EE236
Homework 8
Solution

1) Yariv 8.2

$$\alpha = \frac{\left(+N_1 \frac{g_2}{g_1}\right)\lambda^2 \eta}{8\pi n^2 t_{spont}} g(\nu_{peak}) \quad \text{from } (8.4\text{-}4)$$

take $g_2 = g_1 = 1$. $\eta = 1$ $\quad n = 1$

$$\lambda = \frac{c}{n}\frac{1}{\nu_0} = \frac{3\times10^{10}\ \text{cm/sec}}{3\times10^{14}\ \text{Hz}} = 10^{-4}\text{cm}$$

$$g(\nu) = \frac{2(\ln 2)^{1/2}}{\pi^{1/2}\Delta\nu} e^{-\left[4(\ln 2)(\nu-\nu_0)^2/(\Delta\nu)^2\right]}$$

$$g(\nu=\nu_{peak}) = \frac{2(\ln 2)^{1/2}}{\pi^{1/2}\Delta\nu}$$

$\Delta\nu = c\cdot 400\text{cm}^{-1}$
$= 1.2\times10^{13}\text{Hz}$

$$\alpha = \frac{N_1\lambda^2}{8\pi t_{spont}} \frac{2(\ln 2)^{1/2}}{\pi^{1/2}\Delta\nu}$$

$$\alpha = \frac{10^{18}\text{cm}^{-3}\,(10^{-4}\text{cm})^2}{8\pi\, 10^{-4}\text{sec}} \frac{2(\ln 2)^{1/2}}{\pi^{1/2}(1.2\times10^{13}\text{sec}^{-1})}$$

$$\alpha = \frac{10^{18}\,cm^{-3}(10^{-4}\,cm)^2}{8\pi\, 10^{-4}} \quad 7.8\times 10^{-14}$$

$$\alpha = \frac{1}{8\pi}\cdot 7.8$$

$$\alpha = .31\ cm^{-1}$$

$$\log_{10}\frac{I_{in}}{I_{out}} = \log_{10} e^{-\alpha(\nu) z} \qquad z = 1cm$$

$$= -\alpha(\nu)\log_{10} e$$

$$= -1.3$$

$$W_{spont} = \frac{1}{t_{spont}} = A$$

$$W_{12} = B_{12}\,\rho(\nu)$$

$$\frac{W_{12}}{A} = \frac{c^3}{8\pi n^3 h\nu}\ \frac{8\pi n^2 h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT}-1}\right)$$

$$1 = \frac{W_{12}}{A} = \frac{1}{e^{h\nu/kT}-1}$$

$$e^{h\nu/kT} = 2$$

$$\frac{h\nu}{KT} = \ln 2$$

$$T = \frac{h\nu}{K \ln 2}$$

$$h = 6.626 \times 10^{-34}\ J\text{-}s$$

$$K = 1.38 \times 10^{-23}\ J/^{\circ}K$$

$$T = \frac{(6.6 \times 10^{-34})(3 \times 10^{14})}{(1.38 \times 10^{-23}) \ln 2} \quad (^{\circ}K)$$

$$T = 21000\ ^{\circ}K$$

2) Yariv 8.6

In the limit of negligable saturation, $\Omega \to 0$

$$\chi' = K \frac{1}{1+(\omega-\omega_0)^2 T_2^2}$$

$$\chi'' = K \frac{(\omega_0-\omega) T_2}{1+(\omega-\omega_0)^2 T_2^2}$$

$$(\chi'' = ?) \quad -\frac{K}{\pi} PV \int_{-\infty}^{\infty} \frac{d\omega'}{(\omega'-\omega)(1+(\omega'-\omega_0)^2 T_2^2)}$$

Integrating with mathmatica, we get

$$(\chi'' = ?) \quad \frac{K}{\pi} \lim_{\substack{\varepsilon \to 0 \\ R \to \infty}} \left[-\frac{T_2(\omega-\omega_0)\tan^{-1}[T_2(\omega'-\omega_0)]}{1+(\omega-\omega_0)^2 T_2^2} + \frac{\ln(\omega'-\omega)}{1+(\omega-\omega_0)^2 T_2^2} - \frac{\ln(1+T_2^2(\omega'-\omega_0)^2)}{2[1+(\omega-\omega_0)^2 T_2^2]} \right] \Bigg|_{-R}^{\omega-\varepsilon} \Bigg|_{\omega-\varepsilon}^{R}$$

The second and third terms go to zero as $\omega' \to \pm\infty$

and the $\tan^{-1} \to \pi$

$$(\chi'' = ?) = K \frac{(\omega_0 - \omega) T_2}{1 + (\omega - \omega_0)^2 T_2^2}$$