

EE 240B – Spring 2018

Advanced Analog Integrated Circuits Lecture 4: Electronic Noise



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Electronic Noise

- **Why is noise important?**
 - Sets minimum signals we can deal with
 - Ensuring sufficiently low noise will be another “active constraint” that we will develop a design methodology around
- **Most often care about “signal-to-noise ratio”**
 - How do you make the signal larger?
 - Will see today and next time how to make noise smaller, but can you guess what it takes?

Types of “Noise”

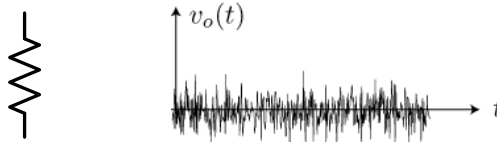
- **Interference**
 - Not actually “noise” – deterministic
 - Signal coupling
 - Capacitive, inductive, substrate, etc.
 - Supply variations
- **Device noise**
 - Caused by discreteness of charge
 - “Fundamental physics” related – **thermal and shot noise**
 - “Technology” related – flicker noise

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Noise in Electrical Circuits



- **Noise is random**
 - Has to be treated statistically – can’t predict actual value
- **Deal with mean (average), variance, spectrum**

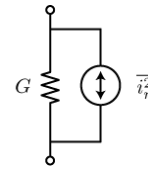
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Origin and Properties of Thermal Noise

- **Origin: Brownian Motion**
 - Thermally agitated particles
 - (Everything that can generate heat has this type of noise)



- **Independent of DC current flow**
- **Zero mean, Gaussian PDF, i.i.d.**
- **Available noise power:** $P_N = k_B T \Delta f$
 - Noise power in bandwidth Δf delivered to a matched load
 - Power spectral density is “white” up to ~THz frequencies
 - $k_B T = 4e-21$ J (T = 290K)

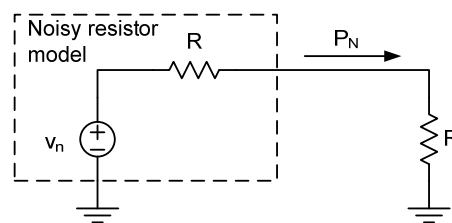
J.B. Johnson, “Thermal Agitation of Electricity in conductors,” Phys. Rev., July 1928

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Resistor Noise Model



$$P_N = k_B T \Delta f = \frac{\overline{v_n^2}}{4R}$$

Mean square noise voltage:

$$\overline{v_n^2} = 4k_B T R \Delta f$$

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Resistor Noise Model (current)

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Thermal Noise in Capacitors?

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Noise Power Spectral Density (PSD)

Noise Calculations

- **Noise calculations**
 - Instantaneous voltages add
 - Power spectral densities add
 - RMS voltages do NOT add
- Example: $R_1 + R_2$ in series

Calculating Noise in Passive Networks

- Capacitors and inductors only shape spectrum:

$$\overline{v_{on,T}^2(f)} = \sum_x |H_x(s)|_{s=2\pi jf}^2 \overline{v_x^2(f)}$$

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Side Note on Shot Noise (Diodes)

- Shot noise

- Zero mean, Gaussian pdf, white
- Proportional to current
- Independent of temperature

$$\overline{i_n^2} = 2qI_D \Delta f$$

- Example:

$$I_D = 1\text{mA} \rightarrow \underline{17.9\text{pA/rt-Hz}}$$

$$1\text{MHz bandwidth} \rightarrow \sigma = \underline{17.9\text{nA}}$$

- Shot noise versus thermal noise

- $g_{\text{diode}} = I_d / (k_b T / q)$
- Thermal noise density: $4k_b T g_{\text{diode}} = 4qI_d$
- Shot noise half of this (current flow in 1 direction)

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Simplified FET Noise Model

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Simplified FET Noise Model

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Word of Caution

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Flicker Noise

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1/f Noise Modeling

$$\overline{i_{1/f}^2} = \frac{K_f I_D \Delta f}{L^2 C_{ox} f}$$

- **Flicker noise**
 - $K_{f,NMOS} = 6 \times 10^{-29} \text{ A}^2\text{F}$
 $K_{f,PMOS} = 3 \times 10^{-29} \text{ A}^2\text{F}$
 - Strongly process dependent
- **Example:** $I_D = 10\mu\text{A}$, $L = 1\mu\text{m}$,
 - $C_{ox} = 15\text{fF}/\mu\text{m}^2$, $f_{hi} = 1\text{MHz}$

$$\begin{array}{ll} f_{lo} = 1\text{Hz} & \rightarrow \sigma = 0.74\text{nA} \\ f_{lo} = 1/\text{year} & \rightarrow \sigma = 1.11\text{nA} \end{array}$$

$$\overline{i_{1/f,total}^2} = \int_{f_{lo}}^{f_{hi}} \frac{K_f I_D df}{L^2 C_{ox} f} = \frac{K_f I_D}{L^2 C_{ox}} \ln \frac{f_{hi}}{f_{lo}}$$

Modified FET Noise PSD

1/f Noise Corner Frequency

- **Definition (MOS)**

$$\frac{K_f I_D \Delta f}{L^2 C_{ox} f_{co}} = 4k_B T_r \gamma g_m \Delta f$$

$$f_{co} = \frac{K_f I_D}{L^2 C_{ox} 4k_B T_r \gamma g_m} \cdot 1$$

$$= \frac{K_f}{4k_B T_r \gamma C_{ox}} \cdot \frac{1}{L^2} \cdot \frac{1}{g_m/I_D}$$

$$= \frac{K_f}{8k_B T_r \gamma C_{ox}} \cdot \frac{V^*}{L^2}$$

- **Example:**

- $V^* = 100\text{mV}$, $\gamma = 1.16$

$L = 100\text{nm}$	→	NMOS	PMOS
		2MHz	233kHz
$L = 300\text{nm}$	→	222kHz	25.9kHz

Noise PSD Calculation in an Amplifier
