Homework 1

1. (a) Find the quality (Q) factor of the cavity.

$$Q = \omega\tau = \frac{2\pi c}{\lambda_0}\tau$$

$$\tau = \frac{n}{\alpha c}$$

$$\alpha = \alpha_m + \alpha_i = \frac{1}{2L}\ln\frac{1}{R_1R_2} + \alpha_i$$

Using $\alpha_i = 10\ \mathrm{cm}^{-1}$, $R_1 = 1$, $R_2 = 0.99$, and $L = 1\mu$m, we find

$$\alpha_m = 50.25\ \mathrm{cm}^{-1},$$

$$\alpha = 60.25\ \mathrm{cm}^{-1}.$$

Plugging in, and using $\lambda_0 = 1\mu$m and $n = 3.5$, we find

$$Q = \frac{2\pi \cancel{c}}{\lambda_0}\frac{n}{\alpha \cancel{c}} = 3650.$$

(b) Find the threshold gain and quantum efficiency of the laser.

$$g_{th} = \frac{\alpha}{\Gamma} = 60.25\ \mathrm{cm}^{-1}$$

using α from before and $\Gamma = 1$.

$$\eta = \frac{\alpha_m}{\alpha_i + \alpha_m} = 0.834$$

using α_m and α_i from before.

2. (a) What is the total Q of the cavity?
Start with the definition of Q:

$$Q = \omega\frac{E}{P_{tot}} = \omega\frac{E}{P_r + P_m + P_s},$$

where P_{tot} is total power lost, and P_r, P_m, and P_s are power lost in radiation, metal, and semiconductor respectively. We also have

$$P_m = \omega\frac{0.3E}{Q_m},$$

$$P_r = \omega\frac{E}{Q_r},$$

$$P_s = 0,$$

where $Q_m = 50$ and $Q_r = 500$. Plugging these in and simplifying, we get

$$\begin{aligned} Q &= \omega \frac{E}{0.3\omega E/Q_m + \omega E/Q_r} \\ &= \frac{1}{0.3/Q_m + 1/Q_r} \\ &= 125. \end{aligned}$$

(b) What is the threshold gain and quantum efficiency of the laser?

$$\begin{aligned} g_{th} &= \frac{\alpha}{\Gamma} \\ &= \frac{2\pi n}{\lambda_0 Q \Gamma} \\ &= 2154 \text{ cm}^{-1}, \end{aligned}$$

using $n = 3$, $\lambda_0 = 1\mu$m, and $\Gamma = 0.7$, since 70% of the energy is in the semiconductor, and only the semiconductor experiences gain.

$$\eta = \frac{P_r}{P_{tot}} = \frac{Q}{Q_r} = 0.25$$

3. (a) Use the energy reference below (i.e, $E_V = 0$ and $E_C = E_g$, the bandgap energy), find E_1 and E_2 as functions of the photon energy, $\hbar\omega$.

$$\begin{aligned} E_1 &= -\frac{\hbar^2 k^2}{2m_h^*} \\ E_2 &= E_g + \frac{\hbar^2 k^2}{2m_e^*} \end{aligned}$$

Subtract E_2 and E_1 and equate to $\hbar\omega$:

$$\begin{aligned} E_2 - E_1 &= \hbar\omega \\ &= E_g + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) \\ &= E_g + \frac{\hbar^2 k^2}{2m_r^*} \end{aligned}$$

Rewriting E_1 and E_2 in terms of m_r^* and using the above equation, we get:

$$\begin{aligned} E_1 &= -\frac{\hbar^2 k^2}{2m_r^*}\frac{m_r^*}{m_h^*} \\ &= -\frac{m_r^*}{m_h^*}(\hbar\omega - E_g) \end{aligned}$$

$$
\begin{aligned}
E_2 &= E_g + \frac{\hbar^2 k^2}{2m_r^*}\frac{m_r^*}{m_e^*} \\
&= E_g + (\hbar\omega - E_g)\frac{m_r^*}{m_e^*}
\end{aligned}
$$

(b) Derive $f_C(E_2(\hbar\omega))$ as a function of $\hbar\omega$.

$$
\begin{aligned}
f_C(E_2) &= \frac{1}{1+e^{(E_2-F_C)/kT}} \\
&= \frac{1}{1+e^{\left(E_g+(\hbar\omega-E_g)\frac{m_r^*}{m_e^*}-F_C\right)/kT}},
\end{aligned}
$$

plugging in for E_2 from above.

(c) Derive $f_V(E_1(\hbar\omega))$ as a function of $\hbar\omega$.

$$
\begin{aligned}
f_V(E_1) &= \frac{1}{1+e^{(E_1-F_V)/kT}} \\
&= \frac{1}{1+e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega-E_g)-F_V\right)/kT}}
\end{aligned}
$$

(d) Calculate and plot the emission probability $f_e(\hbar\omega) = f_C(\hbar\omega)(1 - f_V(\hbar\omega))$ for photon energies from 0.8 to 1.5 eV. Plot for two temperatures: T = 0 and T = 300 K.

Simply plug in f_C and f_V found above:

$$
f_e(\hbar\omega) = \frac{1}{1+e^{\left(E_g+(\hbar\omega-E_g)\frac{m_r^*}{m_e^*}-F_C\right)/kT}}\left(1-\frac{1}{1+e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega-E_g)-F_V\right)/kT}}\right)
$$

To use this equation we must find F_C and F_V.

Since $F_C - F_V > E_g$, F_C and/or F_V is degenerate. Since $m_e^* < m_h^*$, electrons are degenerate: $F_C > E_g$. Assume holes are also degenerate: $F_V < 0$. Then we have

$$
\begin{aligned}
n &\propto N_C(F_C - E_g)^{3/2} \propto m_e^{*3/2}(F_C - E_g)^{3/2} \\
p &\propto N_V(-F_V)^{3/2} \propto m_h^{*3/2}(-F_V)^{3/2}
\end{aligned}
$$

with the same proportionality factor for n and p. Since $n = p$, we have

$$
\begin{aligned}
\frac{m_h^*}{m_e^*} &= \frac{F_C - E_g}{-F_V} \\
4 &= \frac{F_V + 1.2eV - 1eV}{-F_V} \\
-4F_V &= F_V + 0.2eV \\
F_V &= -0.04eV \\
F_C &= F_V + 1.2eV = 1.16eV.
\end{aligned}
$$

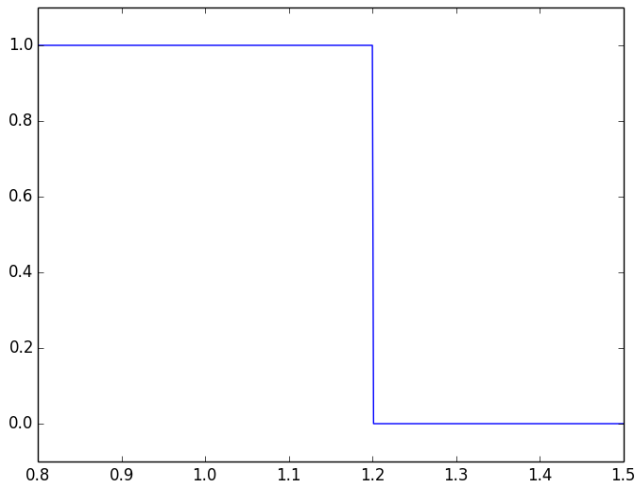

Figure 1: f_e vs $\hbar\omega$ (eV) at $T = 0K$.

Note that $F_V < 0$, justifying our original assumption.

f_e is plotted in Figs. 1 and 2.

(e) Repeat part d) for the Fermi inversion factor $f_g(\hbar\omega) = f_C(\hbar\omega) - f_V(\hbar\omega)$.

$$f_g(\hbar\omega) = \frac{1}{1 + e^{\left(E_g + (\hbar\omega - E_g)\frac{m_r^*}{m_e^*} - F_C\right)/kT}} - \frac{1}{1 + e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega - E_g) - F_V\right)/kT}}$$

f_g is plotted in Figs. 3 and 4.

(f) Plot the gain spectra for T = 0 and T = 300 K for the condition given in d).

$$g(\hbar\omega) = C_0 \left|\mathbf{\hat{e}} \cdot \mathbf{p}\right|^2 \rho_r(\hbar\omega - E_g) f_g(\hbar\omega)$$

$$C_0 = \frac{\pi e^2}{n c \epsilon_0 m_0^2 \omega}$$

$$\left|\mathbf{\hat{e}} \cdot \mathbf{p}\right|^2 = \frac{m_0}{6} E_p, \quad E_p = 25.7eV$$

$$\rho_r(\hbar\omega - E_g) = H(\hbar\omega - E_g) \frac{1}{2\pi^2} \left(\frac{2m_r^*}{\hbar^2}\right)^{3/2} \sqrt{\hbar\omega - E_g}$$

$$f_g(\hbar\omega) = f_C(\hbar\omega) - f_V(\hbar\omega)$$

Assuming $n = 3.5$, $g(\hbar\omega)$ is plotted in Figs. 5 and 6.

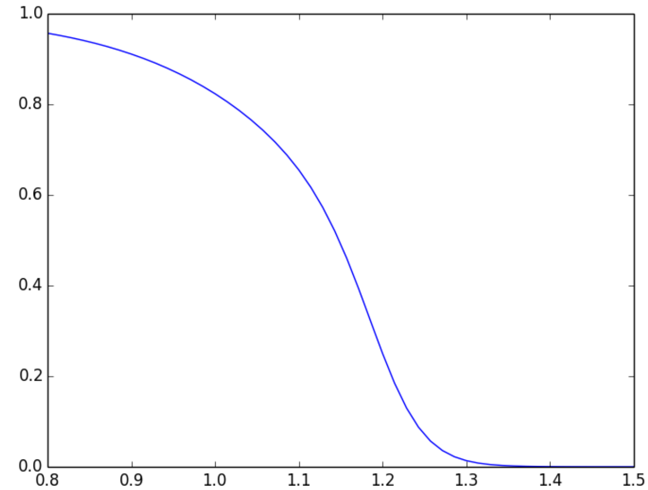

Figure 2: f_e vs $\hbar\omega$ (eV) at $T = 300K$.

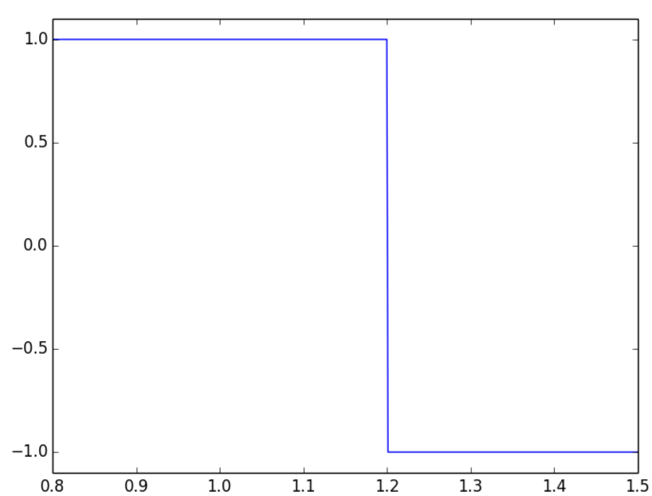

Figure 3: f_g vs $\hbar\omega$ (eV) at $T = 0K$.

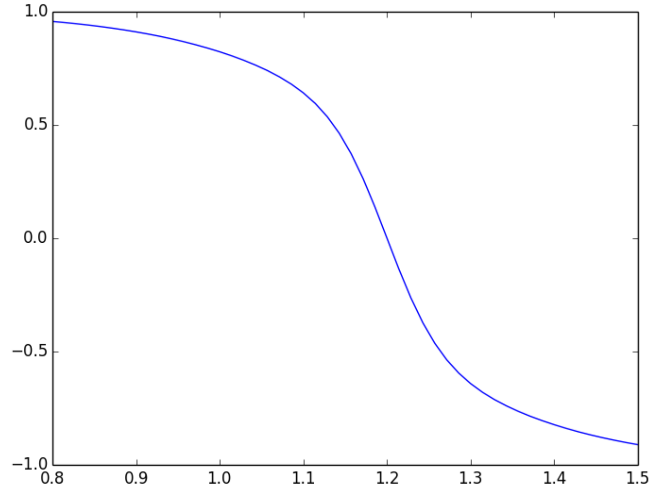

Figure 4: f_g vs $\hbar\omega$ (eV) at $T = 300K$.

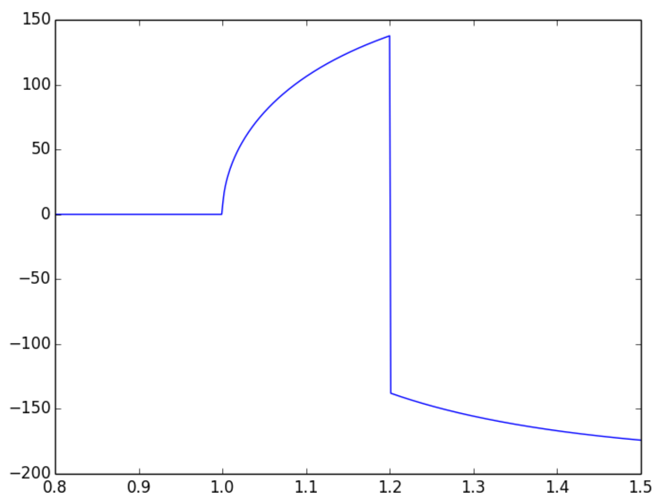

Figure 5: g (cm^{-1}) vs $\hbar\omega$ (eV) at $T = 0K$.

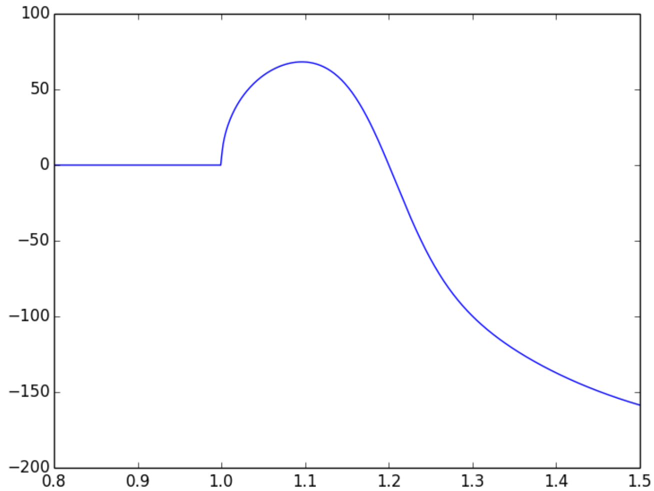

Figure 6: g (cm^{-1}) vs $\hbar\omega$ (eV) at $T = 300K$.

(g) Plot the spontaneous emission spectra for T = 0 and T = 300 K for the condition given in d).

$$r_{\text{spon}}(\hbar\omega) = \frac{8\pi n^2(\hbar\omega)^2}{h^3c^2}\frac{f_e(\hbar\omega)}{f_g(\hbar\omega)}g(\hbar\omega)$$

r_{spon} is plotted in Figs. 7 and 8.

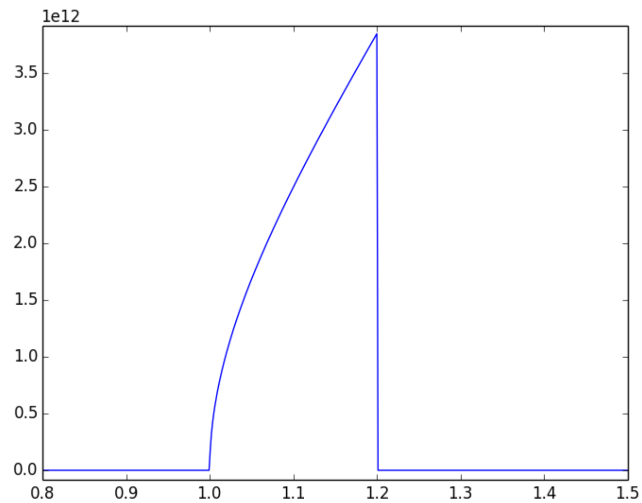

Figure 7: r_{spon} ($\text{eV}^{-1}\text{um}^{-3}\text{s}^{-1}$) vs $\hbar\omega$ (eV) at $T = 0K$.

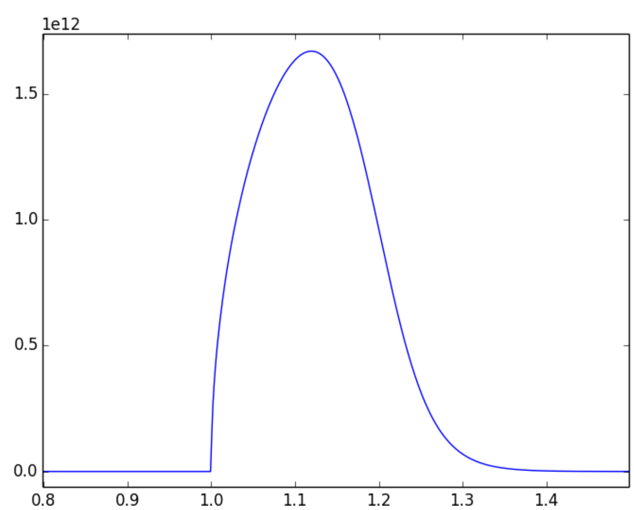

Figure 8: r_{spon} ($\text{eV}^{-1}\text{um}^{-3}\text{s}^{-1}$) vs $\hbar\omega$ (eV) at $T = 300K$.