EE 245: Introduction to MEMS
Lecture 25: Sensing Circuits

- **Announcements**:
- Lecture schedule for the next two weeks:

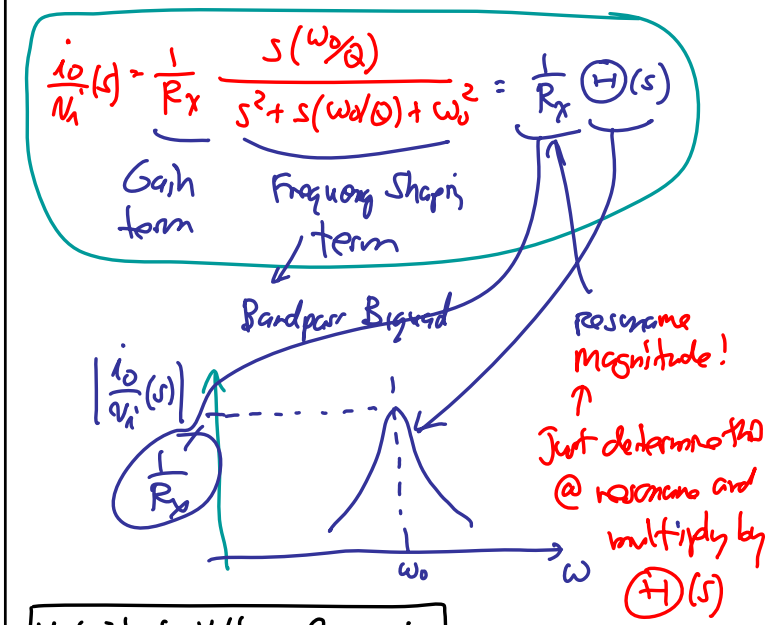
Monday	Tuesday	Wednesday	Thursday	Friday
Nov. 23 Lecture 5-6:30 241 Cory	24 Lecture 3:30-5 Moffitt	25	26	27
30	Dec. 1 Lecture 3:30-5 Moffitt	2	3 Lecture 3:30-5 Moffitt	4

- HW#6 due Tuesday, Nov. 24
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- ---
- **Last Time**:

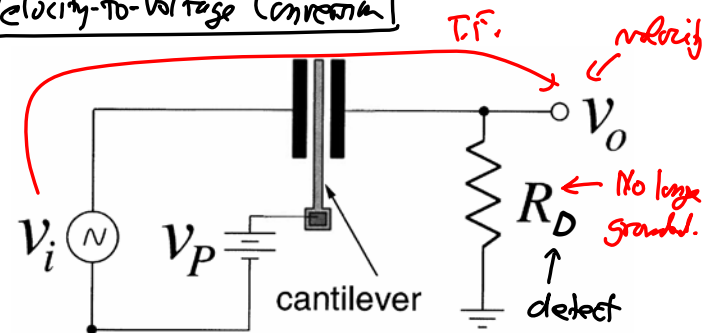
$$\frac{i_o}{v_i}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain term}} \underbrace{\frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}}_{\text{Frequency Shaping term}} = \frac{1}{R_x}\,\Theta(s)$$

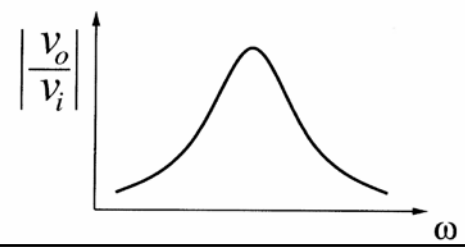
Bandpass Biquad

resonance magnitude!



Velocity-to-Voltage Conversion





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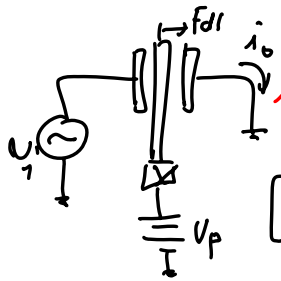


output grounded

$$\frac{\dot{x}}{F_{dl}}(s) = \frac{\omega_o Q}{k}\,\Theta(s)$$

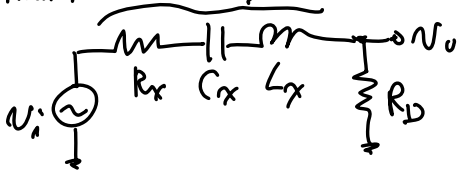
$$\left[F_{dl} = \eta_{e1} v_i\right] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_o Q}{k}\,\Theta(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e1}\frac{\omega_o Q}{k}\,\Theta(s)$$

$$= \frac{\eta_{e1}\eta_{e2}Q}{m\omega_o}\,\Theta(s)$$

$\frac{1}{R_{x12}}$

Now include R_D: $Q = \frac{\omega_o L_x}{R_x}$



$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} \text{ -- math...}$$

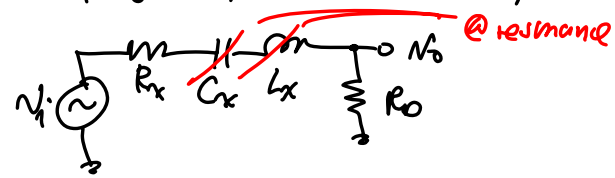
$$= \underbrace{\frac{R_D}{R_x+R_D}}_{\text{Gain term}} \underbrace{\frac{s\left(\frac{R_x+R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x+R_D}{L_x}\right) + \frac{1}{L_xC_x}}}_{\text{Freq. shaping Term}} \quad *$$

$$\left\{Q = \frac{\omega_o L_x}{R_x} \rightarrow Q' = \frac{\omega_o L_x}{R_x+R_D} \rightarrow \frac{R_x+R_D}{L_x} = \frac{\omega_o}{Q'}\right\}$$

$$* \quad \frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D}\,\frac{s(\omega_o/Q')}{s^2 + s(\omega_o/Q') + \omega_o^2}$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D}\cdot\Theta(s, Q') \qquad Q' = Q\left(\frac{R_x}{R_x+R_D}\right)}$$

What you get if you do the whole analysis @ resonance:



@ resonance: $\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}$

Then, to generalize to off-resonance: $\times\,\Theta(s, Q')$

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D}\,\Theta(s, Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

To maximize gain: want R_D big!

But Q drops!

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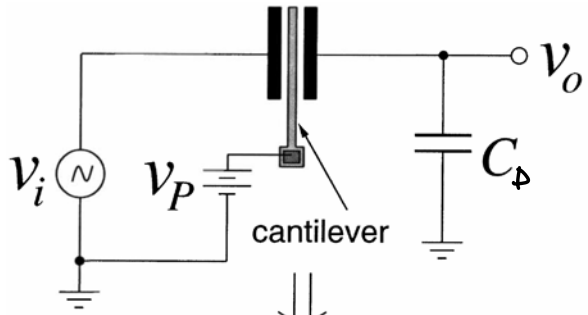
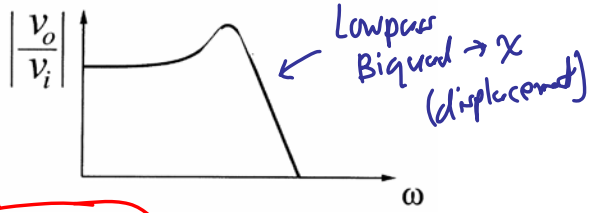
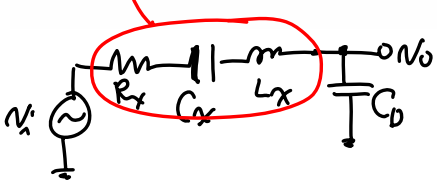
Position-to-Voltage Conversion







$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}} = \frac{\frac{\cancel{s}C_x}{\cancel{s}C_D}}{sR_xC_x + 1 + s^2L_xC_x + \frac{\cancel{s}C_x}{\cancel{s}C_D}}$$

$$= \frac{C_x/C_D}{1+C_x/C_D} \; \frac{1}{1 + \frac{sR_xC_x}{1+C_x/C_D} + s^2\frac{L_xC_x}{1+C_x/C_D}}$$

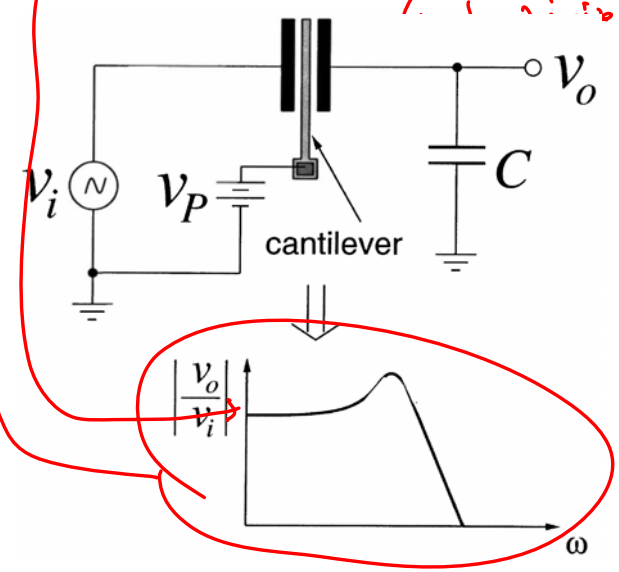
$$= \frac{C_x/C_D}{1+C_x/C_D} \; \frac{\frac{1+C_x/C_D}{L_xC_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1+C_x/C_D)}{L_xC_x}}$$

$$\left[\omega_0^2 = \frac{1}{L_xC_x} \rightarrow (\omega_0')^2 = \omega_0^2(1+C_x/C_D)\right]$$

$$\left[Q' = \frac{\omega_0' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0'}{Q'},\; Q' = Q\sqrt{1+C_x/C_D}\right]$$

$$\frac{v_o}{v_i}(s) = \underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}} \; \underbrace{\frac{(\omega_0')^2}{s^2 + \left(\frac{\omega_0'}{Q'}\right)s + (\omega_0')^2}}_{\text{Lowpass Biquad}}$$

To maximize the gain: → 1, need $C_D \ll C_x$



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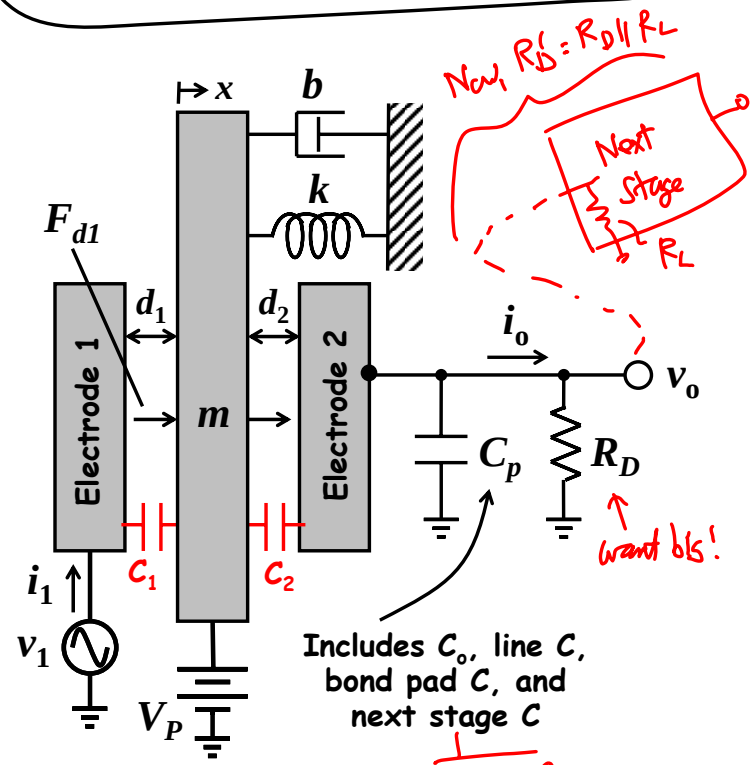
Note: Can use similar short-cut:

① Get DC response → C's dominate

② Then:

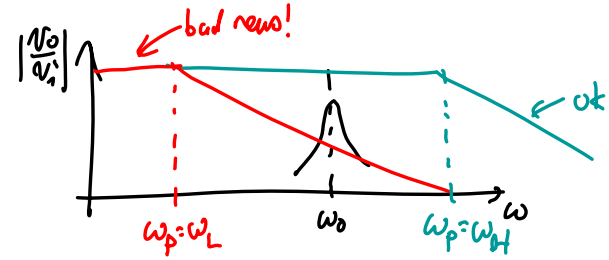
$$\frac{v_o}{v_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, \omega_o', Q') \cdot \omega_o' Q'$$



Now, we get:

$$\omega_p = \frac{1}{(R_x \| R_D) C_p}$$

$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot \Theta(s, \omega_o', Q')$$



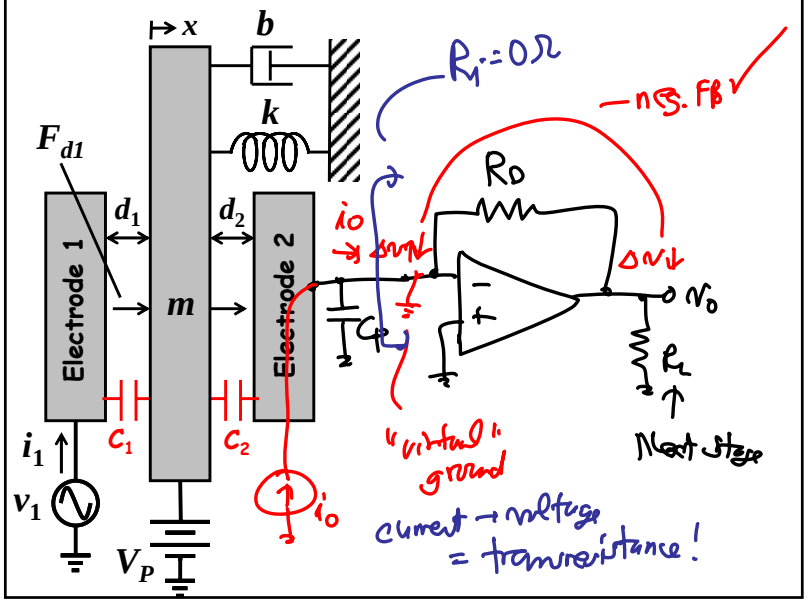
Problems w/ Purely Resistive Detection

① Need large R_D for large gain...but:

② $R_D \uparrow \rightarrow Q \downarrow$

③ $R_D \uparrow \rightarrow \frac{1}{R_D C_p} \downarrow \rightarrow$ get LPF w/ cut-off $\omega_p =$

④ Load R_L from next stage reduces gain...



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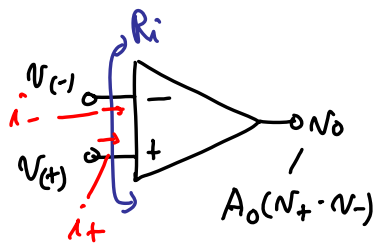
Ideal Op Amp Laws:

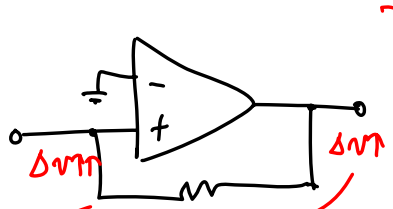
① $R_i = \infty$: $i_- = 0$, $i_+ = 0$

② $R_o = 0$

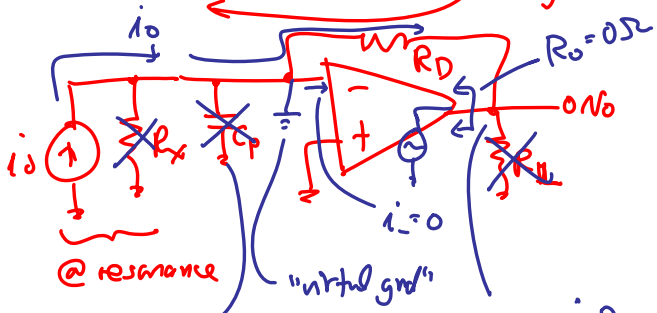
③ $A_o = \infty \rightarrow v_+ = v_-$

neg. FB





Ex. of pos. FB



$v_o = -i_o R_D$

No volts across C_p, ∴ C_p is not there!

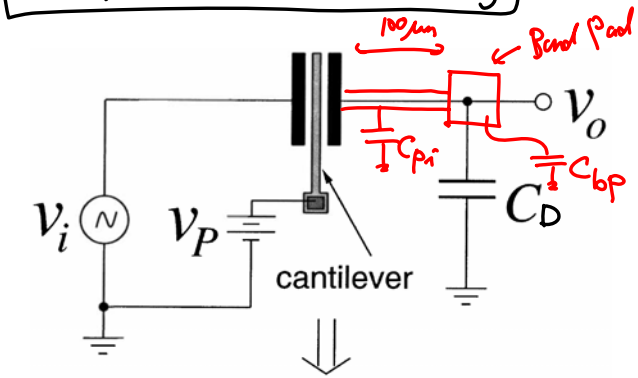
$$\boxed{\frac{v_o}{i_o} = -R_D}$$ @ resonance

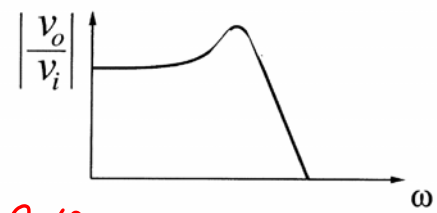
More generally:

$$\boxed{\frac{v_o}{v_i}(s) = -\frac{R_D}{R_x}\,\Theta(s)}$$

no Q degradation (i.e., no Q')

Problems w/ Pure-C Partition Sensing





Start w/:

$$\frac{v_o}{v_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{1}{s} \cdot \Theta(s, \omega_o', Q') \cdot \omega_o' Q'$$

To maximize gain, minimize C_D!

Problem: $C_D \rightarrow C_D + C_{pi} + C_{bp}$

DC Gain: $\dfrac{C_x/(C_D + C_{pi} + C_{bp})}{1 + C_x/(C_D + C_{pi} + C_{bp})}$ smaller

⇓

Need to suppress C_p!

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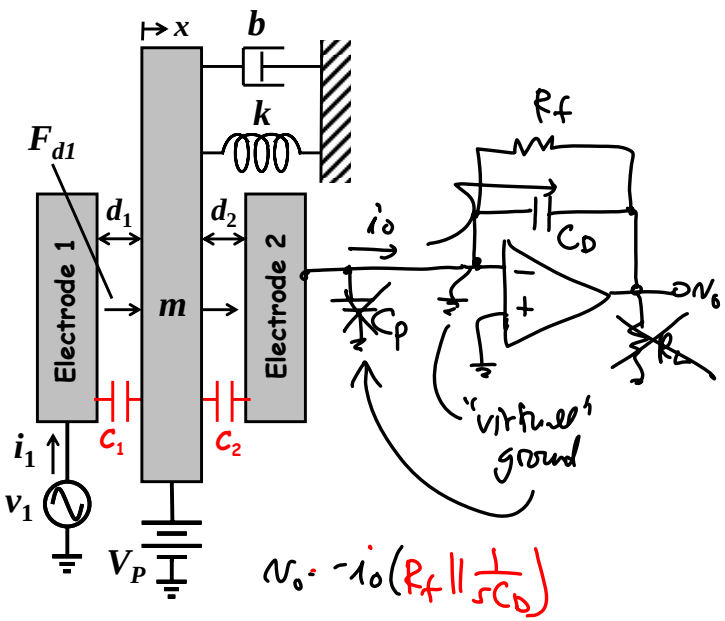

$$v_o = -i_o\left(R_f \parallel \frac{1}{sC_D}\right)$$

$$\left(R_f \gg \frac{1}{sC_D}\right) \Rightarrow = -\frac{v_i}{R_x}\Theta(s)\frac{1}{sC_D}$$

$$\boxed{\frac{v_o}{v_i}(s) = -\frac{1}{R_x C_D}\frac{\Theta(s)}{s}}$$

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