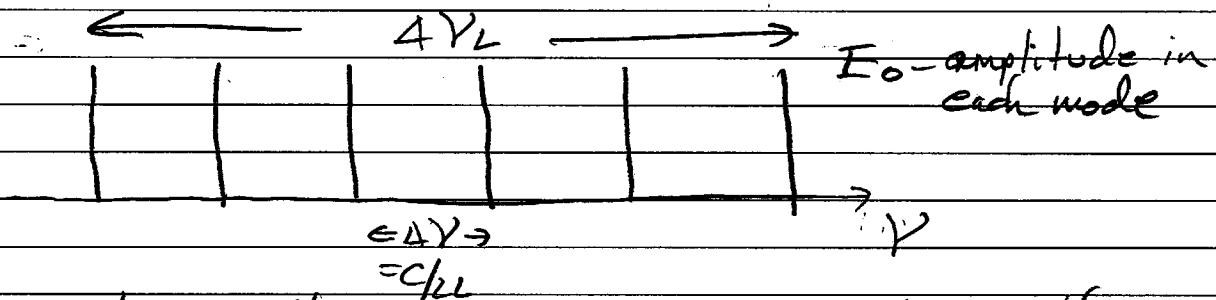


## Mode locking [Read Svelto 8.6]

- Behavior of laser operating on a number of cavity axial modes - We cannot use the simple rate equation model.
- Proper treatment involves equations for amplitude and phase of the electric field for all the participating modes - coupled to the gain as well as any additional components in the cavity (modulator)
- For a constructive, full treatment - start with Appendix F, then go to the references

Here - we will do a descriptive treatment and some analysis.

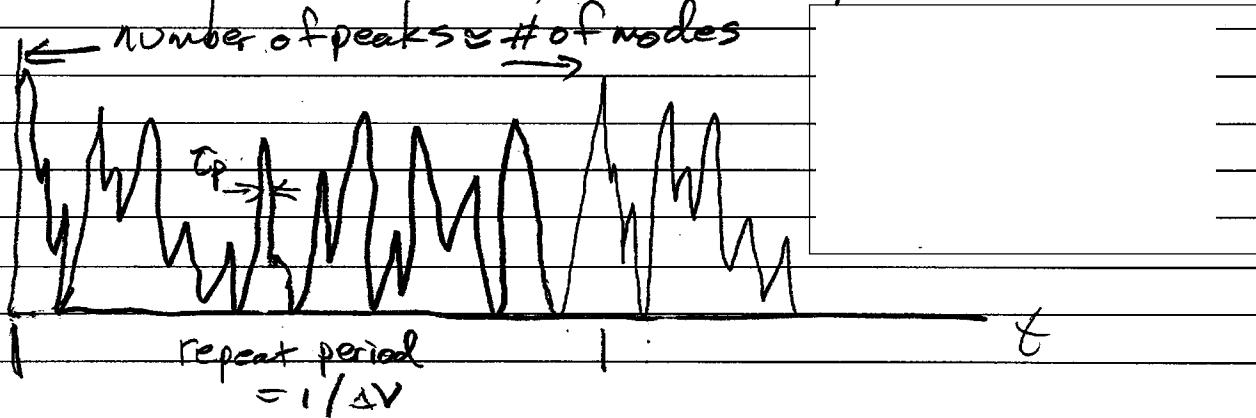
First consider uncontrolled, multimode laser spectrum



$\Delta V = c/2L$  is the mode spacing.  $\Delta V_L$  is the overall spectral bandwidth.

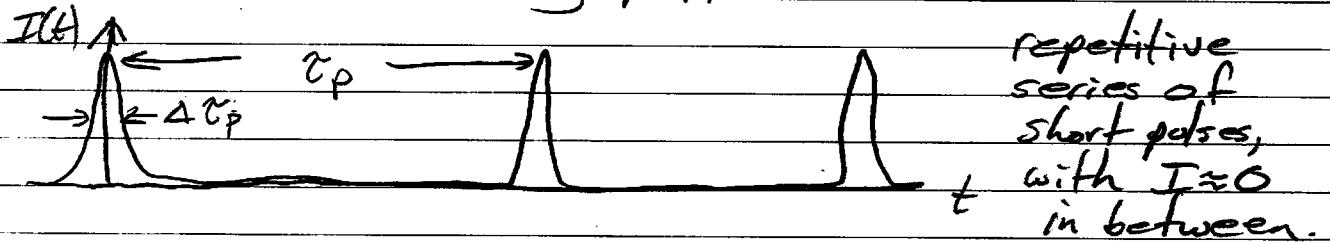
If the relative phases of the individual cavity modes are random, then the time behavior of the intensity will be pseudo-random

$I(t) \leftarrow \text{number of peaks} \leq \# \text{ of modes}$



• This behavior comes about due to "node-beating"-interference between the multiple node fields.

• By forcing a definite phase relation among the mode phases, the intensity can be controlled in a much more interesting pattern.



- Frequency domain picture:

Consider  $2n+1$  modes, equal amplitudes -  $E_0$ , with phases all equal, set to zero

$$\phi_{-n} = \phi_{-n+1} = \phi_{-n+2} = \dots = \phi_{n-1} = \phi_n = 0$$

(slightly less general than in text, but no important difference)

Then

$$= e^{j\omega t} \sum_{l=-n}^n E_0 e^{jl\omega t}$$

The sum can be

done as follows:  $= e^{j\omega t} E_0 \sum_{l=-n}^n x^l$   $x = e^{j\omega t}$

$$\text{let } s = x^{-n} + x^{-n+1} + \dots + x^{-1} + x^n$$

$$xs = x^{-n+1} + x^{-n+2} + \dots + x^{n+1}$$

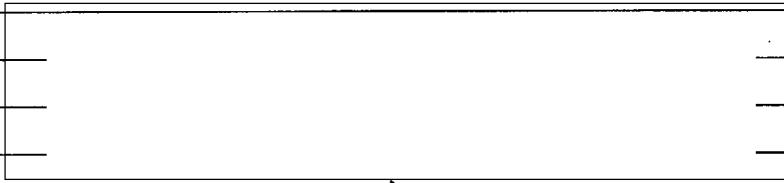
$$\text{then } (1-x)s = x^{-n} - x^{n+1}$$

so

Multiply top + bottom by  $x^{-1/2}$

$$S = \frac{x^{-n-\frac{1}{2}} - x^{n+\frac{1}{2}}}{x^{-1/2} - x^{1/2}}$$

$$= \frac{e^{j(n+\frac{1}{2})\Delta\omega t} - e^{-j(n+\frac{1}{2})\Delta\omega t}}{e^{j\Delta\omega t/2} - e^{-j\Delta\omega t/2}}$$



$$\text{So } E(t) = A(t)e^{j\omega t}$$

with

$$A(t) = S = \frac{\sin[(2n+1)\Delta\omega t/2]}{\sin(\Delta\omega t/2)}$$

(1) Taking  $E(t) = |E(t)|^2$  and looking at the pulse envelope,  $A^2(t)$  governs.

The pulse peaks occur when  $\sin(\Delta\omega t/2) = 0$

or  $\frac{\Delta\omega t}{2} = n\pi$ , so the pulse spacing is

$$\Delta\omega T_p = 2\pi$$

$$T_p = \frac{1}{\Delta\omega} = \frac{2L}{c}$$

The peak intensity of the pulse is

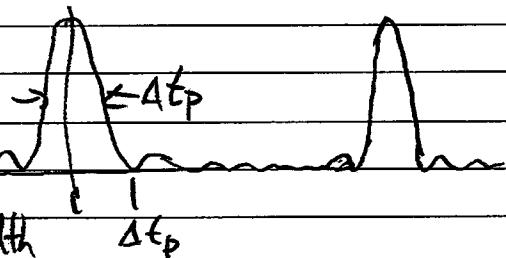
$$I_p = E_0^2 A^2(0) = (2n+1)^2 E_0^2$$

The first zero in the numerator happens at

$$(2n+1)\Delta\omega T_p/2 = \pi$$



$$= \frac{1}{4Y_L - \text{total bandwidth}} \Delta t_p$$

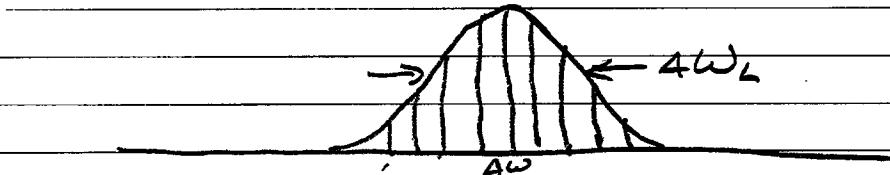


- For a well-designed mode locked laser,  $\Delta V_L$  can span the full gain bandwidth of the active medium

| <u>laser type</u> | <u>pulsewidth</u> |
|-------------------|-------------------|
| semiconductor     | few ps            |
| Nd: YAG           | ~50 ps            |
| Nd: glass         | ~1 ps             |
| dye laser         | ~25 fs            |
| Ti: sapphire      | ~7 fs             |

Peak power is strongly enhanced  $I_p = (2n+1)^2 E_0^2$

For more general mode spectrum (i.e. Gaussian), pulse shape is given by Fourier transform of the envelope spectrum



$$E_p^2 = E_0^2 e^{-\ln(2) \Delta \omega / (\Delta \omega_c)^2}$$



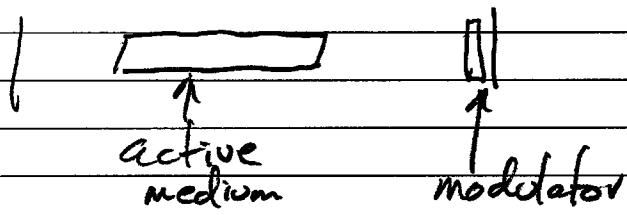
for each pulse

$$\Delta t_p = \frac{0.441}{\Delta V_L}$$

"transform limited pulse":

≈ 0.5 depends on exact shape of spectrum and pulse.

- time domain picture



Modulator transmission is periodic at

- Pulse develops inside cavity, circulates back and forth, with round-trip time  $T_p = 2L/c$ , synchronized to modulator.
- Every round trip, at output mirror a single pulse is coupled out of the cavity.