length of 1 μm, and a diameter of 2 μm. The confinement factor is 100%. The residue loss of the active media is 10cm^{-1} and the refractive index is 3.5. The laser wavelength is 1 μm.



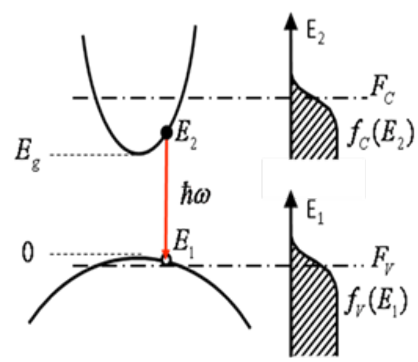
   a. Find the quality factor (Q) of the cavity.
   b. Find the threshold gain and quantum efficiency of the laser.

2. Consider a laser with metallodielectric cavity shown on the right. It consists of a 100-nm-diameter spherical cavity with half metal and half semiconductor. Assume 30% of the energy is in the metal, and the remaining 70% is in the semiconductor. The quality factor of the metal is 50 (note: the Q of a material describes the loss of that material, it is different from the cavity Q). Assume the residue loss in semiconductor is negligible (i.e., its material Q is ∞). The cavity acts like a dipole antenna and radiates optical energy (i.e., output light). Assume the radiation Q is 500 (i.e., the Q associated with the radiation loss is 500). The laser wavelength is 1 μm.



   a. What is the total Q of the cavity?
   b. What is the threshold gain and quantum efficiency of the laser?

3. Refer to the diagram on the right. Under biased condition, both conduction and valence bands are populated. The electron distribution in conduction band is described by Fermi-Dirac distribution, $f_C(E_2)$, with quasi-Fermi energy F_C. The electron distribution in valence band is described by Fermi-Dirac distribution, $f_V(E_1)$, with quasi-Fermi energy F_V. Here, E_1 and E_2 are related by an optical transition (i.e., they have the same k). The optical matrix element is



$$\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 = \frac{m_0}{6}E_p \quad \text{with } E_p = 25.7 \text{ eV}$$

   a. Use the energy reference below (i.e, $E_V = 0$ and $E_C = E_g$, the bandgap energy), find E_1 and E_2 as functions of the photon energy, $\hbar\omega$.
   b. Derive $f_C(E_2(\hbar\omega))$ as a function of $\hbar\omega$.
   c. Derive $f_V(E_1(\hbar\omega))$ as a function of $\hbar\omega$.

d. Assuming E_g = 1 eV, $F_C - F_V$ = 1.2 eV, $m_e^* = 0.1m_0$, $m_h^* = 0.4m_0$. Calculate and plot the emission probability $f_e(\text{h}\omega) = f_C(E_2(\text{h}\omega)) \cdot [1 - f_V(E_1(\text{h}\omega))]$ for photon energies from 0.8 to 1.5 eV. Plot for two temperatures: T = 0 and T = 300 K.

e. Repeat part d) for the Fermi inversion factor: $f_g(\text{h}\omega) = f_C(E_2(\text{h}\omega)) - f_V(E_1(\text{h}\omega))$

f. Plot the gain spectra for T = 0 and T = 300 K for the condition given in d).

g. Plot the spontaneous emission spectra for T = 0 and T = 300 K for the condition given in d).