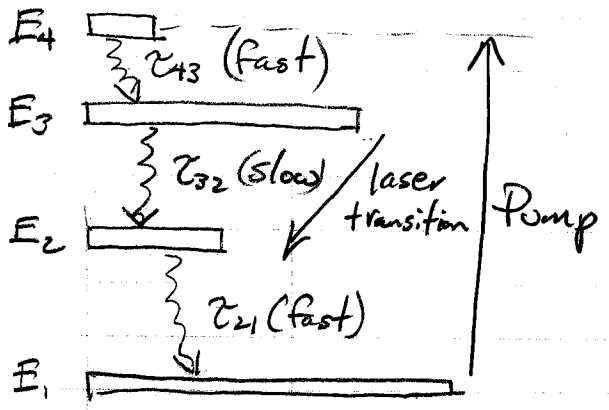
Laser pumping

4-level laser rate equations

How to achieve population inversion



① Pumping process excites atoms from ground state to state 4

② fast decay $4 \to 3$

③ slow decay $3 \to 2$

④ fast decay $2 \to 1$

$\Rightarrow$ build-up of inversion on $3 \to 2$

Rate equation analysis

$$\frac{dN_4}{dt} = R_p (N_1 - N_4) - (\gamma_{43} + \gamma_{42} + \gamma_{41}) N_4$$

pump rate

$$\tau_4 = \gamma_{43} + \gamma_{42} + \gamma_{41}$$

in steady state, $\frac{d}{dt} = 0$

$$N_4 = \frac{R_p \tau_4}{1 + R_p \tau_4} N_1 \cong R_p \tau_4 N_1 \quad \text{if } R_p \tau_4 \ll 1 \text{ often}$$

(pump process not saturated)

$$\frac{dN_3}{dt} = \gamma_{43} N_4 - (\gamma_{32} + \gamma_{31}) N_3 = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3}$$

ss $\to$

② for good laser, $\tau_3 \gg \tau_{43}$

$$\boxed{N_3 \gg N_4}$$

$$\frac{dN_2}{dt} = \gamma_{42}N_4 + \gamma_{32}N_3 - \gamma_{21}N_2 = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$

at ss, use ②

$$= \beta N_3 \qquad \beta = \frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43}\tau_{21}}{\tau_3\tau_{42}}$$

for a good laser, $\gamma_{42} \approx 0$, level 4 feeds primarily into level 3.

$$\beta \cong \frac{\tau_{21}}{\tau_{32}} = \frac{N_2}{N_3} \qquad ③$$

If $\tau_{32} > \tau_{21}$, $\beta < 1 \rightarrow N_2 < N_3$ <u>inversion</u>

depends only on <u>lifetime ratios</u>. Easy to look for good laser candidates.

Magnitude of inversion density $(N_3 - N_2)$?

using ③ <u>So, find N_3</u>

② $\rightarrow N_3 = \frac{\tau_3}{\tau_{43}} N_4$ ① $\rightarrow N_4 = R_p \tau_4 N_1$ $\quad N_2 = \beta N_3$

$$N_1 + N_2 + N_3 + N_4 = N \qquad \text{conservation of atoms}$$

$$N_1 = \frac{1}{R_p \tau_4} \frac{\tau_{43}}{\tau_3} N_3 \qquad \text{using ①}$$

$$N_3 \left[\frac{1}{R_p \tau_4} \frac{\tau_{43}}{\tau_3} + \beta + 1 + \frac{\tau_{43}}{\tau_3} \right] = N$$

$$\therefore \frac{N_3 - N_2}{N} = \frac{(1-\beta) R_p \tau_4}{\left[\frac{\tau_{43}}{\tau_3} + \left(1+\beta + \frac{\tau_{43}}{\tau_3}\right) R_p \tau_4\right]}$$

Quantum efficiency η – $\frac{\text{\# of photons emitted on laser trans}}{\text{\# of pump photons abs}}$

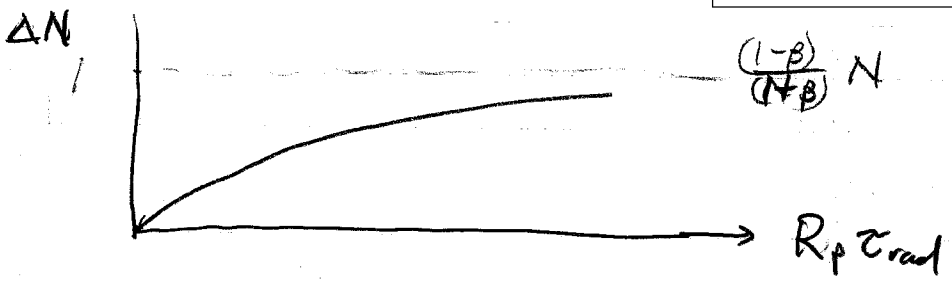
$$\eta = \underbrace{\frac{\gamma_{43}}{\gamma_4}}_{\substack{\text{fraction} \\ \text{of 4} \\ \text{that relaxes} \\ \text{to 3}}} \times \underbrace{\frac{\gamma_{32}^{rad}}{\gamma_3}}_{\substack{\text{fraction of} \\ \text{3 that} \\ \text{radiates}}}$$

$$\frac{N_3 - N_2}{N} = \frac{(1-\beta) R_p \eta \tau_{rad}}{1 + (1+\beta + \tau_{43}/\tau_3) \eta R_p \tau_{rad}}$$

good laser $\tau_{43} << \tau_3$, $\eta \cong 1$

$$\frac{N_3 - N_2}{N} \cong \frac{(1-\beta) R_p \eta \tau_{rad}}{1 + (1+\beta) R_p \eta \tau_{rad}}$$

large $R_p \eta \tau_{rad}$ $\rightarrow$
$\beta << 1$



Review conditions for large inversion

① level 4 efficiently feeds level 3

$\gamma_{42} \simeq 0 \qquad \gamma_4 \cong \gamma_{43}$

$\rightarrow \beta \cong \tau_{21}/\tau_{32}$

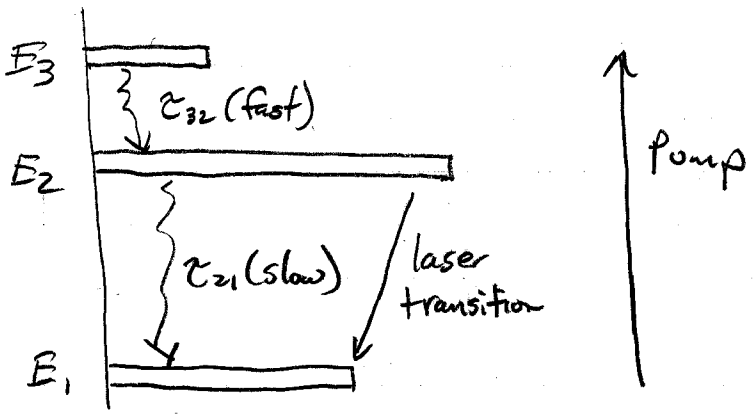
② τ_{21} fast, τ_{32} slow $\rightarrow \beta \ll 1$

③ High fluorescence efficiency

④ Pump rate $\quad R_p \eta \tau_{rad} > 1$

$R_p \gg \gamma_{32}^{rad}$

<u>Three level laser</u>



lower laser level is <u>ground state</u>.

Very tough to invert to ground state.

Have to pump up > half of all atoms

$$\frac{dN_3}{dt} = R_p(N_1 - N_3) - \frac{N_3}{\tau_3}$$

$N_1 + N_2 + N_3 = N \qquad$ conservation of atoms

Fluorescence quantum efficiency

For high efficiency $\tau_{32} \cong \tau_3$ $\tau_{21}^{rad} \cong \tau_{21}$

$\eta \cong 1$

relaxation ratio $\beta \equiv \frac{\tau_{32}}{\tau_{21}}$

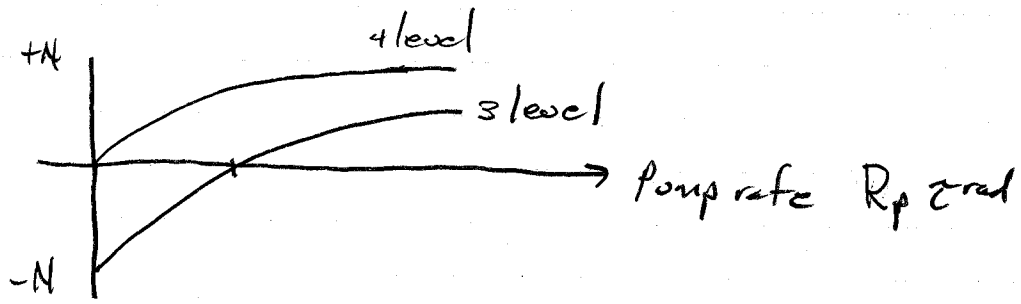
τ_{32} fast τ_{21} slow $\beta \ll 1$

steady state solution

$$\frac{N_2 - N_1}{N} = \frac{(1-\beta)\eta R_p \tau_{21}^{rad} - 1}{(1+2\beta)\eta R_p \tau_{21}^{rad} + 1}$$

even if $\beta < 1$, also need minimum pump rate to get inversion

Compare 3-level + 4 level inversion vs pump



Ruby is best example of 3-level laser
Has close to ideal conditions.
- Broad level 3 : get high W_p using flash lamps
- $\eta \cong 1$
- $\tau_{21} \cong 4$ ms - nearly purely radiative