EE 240B – Spring 2018

Advanced Analog Integrated Circuits
Lecture 5: Noise and SNR Analysis

Elad Alon
Dept. of EECS

General Noise Analysis

- **Method:**
  1) **Create small-signal model**
  2) **All inputs = 0 (linear superposition)**
  3) **Pick output v_o or i_o**
  4) **For each noise source v_x, i_x**
     Calculate $H_x(s) = v_o(s) / v_x(s)$ (… i_o, i_x)
  5) **Total noise at output is:**

$$\overline{v_{on,T}^2(f)} = \sum_x \left|H_x(s)\right|^2_{s=2\pi jf} \overline{v_x^2(f)}$$

$$\overline{v_{on,T}^2} = \int_0^\infty \overline{v_{on,T}^2(f)}\,df$$

Tedious but simple …

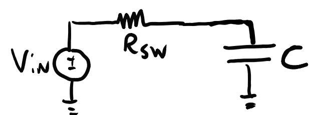
Important Integrals

$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o}} \right|^2 df = \frac{\omega_o}{4}$$

$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \int_0^\infty \left| \frac{\frac{s}{\omega_o}}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}$$

$$\int_0^\infty \left| \frac{\frac{s}{\omega_z}+1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}\left(\frac{\omega_o^{\ 2}}{\omega_z^{\ 2}}+1\right)$$

Noise in a Real Circuit: RC



- **Noise on the capacitor:**

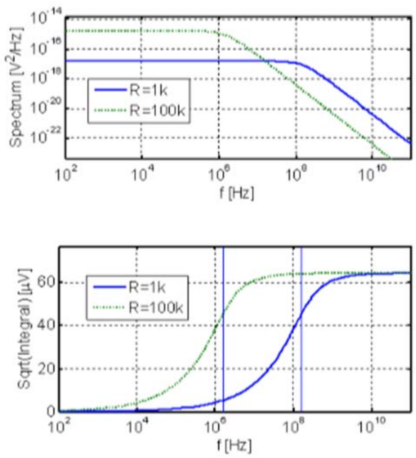
$$\overline{v_{on}^2}(f) = 4k_B TR \left| \frac{1}{1+sRC} \right|^2$$

$$\rightarrow \overline{v_{oT}^2} = 4k_B TR \frac{1}{4RC} = \frac{k_B T}{C}$$

- **Note that effective bandwidth is:**

$$\Delta f = \frac{1}{4RC} = \frac{\omega_o}{4} = \frac{\pi}{2} f_o$$

Noise PSD

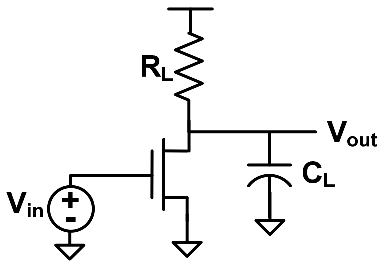


EE 240B Lecture 5 5

Equipartition Theorem

EE 240B Lecture 5 6

CS Amplifier Noise



$$\overline{v_{on}^2(f)} = 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)\left|\frac{R_L}{1+sR_LC_L}\right|^2$$

$$\overline{v_{oT}^2} = 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)R_L^2\int_0^\infty\left|\frac{1}{1+sR_LC_L}\right|^2 df$$

$$= 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)R_L^2\frac{1}{4R_LC_L}$$

$$= \frac{k_BT}{C_L}\left(1+\gamma g_mR_L\right)$$

$$= \frac{k_BT}{C_L}\left(1+\gamma\left|A_{vo}\right|\right)$$

$$= \frac{k_BT}{C_L}n_F$$

Optional: Two-Stage Amplifier

Signal-To-Noise Ratio

- **SNR:** $$SNR = \frac{P_{sig}}{P_{noise}}$$
- **Signal Power (sinusoidal source):**
$$P_{sig} = \tfrac{1}{2}V_{zero-peak}^2$$
- **Noise Power (assuming thermal noise dominates):**
$$P_{noise} = \frac{k_B T}{C} n_f$$
- **So:**
$$\boxed{SNR = \frac{\tfrac{1}{2}CV_{zero-peak}^2}{n_f k_B T}}$$
$$SNR \uparrow +6dB$$
$$\downarrow$$
$$C \uparrow \times 4$$

SNR versus Bits

- **Quantization "noise"**
  - Quantizer step size: Δ
  - Box-car pdf variance: $S_Q = \frac{\Delta^2}{12}$

N	dB
8	50
16	98
24	146

- **SNR of N-Bit sinusoidal signal**
  - Signal power $$P_{sig} = \frac{1}{2}\left(\frac{1}{2}2^{N_{bits}}\Delta\right)^2$$
  - SNR $$SNR = \frac{P_{sig}}{S_Q} = 1.5 \times 2^{2N_{bits}}$$
  - 6.02 dB per Bit $$= \left[1.76 + 6.02 N_{bits}\right] \ \text{dB}$$

SNR versus C_L

- For a 1V sinusoidal signal at 100°C:

Bits	SNR [dB]	C
3.0	20	4.1 aF
6.3	40	412 aF
9.7	60	41 fF
13.0	80	4.1 pF
16.3	100	412 pF
19.6	120	41 nF
23.0	140	4.1 μF

SNR versus Power

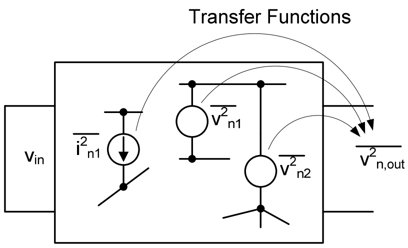
- 1 Bit → 6dB → 4x SNR
- 4x SNR → 4x C
- Circuit bandwidth ~g_m/C → 4x g_m
- Keeping V* constant → 4x I_D, 4x W

- Thermal noise limited circuit:
  - Each bit QUADRUPLES power!

- Comparison vs. digital circuits…

Input and Output Referred Noise

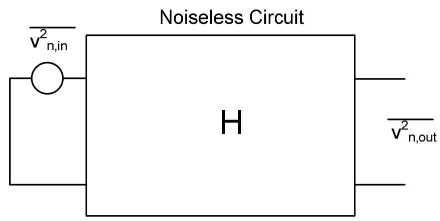
Output



$$v_{out} = H_i \cdot s_{n,i}$$

$$\overline{v^2_{n,out}} = \sum_i |H_i|^2 \overline{s^2_{n,i}}$$

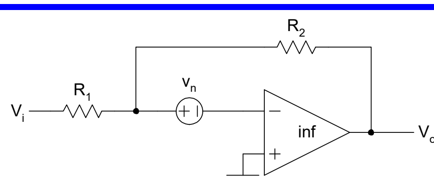
Input



$$v_{out} = H \cdot v_{in}$$

$$\overline{v^2_{n,in}(\omega)} = \frac{\overline{v^2_{n,out}(\omega)}}{|H|^2}$$

Noise Example



- **Ignoring noise from R_1, R_2:**

$$v_o = -v_i \underbrace{\frac{R_2}{R_1}}_{-A_{v0}} + v_n\left(1+\frac{R_2}{R_1}\right) = -v_i \underbrace{\frac{R_2}{R_1}}_{-A_{v0}} + v_n \frac{R_1+R_2}{R_1}$$

$$\overline{v^2_{ieq}} = \overline{v^2_n}\left(\frac{R_1+R_2}{R_1}\frac{R_1}{R_2}\right)^2 = \overline{v^2_n}\left(\frac{R_1+R_2}{R_2}\right)^2 = \overline{v^2_n}\left(1+\frac{1}{|A_{v0}|}\right)^2$$

- **"Ideal" feedback, why is $v_{i,eq}^2 > v_n^2$?**

Source Impedance