EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

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Lecture 14: Beam Bending

Announcements

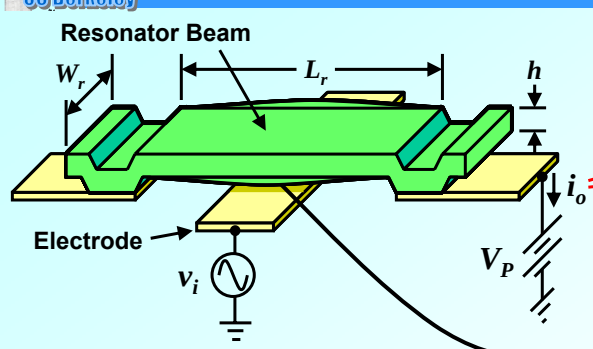
- Move midterm date to Thursday, Nov. 1 (1 week later than original tentative date)
- Website issues

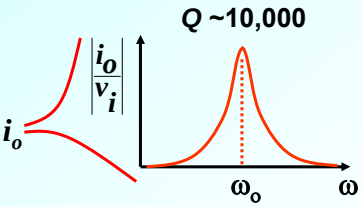
Lecture Outline

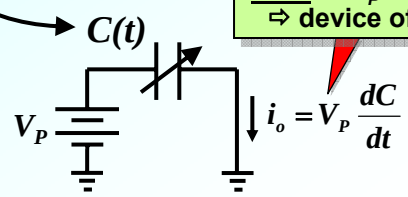
- Reading: Senturia Chpts. 8, 9
- Lecture Topics:
  - Material Properties
    - Young's modulus
    - Yield strength
    - Quality factor
  - On-chip Measurement of Material Properties
  - Anisotropic Material Properties
  - Beam Bending

Quality Factor (or Q)

Clamped-Clamped Beam μResonator







Note: If $V_P = 0V$ ⇨ device off

Frequency:

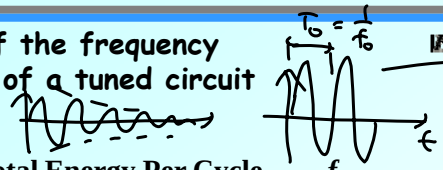
Stiffness — Young's Modulus — Density — Mass

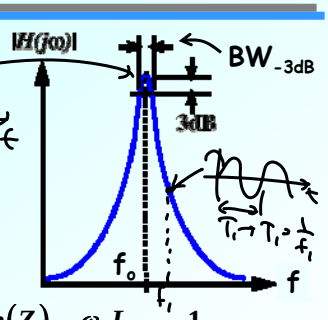
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

(e.g., $m_r = 10^{-13}$ kg) ⇨ Smaller mass ⇨ higher freq. range and lower series R_x

Quality Factor (or Q)

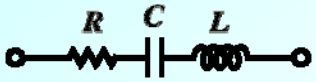
- Measure of the frequency selectivity of a tuned circuit
- Definition:





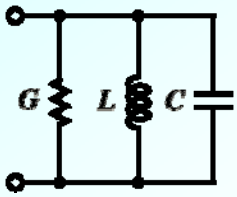
$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

- Example: series LCR circuit



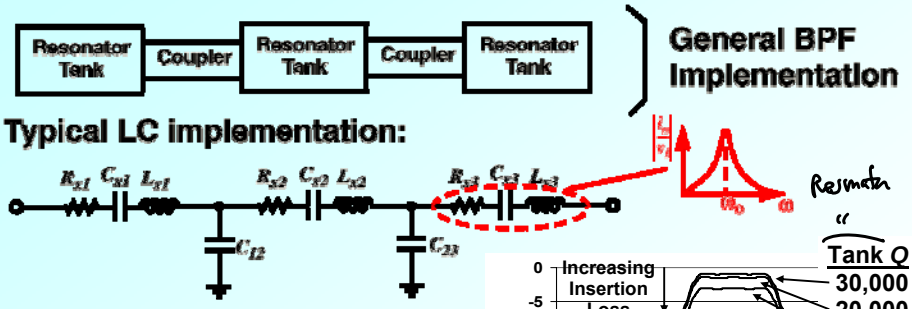
⇨ $$Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit

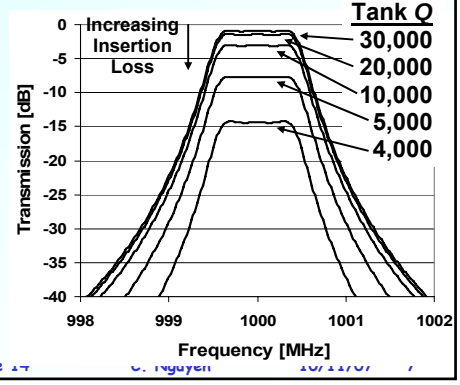


⇨ $$Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

Selective Low-Loss Filters: Need Q



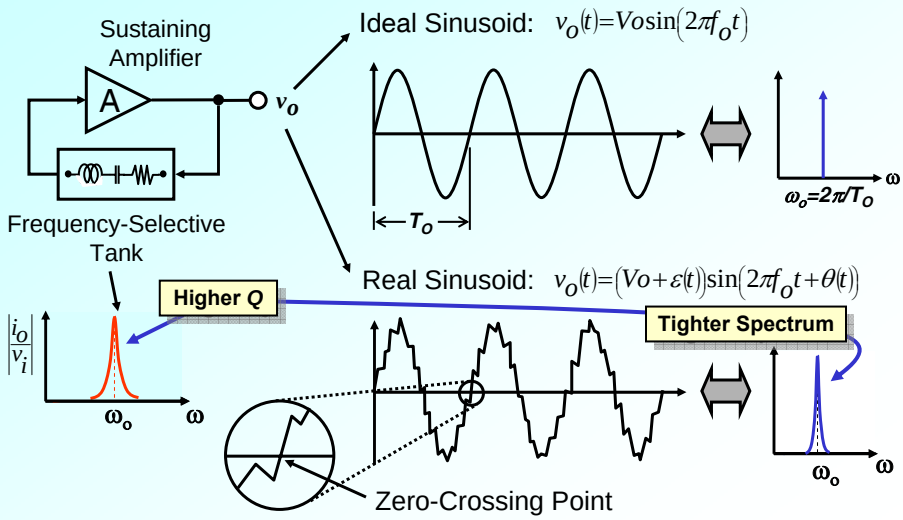
Resonator "Tank Q"



- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000

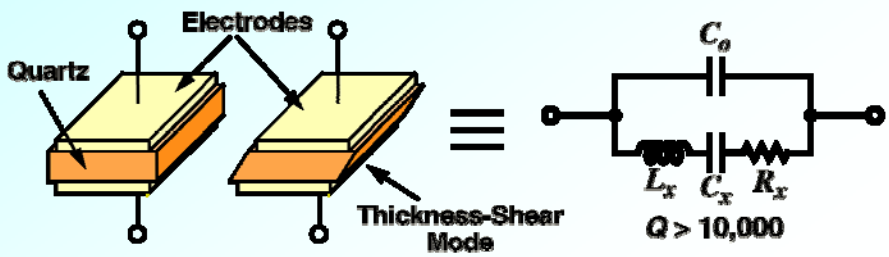
Oscillator: Need for High Q

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability



Attaining High Q

- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode



Energy Dissipation and Resonator Q

Material Defect Losses

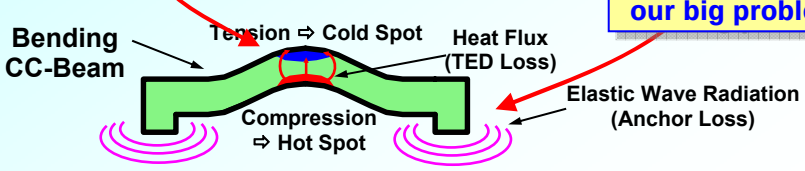
Gas Damping

$$\frac{1}{Q} = \frac{1}{Q_{defects}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{viscous}} + \frac{1}{Q_{support}}$$

Thermoelastic Damping (TED)

Anchor Losses

At high frequency, this is our big problem!



Thermoelastic Damping (TED)

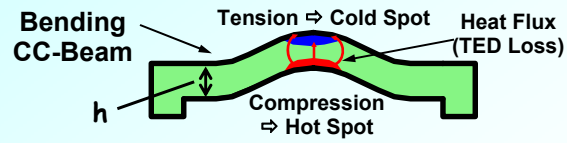
- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED} f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$



ζ = thermoelastic damping factor
α = thermal expansion coefficient
T = beam temperature
E = elastic modulus
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
f_{TED} = characteristic TED frequency

TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

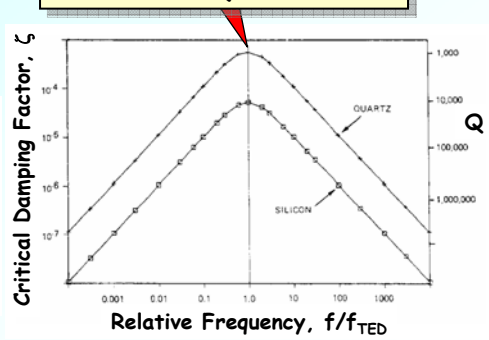
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm^2
Material density	2.33	2.60	g/cm^3
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

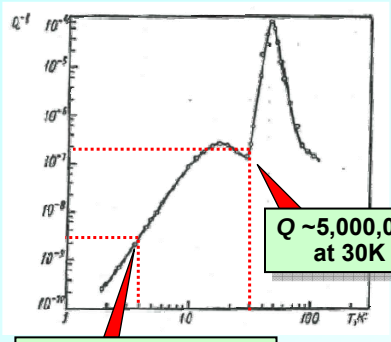
Peak where Q is minimized



[from Roszhart, Hilton Head 1990]

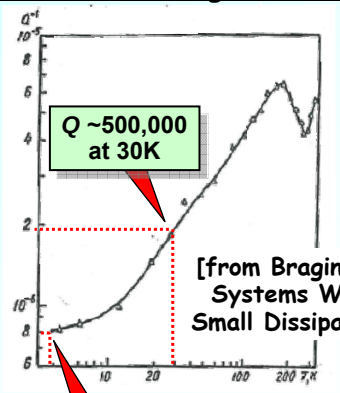
Q vs. Temperature

Quartz Crystal



Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle

Aluminum Vibrating Resonator

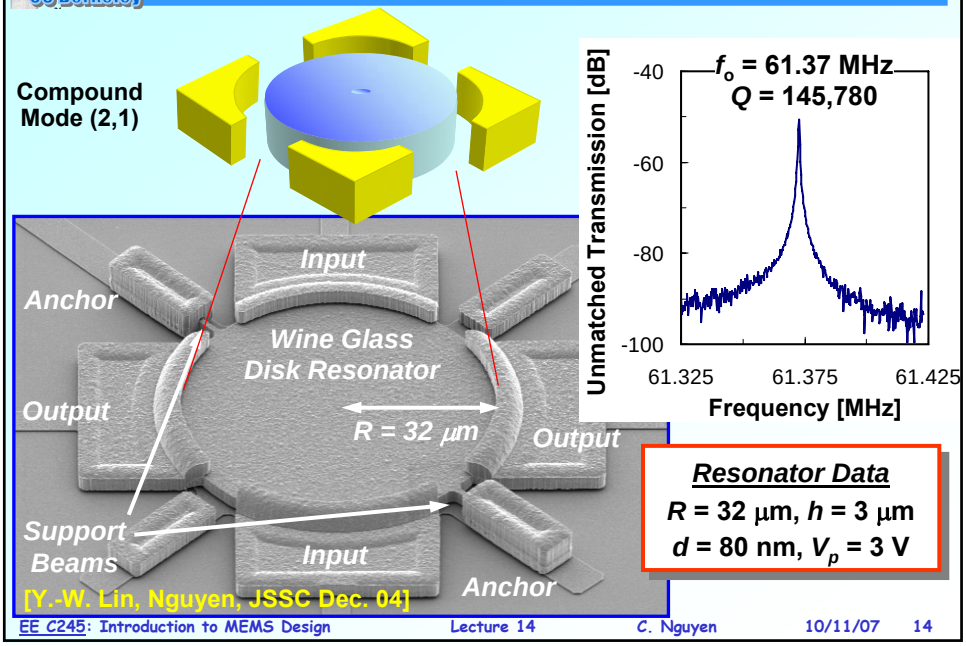


[from Braginsky, Systems With Small Dissipation]

Even aluminum achieves exceptional Q's at cryogenic temperatures

Polysilicon Wine-Glass Disk Resonator

Compound Mode (2,1)



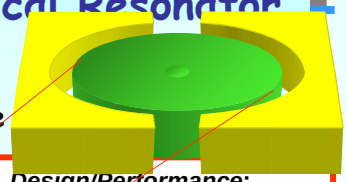
Resonator Data

R = 32 μm, h = 3 μm

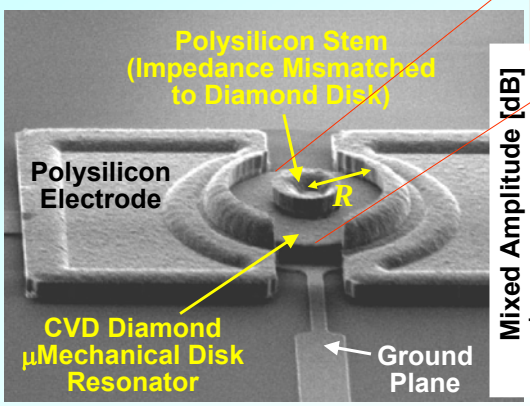
d = 80 nm, V_p = 3 V

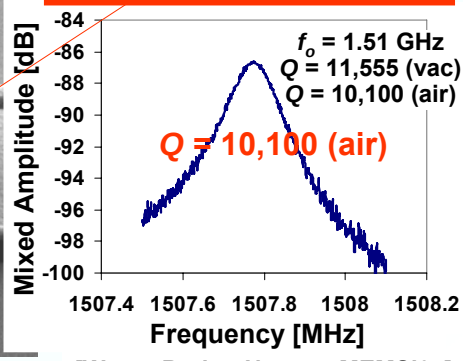
1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- ***Below***: 20 μm diameter disk



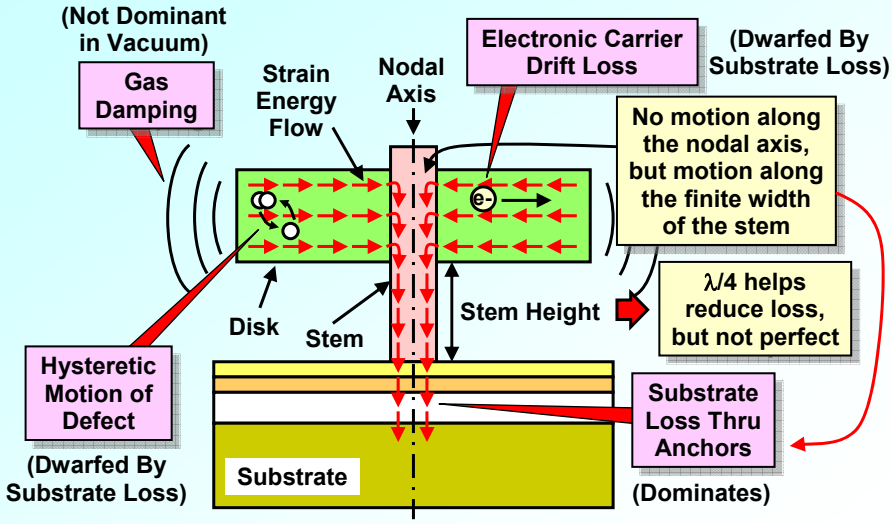
Design/Performance:
R=10μm, t=2.2μm, d=800Å, V_P=7V
f_o=1.51 GHz (2nd mode), Q=11,555





[Wang, Butler, Nguyen MEMS'04]

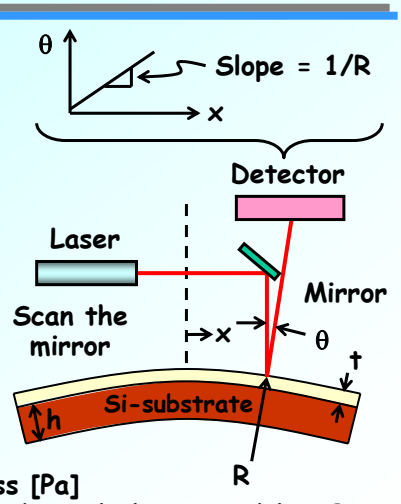
Disk Resonator Loss Mechanisms



MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:



$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]

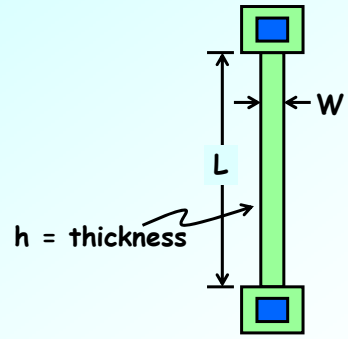
MEMS Stress Test Structure

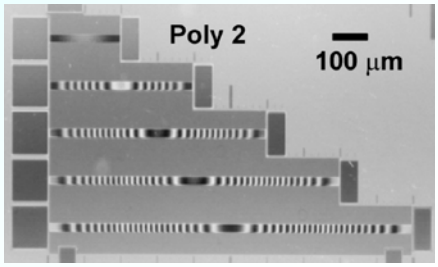
- Simple Approach: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

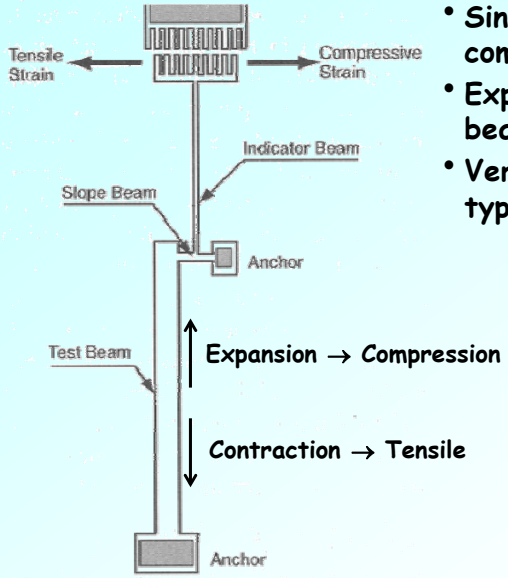
E = Young's modulus [Pa]
I = (1/12)Wh³ = moment of inertia
L, W, h indicated in the figure

  - Limitation: Only compressive stress is measurable

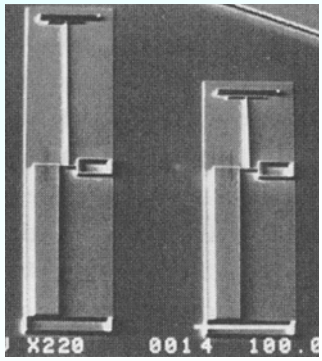




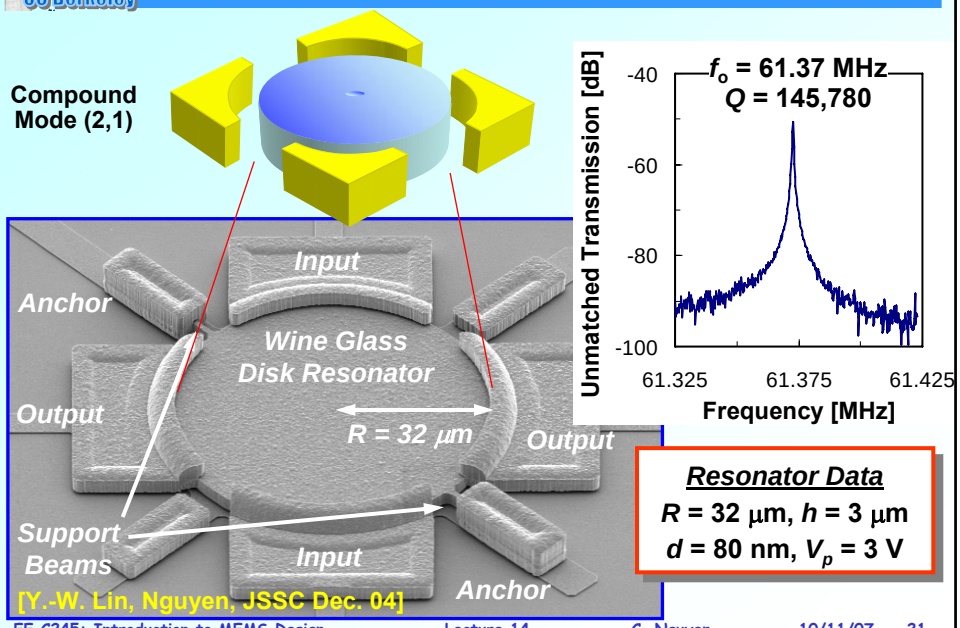
More Effective Stress Diagnostic



- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress



Q Measurement Using Resonators



Resonator Data
R = 32 μm, h = 3 μm
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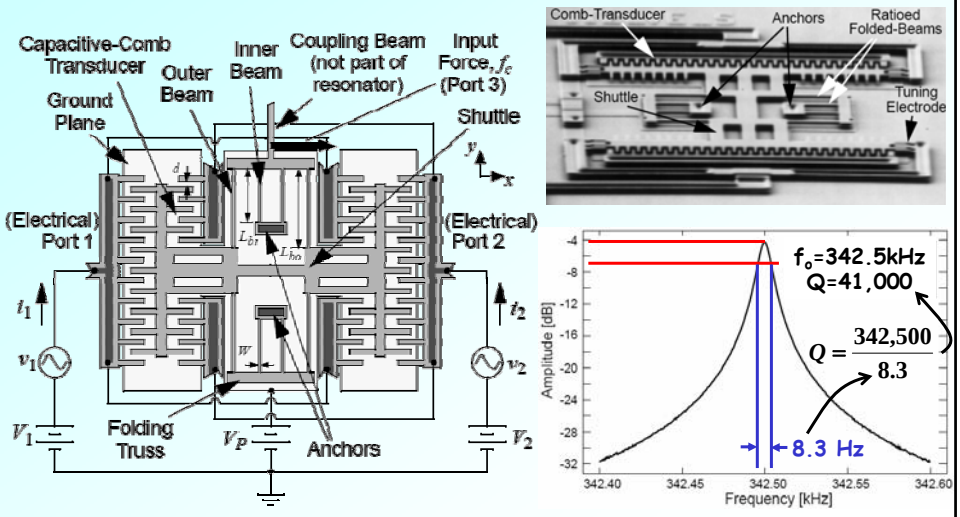
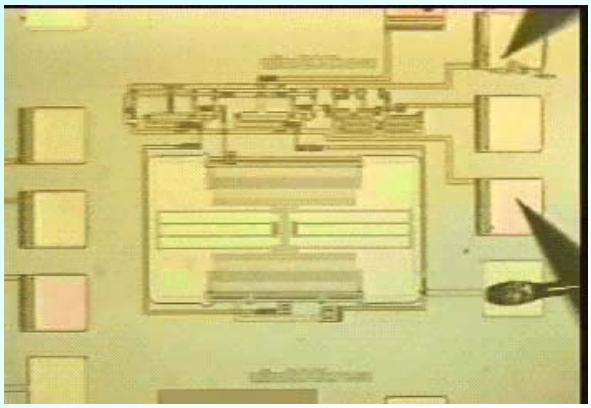
Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator



Comb-Drive Resonator in Action

- **Below**: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



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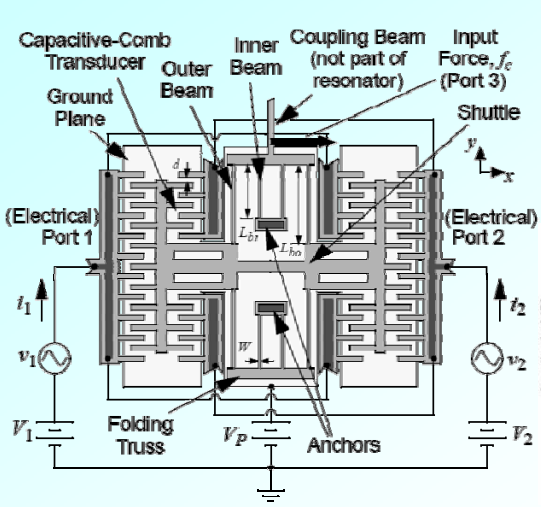
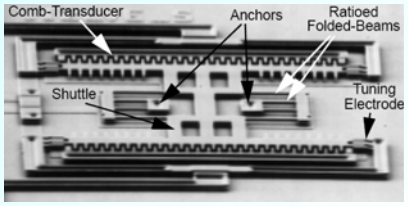
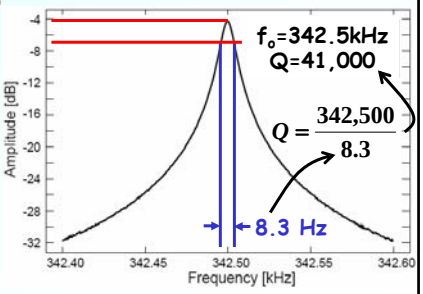






Measurement of Young's Modulus

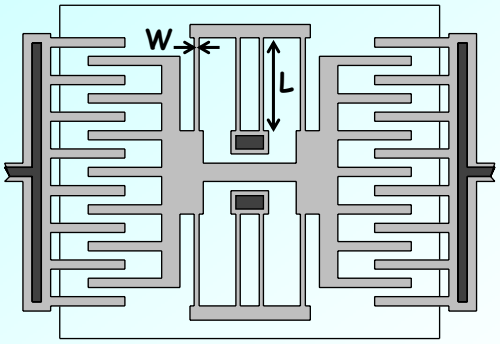
- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

Young's modulus

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Equivalent mass

h = thickness



- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}}_{\text{Stiffness Coefficients}} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

Stresses Strains

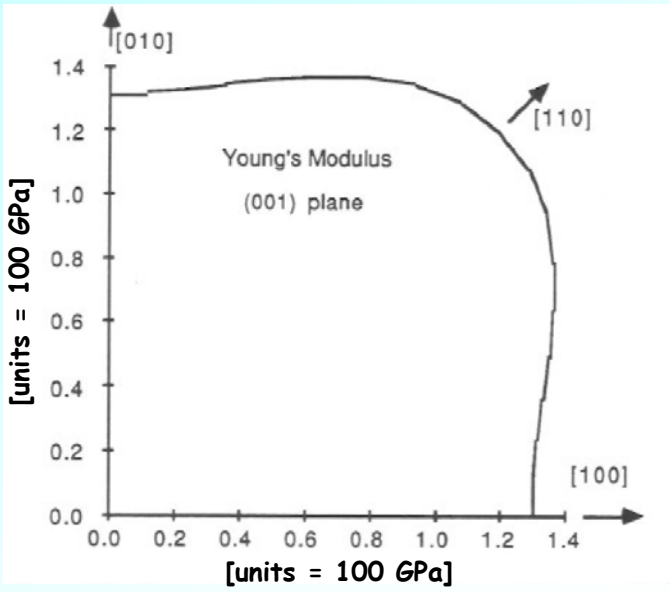
Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

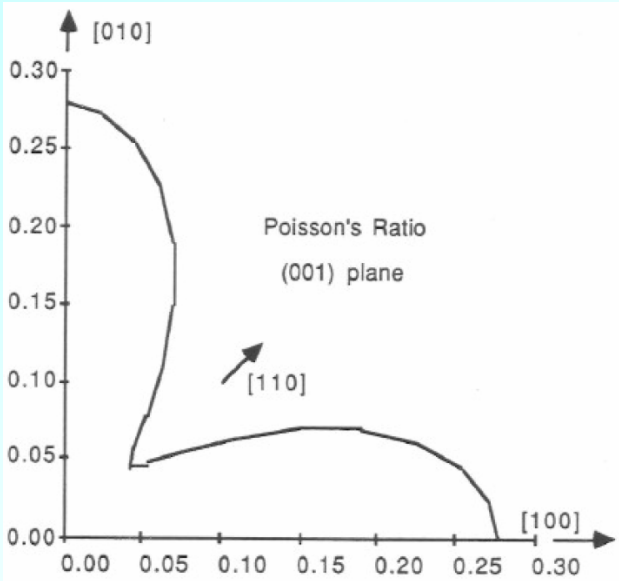
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

where $\begin{cases} C_{11} = 165.7 \text{ GPa} \\ C_{12} = 63.9 \text{ GPa} \\ C_{44} = 79.6 \text{ GPa} \end{cases}$

Young's Modulus in the (001) Plane

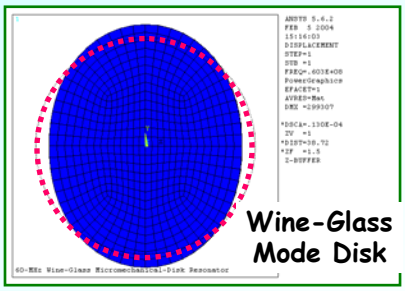


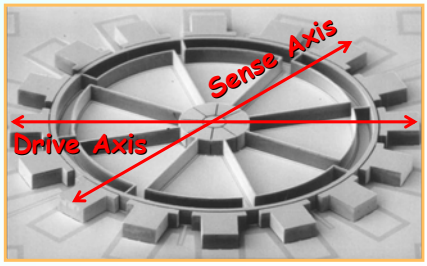
Poisson Ratio in (001) Plane



Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here



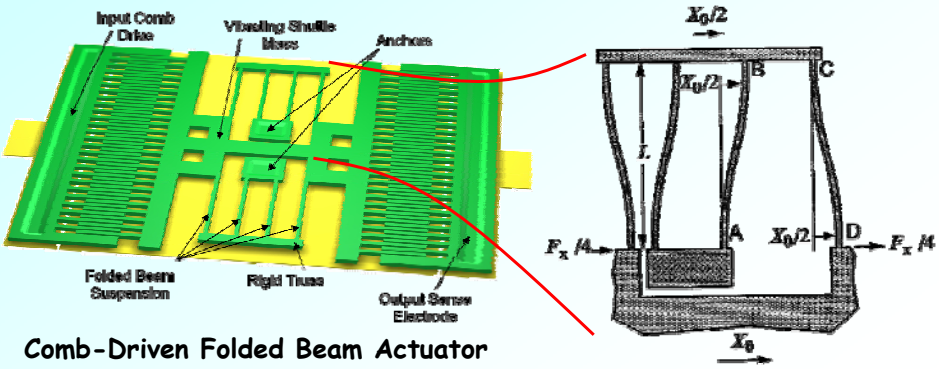


Ring Gyroscope

Bending of Beams

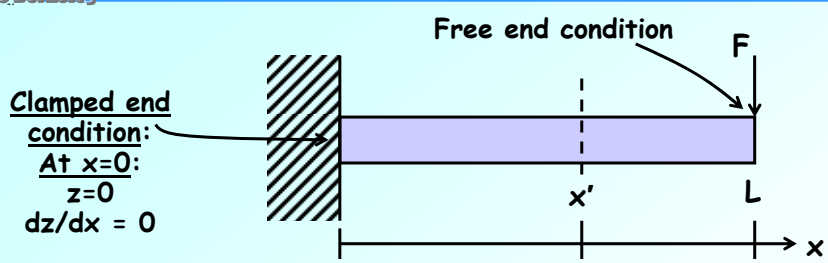
Beams: The Springs of Most MEMS

- Springs and suspensions very common in MEMS
  - Coils are popular in the macro-world; but not easy to make in the micro-world
  - Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS



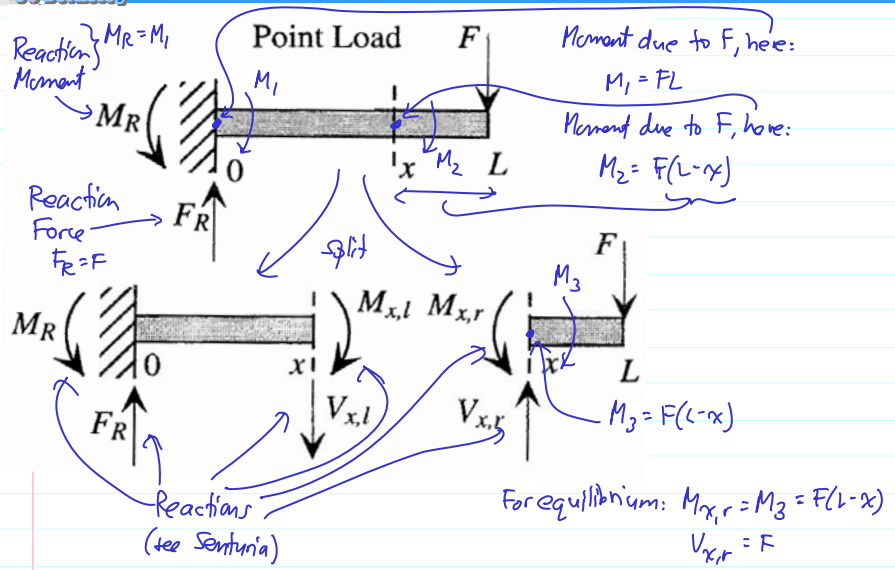
Comb-Driven Folded Beam Actuator

Bending a Cantilever Beam

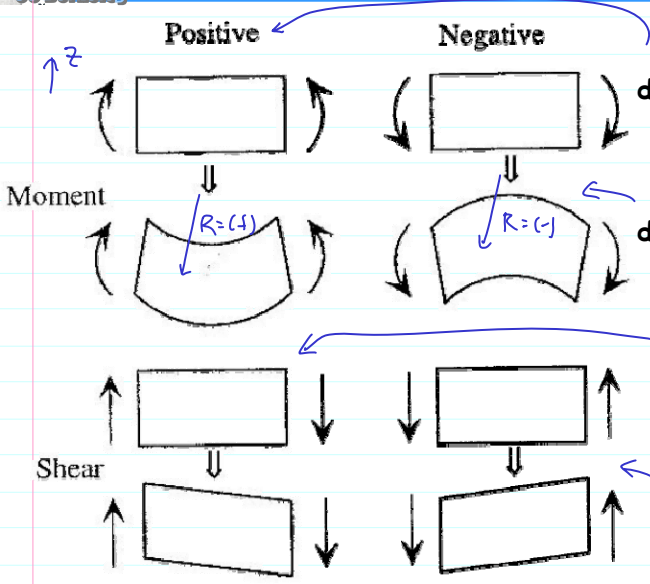


- Objective: Find relation between tip deflection y(x=L) and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible

Reaction Forces and Moments



Sign Conventions for Moments & Shear Forces



(+) moment leads to deformation with a (+) radius of curvature (i.e., upwards)

(-) moment leads to deformation with a (-) radius of curvature (i.e., downwards)

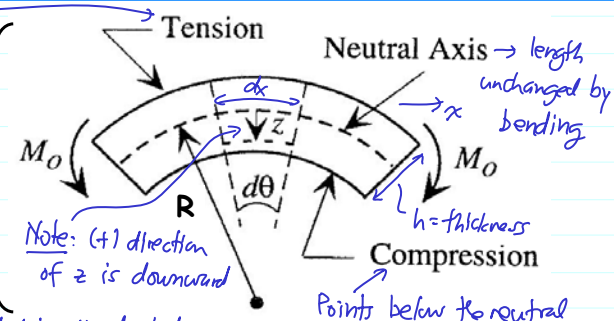
(+) shear forces produce clockwise rotation

(-) shear forces produce counter-clockwise rotation

Beam Segment in Pure Bending

Points above the neutral axis go into tension

Small section of a beam bent in response to a tranverse load



Note: (+) direction of z is downward

Points below the neutral axis go into compression

Consider a segment bounded by the dashed lines defined by $d\theta$:

At $z=0$: (i.e., at the neutral axis) $\Rightarrow$ segment length $= dx = R\,d\theta$ (1)

At any z: segment length $= dL = (R-z)\,d\theta$ (2)

Combining (1) & (2): $dL = dx - z\,d\theta = dx - \frac{z}{R}\,dx$

Beam Segment in Pure Bending (cont.)

Thus, the axial strain @ z:

[dx = original (unstressed) segment length]

$$\epsilon_x = \frac{dL - dx}{dx} = -\frac{z}{R} \Rightarrow \boxed{\epsilon_x = -\frac{z}{R}}$$