

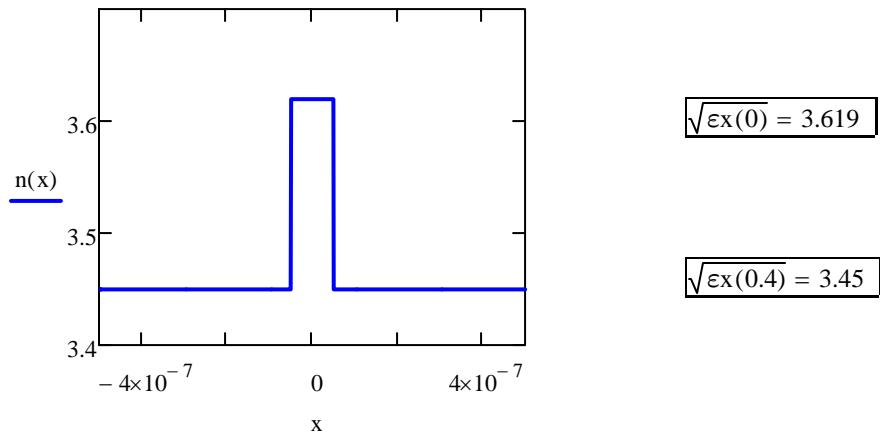
$$\begin{aligned}
 h_{\text{bar}} &:= 1.05459 \cdot 10^{-34} \cdot \text{J} \cdot \text{s} & q &:= 1.6 \cdot 10^{-19} \cdot \text{C} & m_0 &:= 9.11 \cdot 10^{-31} \cdot \text{kg} \\
 m_e &:= 0.067 \cdot m_0 & eV &:= q \cdot V & meV &:= 10^{-3} \text{ eV} \\
 k_B &:= 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}} & T_{\text{green}} &:= 300 \text{ K} & kB \cdot T &= 0.026 \cdot \text{eV} \\
 \epsilon_0 &:= 8.854 \cdot 10^{-12} \frac{\text{F}}{\text{m}} & \mu_0 &:= 4\pi \cdot 10^{-7} \cdot \frac{\text{H}}{\text{m}}
 \end{aligned}$$

$$1. \quad Egx(x) := (1.424 + 1.247 \cdot x) \text{ eV} \quad Egx(0.4) = 1.923 \cdot \text{eV}$$

$$m_{\text{ex}}(x) := (0.067 + 0.083 \cdot x) \cdot m_0 \quad m_{\text{hx}}(x) := (0.5 + 0.29 \cdot x) \cdot m_0$$

$$\epsilon_x(x) := 13.1 - 3 \cdot x$$

$$(a) \quad n(x) := \text{if}(-0.05 \mu\text{m} < x < 0.05 \mu\text{m}, \sqrt{\epsilon_x(0)}, \sqrt{\epsilon_x(0.4)})$$



$$\phi(x) = \begin{cases} C_0 e^{-\alpha(|x|-d/2)}, & |x| \geq d/2 \\ C_1 \cos(k_x x), & |x| < d/2 \end{cases}$$

$$k_x^2 + k_z^2 = \omega^2 \mu \epsilon_1 = \omega^2 \mu \epsilon_0 \epsilon_{r1} = k_0^2 \epsilon_{rcore}$$

$$-\alpha^2 + k_z^2 = \omega^2 \mu \epsilon = k_0^2 \epsilon_{rclad}$$

$$\text{Let } k_z = k_0 n_{\text{eff}}$$

$$k_x = k_0 \sqrt{n_{\text{core}}^2 - n_{\text{eff}}^2}$$

$$\alpha = k_0 \sqrt{n_{\text{eff}}^2 - n_{\text{clad}}^2}$$

Boundary Condition:

$$\alpha = k_x \tan(k_x d / 2)$$

$$\lambda := \frac{1.24\text{eV}\cdot\mu\text{m}}{\text{Eg}(0)} \quad \lambda = 0.871\cdot\mu\text{m} \quad \omega := 2\pi\cdot\frac{c}{\lambda} \quad \omega = 2.163 \times 10^{15} \frac{1}{\text{s}}$$

$$d := 0.1\mu\text{m} \quad k_0 := \frac{2\cdot\pi}{\lambda} \quad k_0 = 7.216 \times 10^6 \frac{1}{\text{m}}$$

$$\alpha(\text{neff}) := k_0\cdot\sqrt{\text{neff}^2 - \varepsilon_x(0.4)}$$

$$k_x(\text{neff}) := k_0\cdot\sqrt{\varepsilon_x(0) - \text{neff}^2}$$

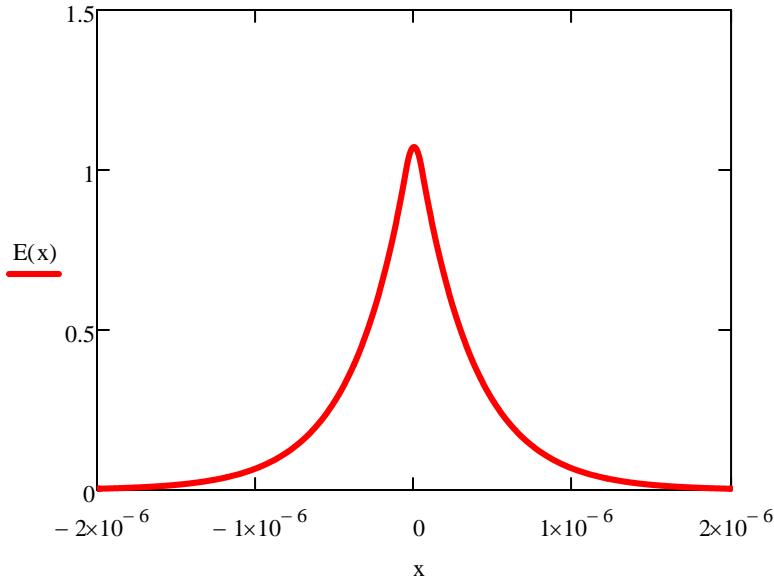
$$\text{Guess: } \text{neff} := 3.5$$

Given

$$\alpha(\text{neff}) = k_x(\text{neff})\cdot\tan\left(k_x(\text{neff})\cdot\frac{d}{2}\right)$$

$$\text{neff} := \text{Find(neff)} \quad \boxed{\text{neff} = 3.472}$$

$$E(x) := \text{if} \left[|x| \geq \frac{d}{2}, \exp\left[-\alpha(\text{neff})\cdot\left(|x| - \frac{d}{2}\right)\right], \frac{\cos(k_x(\text{neff})\cdot x)}{\cos(k_x(\text{neff})\cdot\frac{d}{2})} \right]$$



$$(b) \quad \Gamma_{\text{av}} := \frac{\int_{-\frac{d}{2}}^{\frac{d}{2}} (|E(x)|)^2 dx}{\int_{-10\mu\text{m}}^{10\mu\text{m}} (|E(x)|)^2 dx} \quad \boxed{\Gamma = 0.239}$$

$$(c) \quad V_{\text{green}} := k_0 \cdot d \cdot \sqrt{\varepsilon_x(0) - \varepsilon_x(0.4)} \quad V = 0.79$$

It is single mode waveguide since $V < \pi$

$$(d) \quad \Gamma_1 := \frac{V^2}{2 + V^2} \quad \boxed{\Gamma_1 = 0.238}$$

$$(e) \quad \begin{aligned} R_{\text{green}} &:= 30\% & R_1 &:= R & R_2 &:= R & n_r &:= n_{\text{eff}} & N_{\text{tr}} &:= 10^{18} \text{ cm}^{-3} \\ a &:= 10^{-16} \text{ cm}^2 & L_{\text{green}} &:= 200 \mu\text{m} & w &:= 1 \mu\text{m} & d &:= 0.1 \mu\text{m} & \text{Volume} &:= L \cdot w \cdot d \\ \eta_i &:= 100\% & v_g &:= \frac{c}{n_r} & \alpha_i &:= 20 \text{ cm}^{-1} \end{aligned}$$

$$\tau := 1 \text{ ns}$$

$$\alpha_m := \frac{1}{2 \cdot L} \cdot \ln \left(\frac{1}{R_1 \cdot R_2} \right) \quad \alpha_m = 60.199 \cdot \text{cm}^{-1}$$

$$\tau_p := \frac{1}{v_g(\alpha_i + \alpha_m)} \quad \tau_p = 1.444 \times 10^{-12} \text{ s}$$

$$g_{\text{th}} := \frac{\alpha_m + \alpha_i}{\Gamma} \quad \boxed{g_{\text{th}} = 336.075 \cdot \text{cm}^{-1}}$$

$$N_{\text{th}} := \frac{g_{\text{th}}}{a} + N_{\text{tr}} \quad N_{\text{th}} = 4.361 \times 10^{18} \cdot \text{cm}^{-3}$$

$$I_{\text{th}} := \frac{N_{\text{th}}}{\tau} \cdot q \cdot \text{Volume} \quad \boxed{I_{\text{th}} = 13.954 \cdot \text{mA}}$$

$$(f) \quad \eta_e := \frac{\alpha_m}{\alpha_i + \alpha_m} \quad \boxed{\eta_e = 75.062\%}$$

$$\eta_{e0} := \eta_e \cdot \frac{h_{\text{bar}} \cdot \omega}{q} \quad \boxed{\eta_{e0} = 1.07 \cdot \frac{W}{A}}$$

$$(g) \quad \Delta\lambda := \frac{\lambda^2}{2 \cdot n_{\text{eff}} \cdot L} \quad \boxed{\Delta\lambda = 0.546 \cdot \text{nm}}$$

2 (a)-(c): See Lecture Notes 232-13

$$\begin{aligned} R_1 &:= 30\% & R_2 &:= 30\% & n_r &:= 3.5 & N_{\text{tr}} &:= 10^{18} \text{ cm}^{-3} \\ a &:= 10^{-16} \text{ cm}^2 & \lambda_p &:= 1.55 \cdot \mu\text{m} & \Gamma &:= 50\% \\ \beta_1 &:= 10^{-3} & L_{\text{green}} &:= 300 \mu\text{m} & w &:= 1 \mu\text{m} & d &:= 0.1 \mu\text{m} \\ \eta_i &:= 100\% & \eta_r &:= 90\% & V_{\text{green}} &:= L \cdot w \cdot d & v_g &:= \frac{c}{n_r} \\ A &:= 2 \cdot 10^8 \text{ s}^{-1} & B &:= 10^{-9} \text{ s}^{-1} \cdot \text{cm}^3 & \alpha_i &:= 10 \text{ cm}^{-1} \end{aligned}$$

$$\pi_{\text{green}}^{(N)} := (A + B \cdot N)^{-1}$$

$$g(N) := a \cdot (N - N_{tr})$$

$$\alpha_m := \frac{1}{2 \cdot L} \cdot \ln \left(\frac{1}{R1 \cdot R2} \right) \quad \alpha_m = 40.132 \cdot \text{cm}^{-1}$$

$$\tau_p := \frac{1}{v_g(\alpha_i + \alpha_m)} \quad \tau_p = 2.329 \times 10^{-12} \text{ s}$$

$$g_{th} := \frac{\alpha_m + \alpha_i}{\Gamma} \quad g_{th} = 100.265 \cdot \text{cm}^{-1}$$

$$N_{th} := \frac{g_{th}}{a} + N_{tr} \quad N_{th} = 2.003 \times 10^{18} \cdot \text{cm}^{-3}$$

$$Rsp(N) := B \cdot N^2$$

$$S(N, \beta) := \frac{\beta \cdot Rsp(N)}{\frac{1}{\tau_p} - \frac{c}{n_r} \cdot \Gamma \cdot g(N)}$$

$$I(N, \beta) := \frac{q \cdot V}{\eta_i} \cdot \left(\frac{N}{\tau(N)} + \frac{c}{n_r} \cdot g(N) \cdot S(N, \beta) \right)$$

$$imax := 300$$

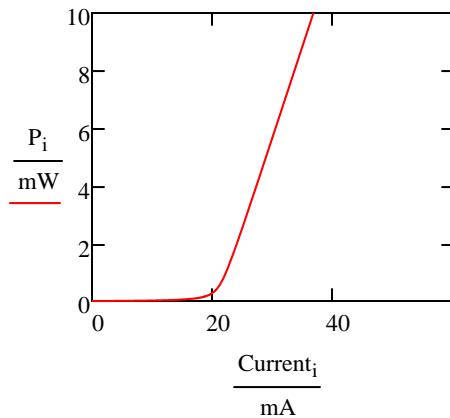
$$i := 1 .. imax$$

$$N_i := \frac{\ln(i)}{\ln(imax + 1)} \cdot N_{th}$$

$$P_i := \left[\left(\frac{1.24 \text{eV} \cdot \mu\text{m}}{\lambda_p} \right) \left(\frac{V}{\Gamma} \cdot \alpha_m \cdot v_g \cdot S(N_i, \beta_1) \right) \right]$$

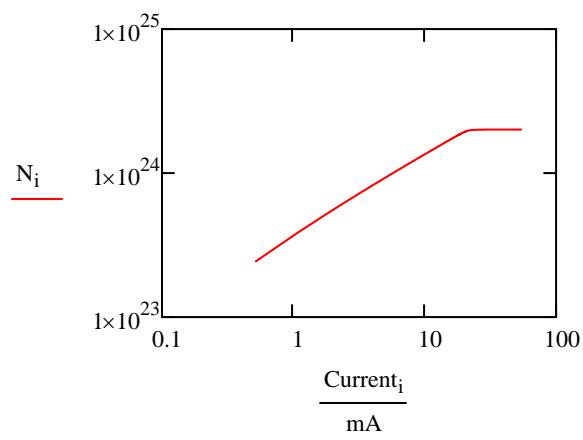
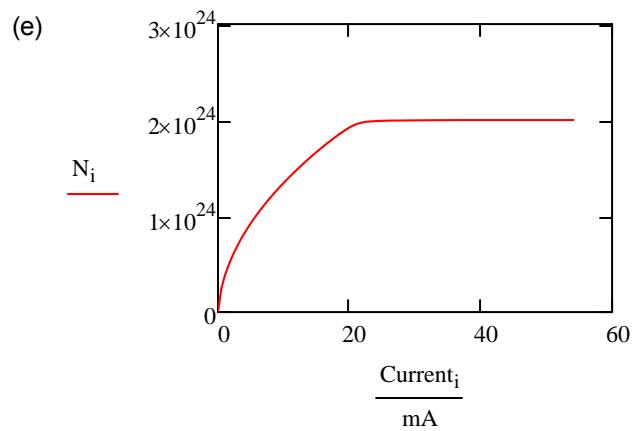
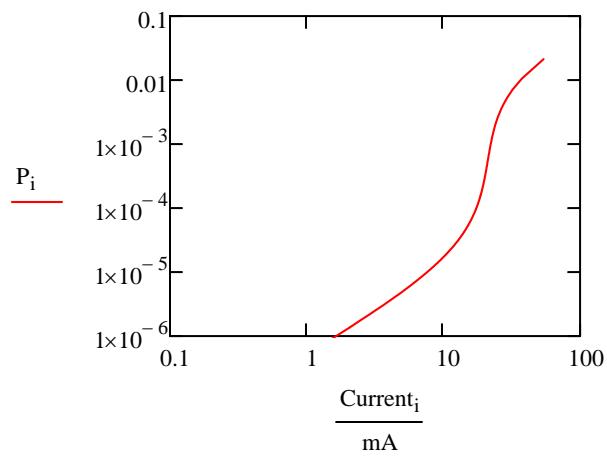
$$\frac{N_{imax}}{N_{th}} = 0.999$$

$$\text{Current}_i := I(N_i, \beta_1)$$



$$I_{th} := \frac{q \cdot V}{\eta_i} \cdot \frac{N_{th}}{\tau(N_{th})}$$

$$I_{th} = 21.173 \cdot \text{mA}$$



$$2. \quad j := 0..2$$

$$\beta_j := 10^{-1-j}$$

$$\text{imax} := 300$$

$$\beta = \begin{pmatrix} 0.1 \\ 0.01 \\ 1 \times 10^{-3} \end{pmatrix}$$

$$i := 1.. \text{imax}$$

$$N_i := \frac{\ln(i)}{\ln(\text{imax} + 1)} \cdot N_{\text{th}}$$

$$P_{\text{out},i,j} := \left[\left(\frac{1.24eV \cdot \mu m}{\lambda p} \right) \left(\frac{V}{\Gamma} \cdot \alpha m \cdot v g \cdot S(N_i, \beta_j) \right) \right]$$

$$L_{i,j} := I(N_i, \beta_j)$$

