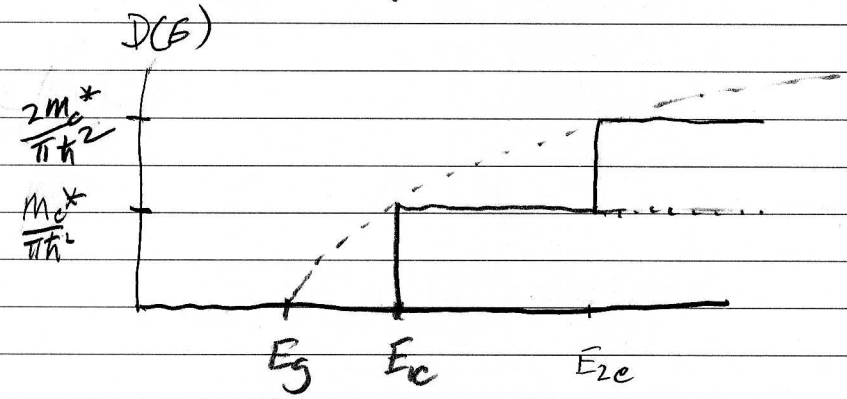
This is the density of states in each sub-band. They add up as energy increases to include more sub-bands



Selection rules for quantum wells

examine matrix element for transition from valence to conduction band

$$\boxed{}$$

Most common configuration is light beam normal to surface, so $\vec{E}$ is in the plane of the well.
Then $\vec{r}\cdot\vec{E} = r_\perp E(r_\perp)$. Since the well thickness is tiny compared to the wavelength, take E as constant in z-direction. So integral factors:

$$H_{12} = -e\int U_c^*(r_\perp)\, e^{-j\vec{k}_{c\perp}\cdot\vec{r}_\perp}\,(r_\perp E e^{jk_r\cdot r_\perp})\, U_v(r_\perp)\, e^{j\vec{k}_v\cdot\vec{r}_\perp}\, dx\,dy$$

$$\times \int \sin\left(\frac{n_c\pi z}{L_z}\right)\sin\left(\frac{n_v\pi z}{L_z}\right)dz$$

Similar to bulk case, $\vec{k}_{v\perp}+\vec{k}_{r\perp}-\vec{k}_{c\perp}=0$.
Neglecting $|k_{r\perp}| \ll |k_{c\perp}, k_{v\perp}|$, we have

$$\boxed{}$$

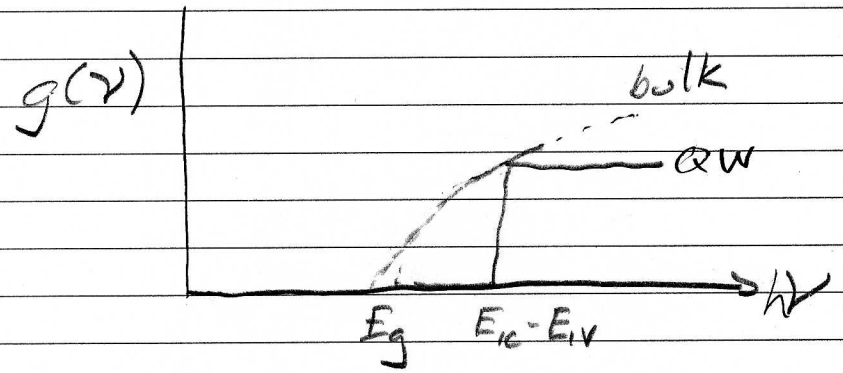
The z-integral requires $n_c = n_v$ or $\Delta n = 0$.

Absorption + gain coefficient

In quantum wells, the joint density of states per unit volume becomes:

$$\frac{1}{L_z} \cdot \text{JDOS/unit area}$$

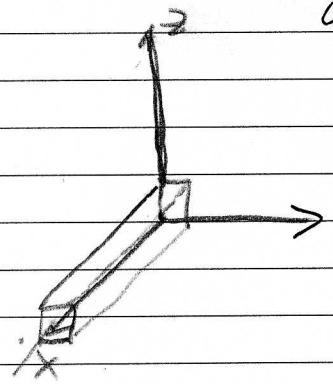
The gain spectrum is modified. Compared to bulk, the gain is increased, especially at the band edge.



Quantum wires + dots

The main effect of quantum confinement on the optical properties of semiconductors is to modify the joint DOS.

Quantum wires: in 1-D wires, we get particle-in-box wavefunctions in z and y directions. In x direction, electrons are still free.



$$\psi_{yz}(y,z) = \sin\left(\frac{n\pi z}{L_z}\right)\sin\left(\frac{m\pi y}{L_y}\right)$$

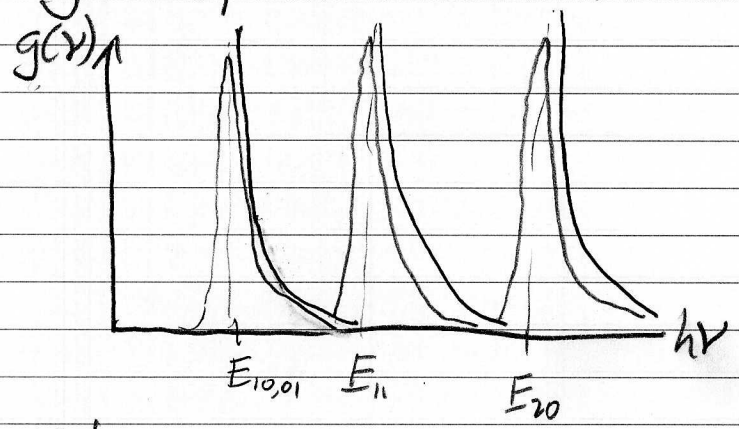
$$E_{(n,m)c} = \frac{\hbar^2\pi^2}{2m_c^*}\left(\frac{n^2}{L_z^2} + \frac{m^2}{L_y^2}\right)$$

Now we count states along a line: length of line is $2k_x$ (+ and −)

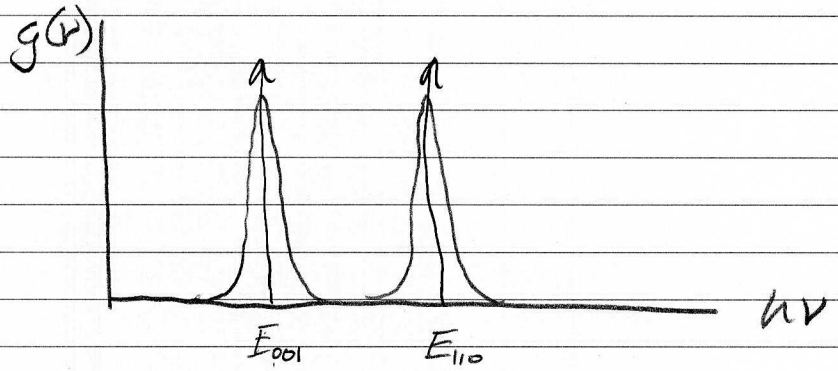
$$N = 2 \cdot \frac{2k_x}{(2\pi/L)} = L \cdot \frac{2k_x}{\pi}$$

$$\frac{dN}{dE} = \frac{2L}{\pi\hbar}(2m^*)^{1/2}\frac{1}{2}E^{-1/2} = \frac{(2m^*)^{1/2}L}{\pi\hbar}E^{-1/2}$$

Now the gain spectrum has an infinite singularity!



Broadening mechanisms of course make the gain finite

Quantum dots:

Now the electrons are fully confined in all 3 dimensions. The electronic states become discrete, just like atoms. Gain now occurs at discrete frequencies