EE 232, HW#2 Solution

(a) Use the energy reference below (i.e, $E_V = 0$ and $E_C = E_g$, the bandgap energy), find E_1 and E_2 as functions of the photon energy, $\hbar\omega$.

$$E_1 = -\frac{\hbar^2 k^2}{2m_h^*}$$
$$E_2 = E_g + \frac{\hbar^2 k^2}{2m_e^*}$$

Subtract E_2 and E_1 and equate to $\hbar\omega$:

$$\begin{aligned} E_2 - E_1 &= \hbar\omega \\ &= E_g + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) \\ &= E_g + \frac{\hbar^2 k^2}{2m_r^*} \end{aligned}$$

Rewriting E_1 and E_2 in terms of m_r^* and using the above equation, we get:

$$\begin{aligned} E_1 &= -\frac{\hbar^2 k^2}{2m_r^*}\frac{m_r^*}{m_h^*} \\ &= -\frac{m_r^*}{m_h^*}(\hbar\omega - E_g) \end{aligned}$$

$$E_2 = E_g + \frac{\hbar^2 k^2}{2m_r^*}\frac{m_r^*}{m_e^*}$$
$$= E_g + (\hbar\omega - E_g)\frac{m_r^*}{m_e^*}$$

(b) Derive $f_C(E_2(\hbar\omega))$ as a function of $\hbar\omega$.

$$f_C(E_2) = \frac{1}{1+e^{(E_2-F_C)/kT}}$$
$$= \frac{1}{1+e^{\left(E_g+(\hbar\omega-E_g)\frac{m_r^*}{m_e^*}-F_C\right)/kT}},$$

plugging in for E_2 from above.

(c) Derive $f_V(E_1(\hbar\omega))$ as a function of $\hbar\omega$.

$$f_V(E_1) = \frac{1}{1+e^{(E_1-F_V)/kT}}$$
$$= \frac{1}{1+e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega-E_g)-F_V\right)/kT}}$$

(d) Calculate and plot the emission probability $f_e(\hbar\omega) = f_C(\hbar\omega)(1 - f_V(\hbar\omega))$ for photon energies from 0.8 to 1.5 eV. Plot for two temperatures: T = 0 and T = 300 K.

Simply plug in f_C and f_V found above:

$$f_e(\hbar\omega) = \frac{1}{1+e^{\left(E_g+(\hbar\omega-E_g)\frac{m_r^*}{m_e^*}-F_C\right)/kT}}\left(1-\frac{1}{1+e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega-E_g)-F_V\right)/kT}}\right)$$

To use this equation we must find F_C and F_V.

Since $F_C - F_V > E_g$, F_C and/or F_V is degenerate. Since $m_e^* < m_h^*$, electrons are degenerate: $F_C > E_g$. Assume holes are also degenerate: $F_V < 0$. Then we have

$$n \propto N_C(F_C - E_g)^{3/2} \propto m_e^{*3/2}(F_C - E_g)^{3/2}$$
$$p \propto N_V(-F_V)^{3/2} \propto m_h^{*3/2}(-F_V)^{3/2}$$

with the same proportionality factor for n and p. Since $n = p$, we have

$$\frac{m_h^*}{m_e^*} = \frac{F_C - E_g}{-F_V}$$
$$4 = \frac{F_V + 1.2eV - 1eV}{-F_V}$$
$$-4F_V = F_V + 0.2eV$$
$$F_V = -0.04eV$$
$$F_C = F_V + 1.2eV = 1.16eV.$$

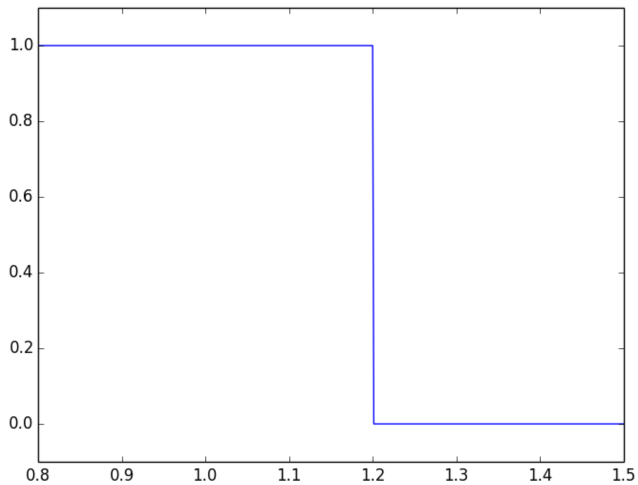

Figure 1: f_e vs $\hbar\omega$ (eV) at $T = 0K$.

Note that $F_V < 0$, justifying our original assumption.

f_e is plotted in Figs. 1 and 2.

(e) Repeat part d) for the Fermi inversion factor $f_g(\hbar\omega) = f_C(\hbar\omega) - f_V(\hbar\omega)$.

$$f_g(\hbar\omega) = \frac{1}{1 + e^{\left(E_g + (\hbar\omega - E_g)\frac{m_r^*}{m_e^*} - F_C\right)/kT}} - \frac{1}{1 + e^{\left(-\frac{m_r^*}{m_h^*}(\hbar\omega - E_g) - F_V\right)/kT}}$$

f_g is plotted in Figs. 3 and 4.

(f) Plot the gain spectra for T = 0 and T = 300 K for the condition given in d).

$$g(\hbar\omega) = C_0 \left|\hat{\mathbf{e}} \cdot \mathbf{p}\right|^2 \rho_r(\hbar\omega - E_g) f_g(\hbar\omega)$$

$$C_0 = \frac{\pi e^2}{n c \epsilon_0 m_0^2 \omega}$$

$$\left|\hat{\mathbf{e}} \cdot \mathbf{p}\right|^2 = \frac{m_0}{6} E_p, \quad E_p = 25.7eV$$

$$\rho_r(\hbar\omega - E_g) = H(\hbar\omega - E_g)\frac{1}{2\pi^2}\left(\frac{2m_r^*}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}$$

$$f_g(\hbar\omega) = f_C(\hbar\omega) - f_V(\hbar\omega)$$

Assuming $n = 3.5$, $g(\hbar\omega)$ is plotted in Figs. 5 and 6.

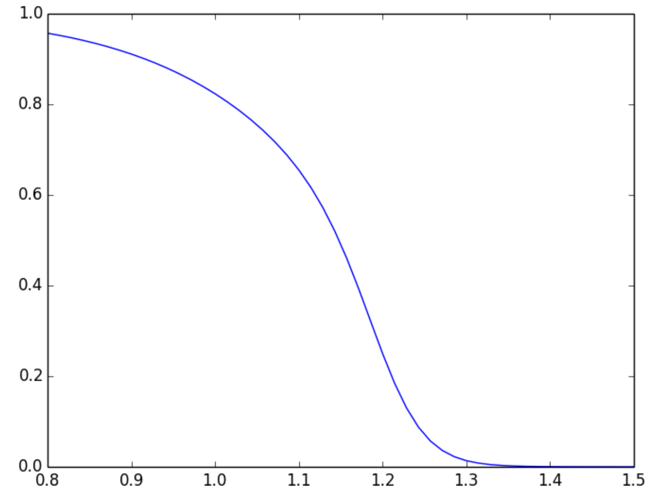

Figure 2: f_e vs $\hbar\omega$ (eV) at $T = 300K$.

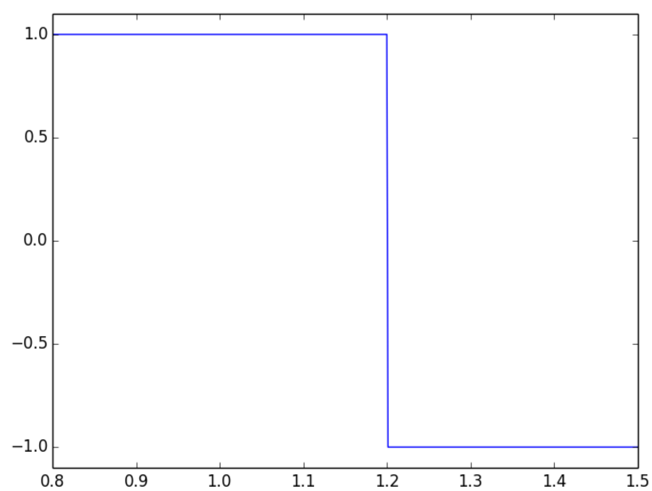

Figure 3: f_g vs $\hbar\omega$ (eV) at $T = 0K$.

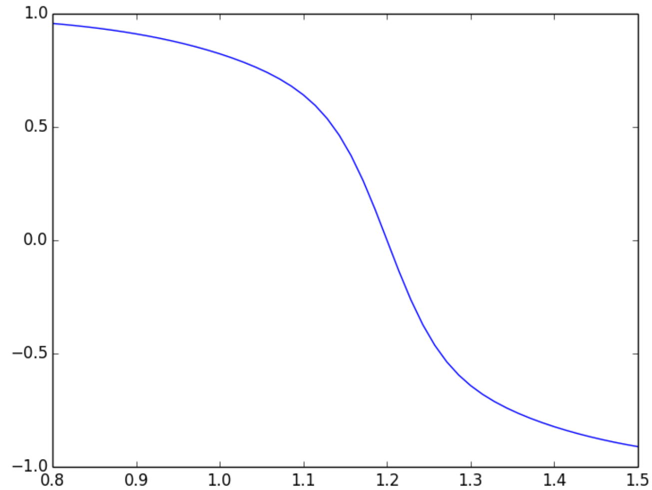

Figure 4: f_g vs $\hbar\omega$ (eV) at $T = 300K$.

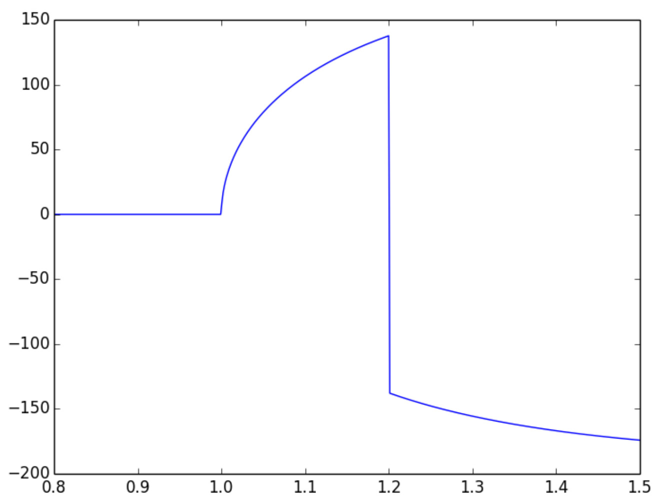

Figure 5: g (cm^{-1}) vs $\hbar\omega$ (eV) at $T = 0K$.

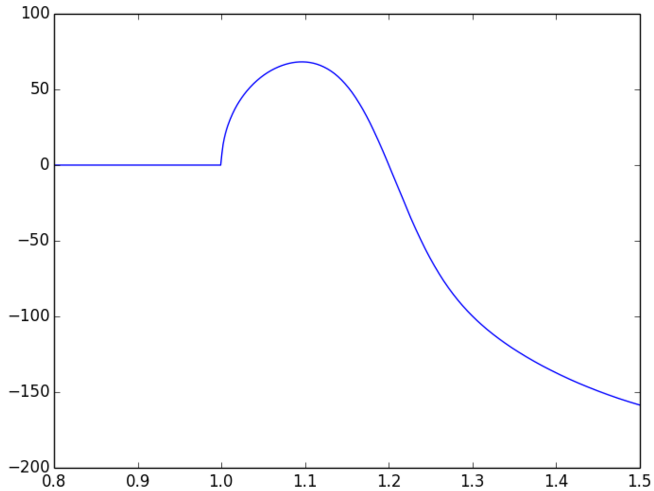

Figure 6: g (cm^{-1}) vs $\hbar\omega$ (eV) at $T = 300K$.

(g) Plot the spontaneous emission spectra for T = 0 and T = 300 K for the condition given in d).

$$r_{\text{spon}}(\hbar\omega) = \frac{8\pi n^2(\hbar\omega)^2}{h^3c^2}\frac{f_e(\hbar\omega)}{f_g(\hbar\omega)}g(\hbar\omega)$$

r_{spon} is plotted in Figs. 7 and 8.

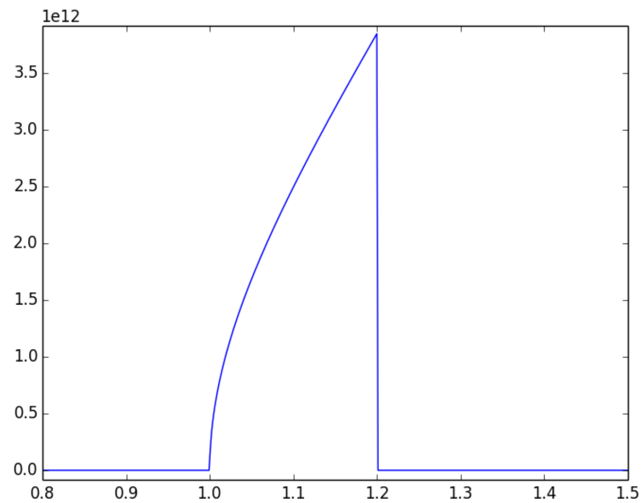

Figure 7: r_{spon} ($\text{eV}^{-1}\text{um}^{-3}\text{s}^{-1}$) vs $\hbar\omega$ (eV) at $T = 0K$.

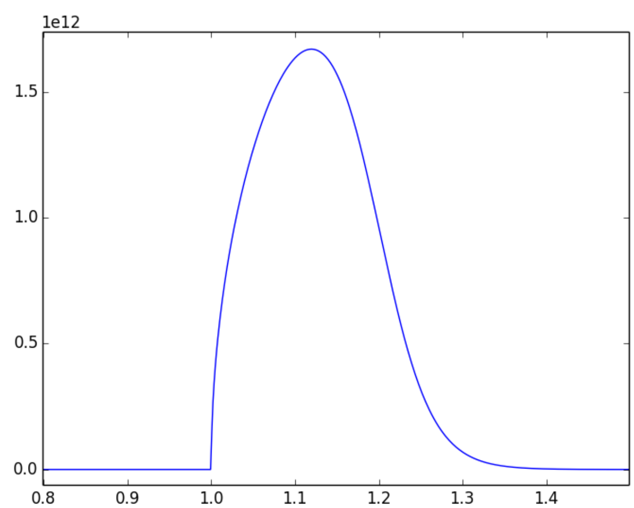

Figure 8: r_{spon} ($\text{eV}^{-1}\text{um}^{-3}\text{s}^{-1}$) vs $\hbar\omega$ (eV) at $T = 300K$.