EE 245: Introduction to MEMS
Lecture 23: Comb Drive

- **Announcements**:
- Lecture schedule for the next 3 weeks:

Monday	Tuesday	Wednesday	Thursday	Friday
9 Lecture 5-6:30 241 Cory	10 Lecture 3:30-5 Moffitt	11	15 Lecture 3:30-5 Moffitt	13
16 Discuss 3:30-5 241 Cory	17	18	19	20
23 Lecture 5-6:30 241 Cory	24 Lecture 3:30-5 Moffitt	25	26	27

- HW#6 due Tuesday, Nov. 24
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling

- **Last Time**:

$v_1 = |v_1| \cos\omega_0 t \rightarrow x = |x| \sin\omega_0 t$

90° phase shifted from v_1

Force terms only @ ω_0:

$$F_{d1}|_{\omega_0} = \underbrace{V_{P1}\frac{C_{01}}{d_1}|v_1|\cos\omega_0 t}_{\text{forcing term}} + \overbrace{V_{P1}^2\frac{C_{01}}{d_1^2}}^{k_e \rightarrow \text{electrical stiffness}}|x|\sin\omega_0 t$$

proportional to x $\quad$ 90°

Electrical Stiffness:

1. A negative spring constant!
2. Derived from V_P

$$\boxed{k_e = V_{P1}^2\frac{C_{01}}{d_1^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

What does this do for us?

↳ it affects resonance freq.!

$\omega_0 = \sqrt{\frac{k_m}{m}}$ ← mechanical spring constant, ← mass

↳ get this w/o V_P

Apply V_P:

$$\omega_0' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}} = \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_0' = \omega_0\left(1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right)^{1/2}}$$

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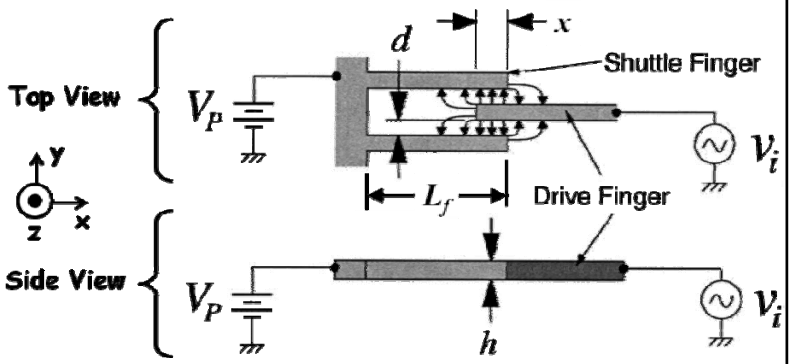
Electrostatic Comb Drive



$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$$

no x dependence! (ideally)

$$\left[C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d}\right]$$

$$F_d = \frac{2}{2}\frac{\epsilon_0 h}{d}\left(V_P^2 - 2V_P v_i + \cancel{v_i^2}\right) \approx \boxed{-2V_P\frac{\epsilon_0 h}{d}v_i = F_d}$$

can balance out

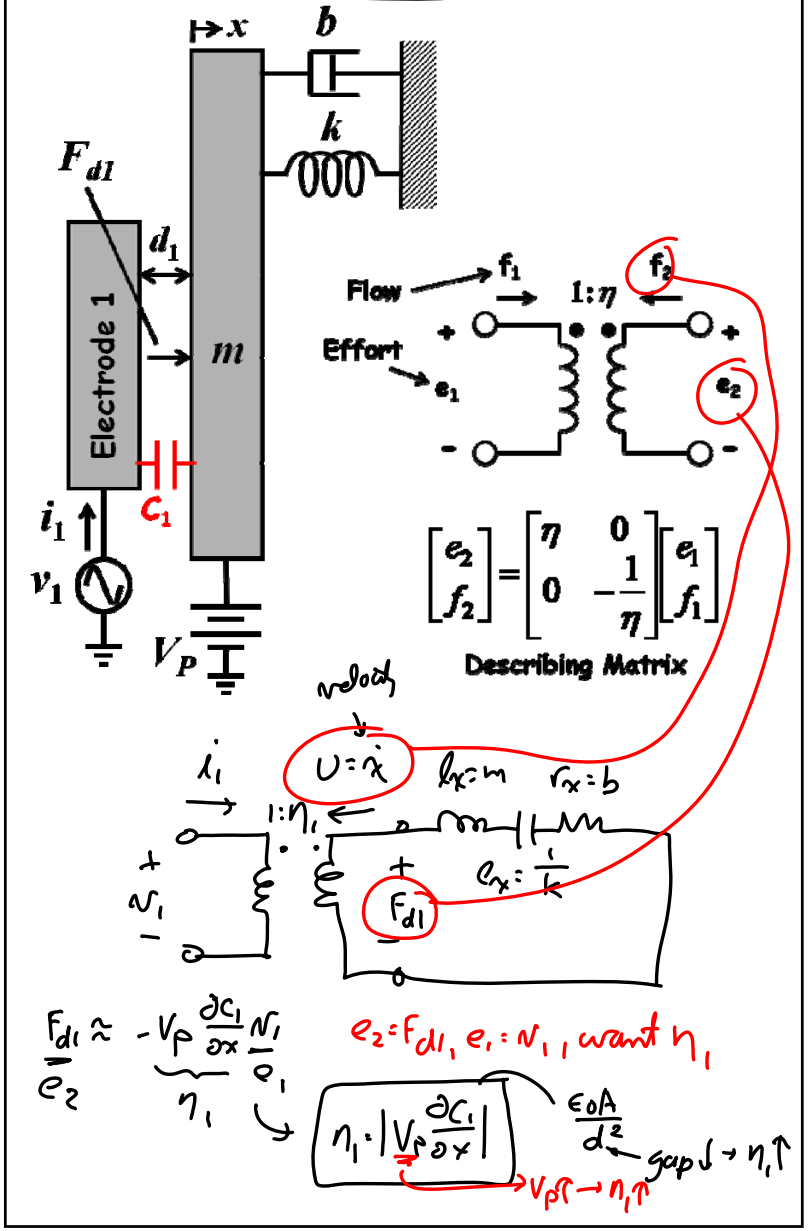
small

when $v_i = (+)$, $F_d = (-)$

∴ no electrical stiffness! (No k_e!)

- Go through remaniing comb-drive slides in Module 12
- Start Module 13: go thru first few slides up to transformer definition
- Then …

Input Electrical Equivalent Ckt.



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

$$\frac{F_{d1}}{e_2} \approx \underbrace{-V_P\frac{\partial C_1}{\partial x}}_{\eta_1}\frac{v_1}{e_1}$$

$e_2 = F_{d1}$, $e_1 = v_1$, want η_1

$$\boxed{\eta_1 = \left|V_P\frac{\partial C_1}{\partial x}\right|}$$

$\frac{\epsilon_0 A}{d^2}$ ← gap↓ → η_1↑

V_P↑ → η_1↑

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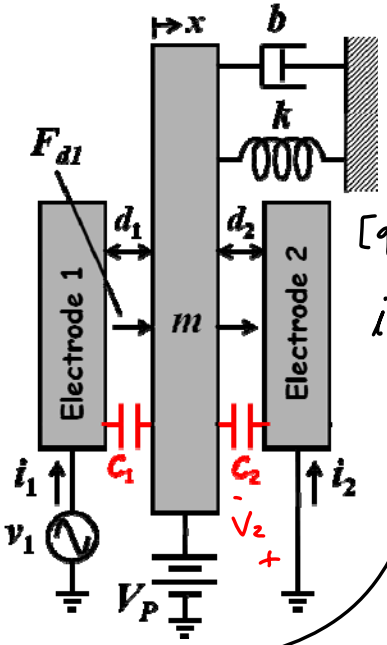
Output Current Into Ground



$[q = CV]$

$\Downarrow$

$i = \frac{dq}{dt} = C\frac{\partial V}{\partial t} + V\frac{\partial C}{\partial t}$

$i_2 = \cancel{C_2(x,t)\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$

$[V_2(t) = -V_P] \rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$

In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

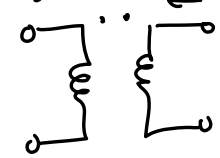
$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

motional current!

$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C}{\partial x}U$

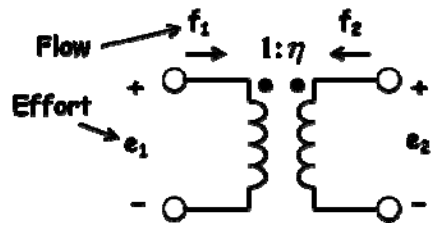
90° phase lag (+) (+) → $I_2 = (-)$ when $x = (+)$ ✓

Velocity → $U = \dot{x}$ → $\eta_2 : 1$ ← I_2



$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$

$[f_1 = I_2, f_2 = U] \rightarrow I_2 = -\eta_2 U \therefore \boxed{\eta_2 = \left|V_P\frac{\partial C}{\partial x}\right|}$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

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