- Announcements:
- Lecture schedule for the next 4 weeks:

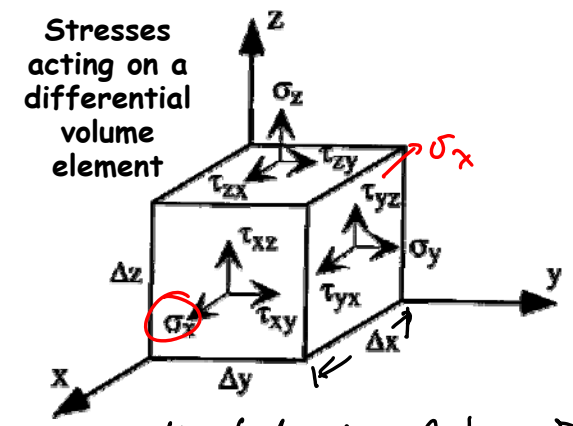
Monday	Tuesday	Wednesday	Thursday	Friday
Oct. 5 Lecture 5-6:30 241 Cory	6 Lecture 3:30-5 Moffitt	7	8 Discuss 3:30-5 Moffitt	9
12 Lecture 5-6:30 241 Cory	13	14	15 Discuss 3:30-5 Moffitt	16
19 Lecture 5-6:30 241 Cory	20 Lecture 3:30-5 Moffitt	21	22 Lecture 3:30-5 Moffitt	23
26 Discuss 3:30-5 Moffitt	27	28	29 Lecture 3:30-5 Moffitt	30
Nov. 2 Lecture 5-6:30 241 Cory	3 Midterm 3:30-5 Moffitt	4	5 Lecture 3:30-5 Moffitt	6

- Midterm Exam:
  - Tu, Nov. 3, during regular lecture
  - 1.5 hours, but might go longer
- ----------

- Today:
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ----------

Volume Change for a Uniaxial Stress (along x-direction)

Stresses acting on a differential volume element



Given an x-directed uniaxial stress σ_x:

$$\Delta x \longrightarrow \Delta x(1+\epsilon_x)$$

$$\left.\begin{aligned} \Delta y &\longrightarrow \Delta y(1-\nu\epsilon_x) \\ \Delta z &\longrightarrow \Delta z(1-\nu\epsilon_x) \end{aligned}\right\}$$

assume isotropic material & same ν along y & z

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The resulting change in volume: ΔV

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after application of } \sigma_x} - \Delta x \Delta y \Delta z$$

$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] $\rightarrow (1+mx)^n \approx 1+nmx$

$$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$$

$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) $\rightarrow$ no ΔV!

$\nu < 0.5 \rightarrow$ finite ΔV

For isotropic materials: (3D) $\rightarrow$ complete stress-strain relations

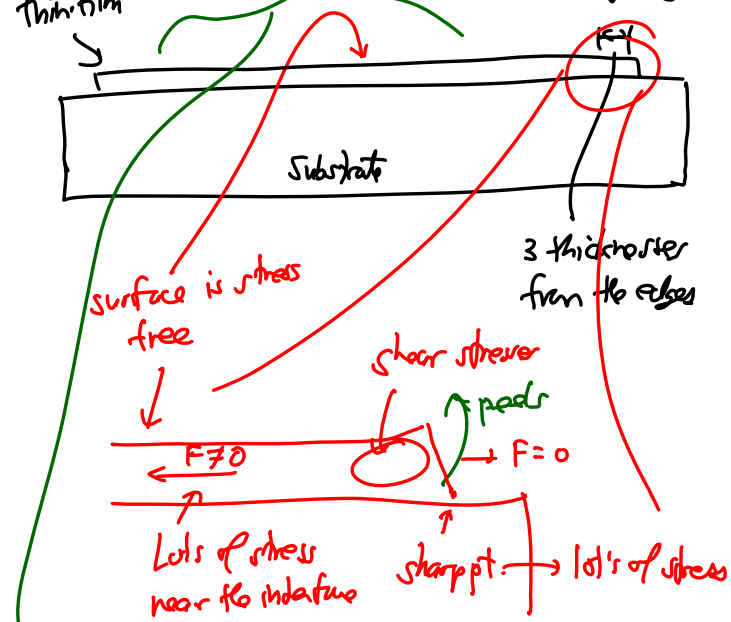
$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right], \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \quad \vdots$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Important Case: Plane Stress

$\Rightarrow$ common case for a thin-film coating on a rigid substrate: $\sigma_z = 0$



Get four components of stress:

$$\epsilon_x = \left(\frac{1}{E}\right)\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \left(\frac{1}{E}\right)\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

Plane stress $\rightarrow$ isotropic $\rightarrow$ $\sigma_x = \sigma_y = \sigma$
(symmetry in the xy-plane)

$$\epsilon_x = \epsilon_y = \epsilon$$

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$$\epsilon_x = \left(\frac{1}{E}\right)[\sigma - \nu\sigma] = \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} = \frac{\sigma}{E'}$$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. linear thermal expansion coefficient

$$\boxed{\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coeff.}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{kelvin}^{-1}]}$$

Remarks.

① α_T values tend to be in 10^{-6} to 10^{-7} range

② $10^{-6}K^{-1} = 1\,\mu\text{strain}/K$

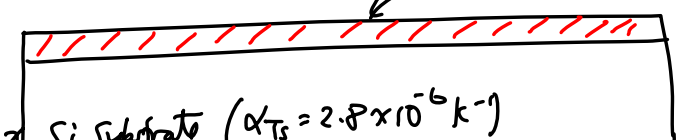
③ In 3D, get volume thermal expansion coeff.

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate T's → α_T can be treated as a const. of material

but actually, it's a function of T!

Ex. Thin-Film Thermal Stress



Assume: ① substrate is much thicker than the film.

② film is deposited stress free @ T_{dep}

③ then whole thing is cooled to room temp, T_r

substrate wins

Thermal Strain of the Substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate:

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film is attached → so the actual strain is that of the substrate

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note this is biaxial strain (assuming isotropic film)

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$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{=E'} \epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide

$\alpha_{Tf} = 70 \times 10^{-6} K^{-1}$, $E' = 4$ GPa

$\underbrace{\text{deposited @ } 250°C, \text{ then cooled to RT} = 25°C}_{\Delta T = 225 K}$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5 \times 10^{-2}$$

$[\mu = 10^{-6}, m = 10^{-3}, k = 10^{3}, G = 10^{9}]$

$$\sigma_{f,mismatch} = (4G)(1.5 \times 10^{-2}) = 60.5 \text{ MPa}$$

$G \uparrow 10^9$

stress is (+) ∴ tensile

[(−) would be compressive]

↑ SiO_2

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