

PROBLEM SET #7

Issued: Wednesday, Nov. 23, 2011.

Due (at 7 p.m.): Thursday, Dec. 8, 2011, in the EE C245 HW box in 240 Cory.

1. Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), cell phones, and game controllers (e.g., the Wii). Of these applications, the last four already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best further explained via example.

This problem involves the MEMS-based micro-gyroscope shown in Fig. 1. In this device, momentum is generated by driving the proof mass into resonance vibration using the capacitive comb fingers along the x -axis. When the device is rotated about the z -axis (indicated in Fig. 1), the vibrating mass will attempt to preserve its momentum in the original x -direction, which will then make the mass appear to deflect in the y -direction. This y -directed motion is then sensed by parallel-plate capacitors to determine rotation rate. In quantitative terms, the angular velocity is sensed about z -axis, which then generates a *Coriolis Force* ($F = 2m_s\dot{x}_d \times \Omega$) that acts on the proof mass along the y -direction, which is then picked up by the varying gap capacitances.

Several figures are provided to support the questions that follow. In particular, Table 1 provides measured or target parameter values for the fabricated device. Figs. 2-7 then identify different parts of the structure, indicate which portions are freely suspended and which are anchored (the blue regions are anchored), and provide key dimensions. As indicated, the gaps of comb fingers are all $1\mu\text{m}$ and the gaps of parallel-plate capacitive fingers are all $1\mu\text{m}$. The thicknesses of the structures are all $20\mu\text{m}$. The applied DC voltage, V_P is 20 V. Use a density $\rho = 2300\text{ kg/m}^3$ and Young's Modulus $E = 150\text{ GPa}$.

Table 1: Gyroscope Modeling Parameters

	Target Resonant Frequency (f_0)	Measured Quality Factor (Q)
Drive Mode	9,000 Hz	150
Sense Mode	10,000 Hz	100

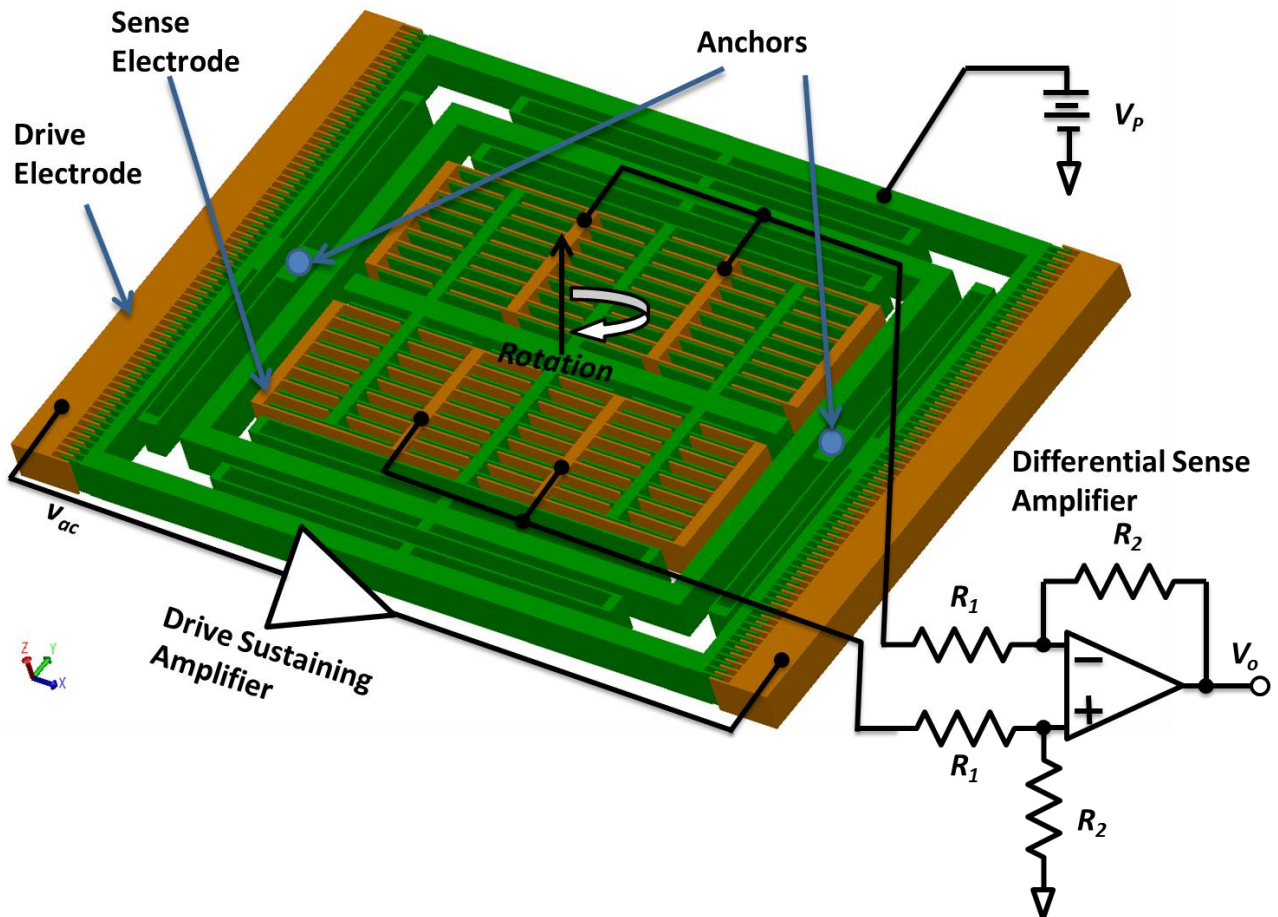


Figure 1. Isometric View of a surface micromachined MEMS gyroscope with drive and sense circuitry.

- Use the surface area information given in the figures to determine the needed spring constants at the electrode locations for (i) the drive and (ii) the sense modes. Ignore the masses of the springs in this problem.
- With a beam width, W of $1\mu\text{m}$, What suspension beam lengths, L_D and L_S , are required to achieve the needed spring constants for (i) the drive and (ii) the sense modes determined in (a)?
- Identify the electrodes for the (i) drive and (ii) sense modes and determine the capacitance and change in capacitance per unit displacement for each.
- Draw and specify (numerically) all element values in the equivalent circuits (transformers + LCR) modeling the (i) drive mode and (ii) the sense mode in (a).
- Code the equivalent circuits in (d) into SPICE netlists, add the necessary elements (e.g., a voltage source, a resistor, a capacitor, or an inductor) to drive at one end and detect velocity at the other, and simulate Bode plots for the voltage-to-velocity transfer functions using SPICE that include the low frequency and resonance responses of the structures.
- Assume that during steady-state oscillation along the drive axis the drive amplifier delivers an ac voltage v_d with an amplitude of 1.5V and a frequency equal to the resonance frequency of the drive mode. Also, assume that the input of the drive amplifier detects velocity and that

its input resistance is very small. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.

- (g) If the device is subjected to velocity rotation rate along the indicated axis of $\Omega = 0.5$ radians/second, what is the magnitude of the output voltage V_o if R_1 is set to 0Ω and R_2 is set to $1 \text{ k}\Omega$. Assume the differential amplifier employs an ideal op-amp.
- (h) Repeat (g) with both R_1 and R_2 set to $15 \text{ k}\Omega$.
- (i) Assuming you drove the device along its sense axis with an input force, sketch the sense axis input force-to-output voltage Bode plots for parts (g) and (h) on one graph. Give numerical values for important markers, like resonance frequency, Q , and the intersection with the y-axis.

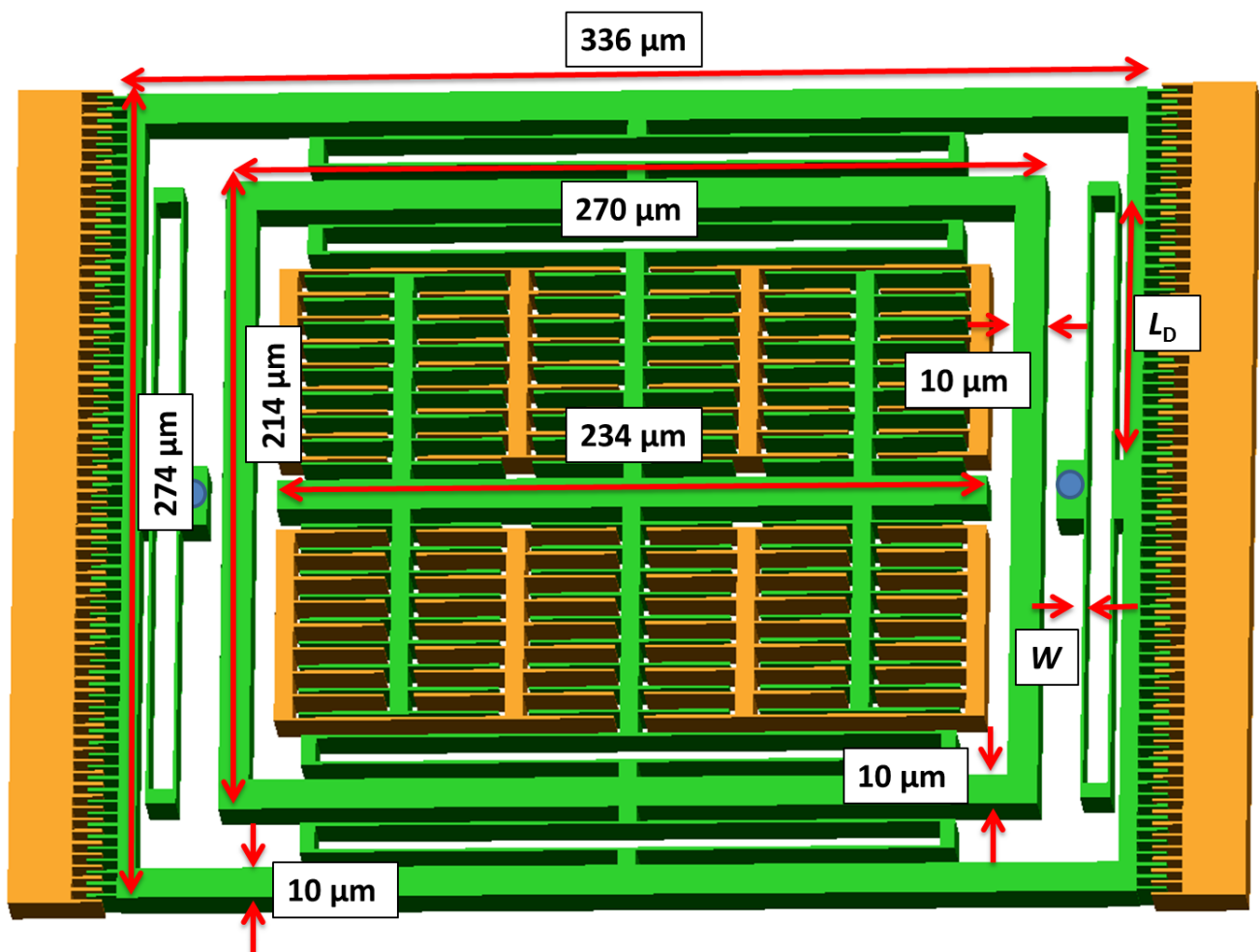


Figure 2: Top View with Proof Mass Dimensions

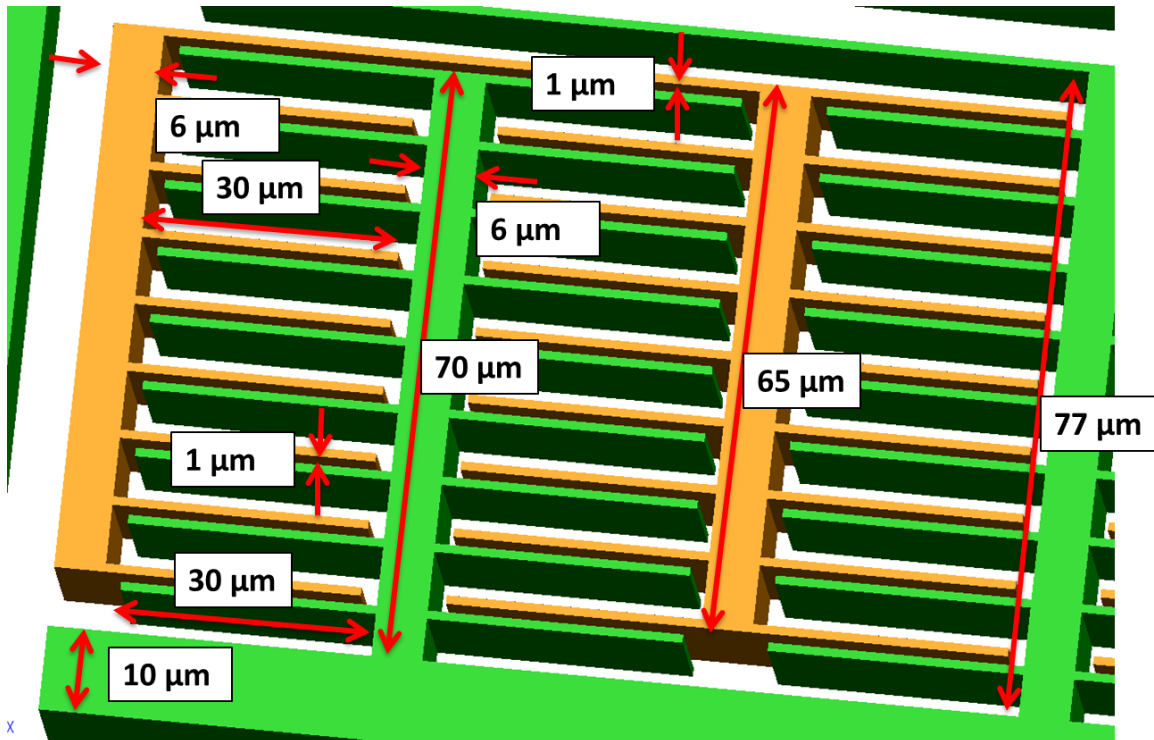


Figure 3: Sense Axis Detail

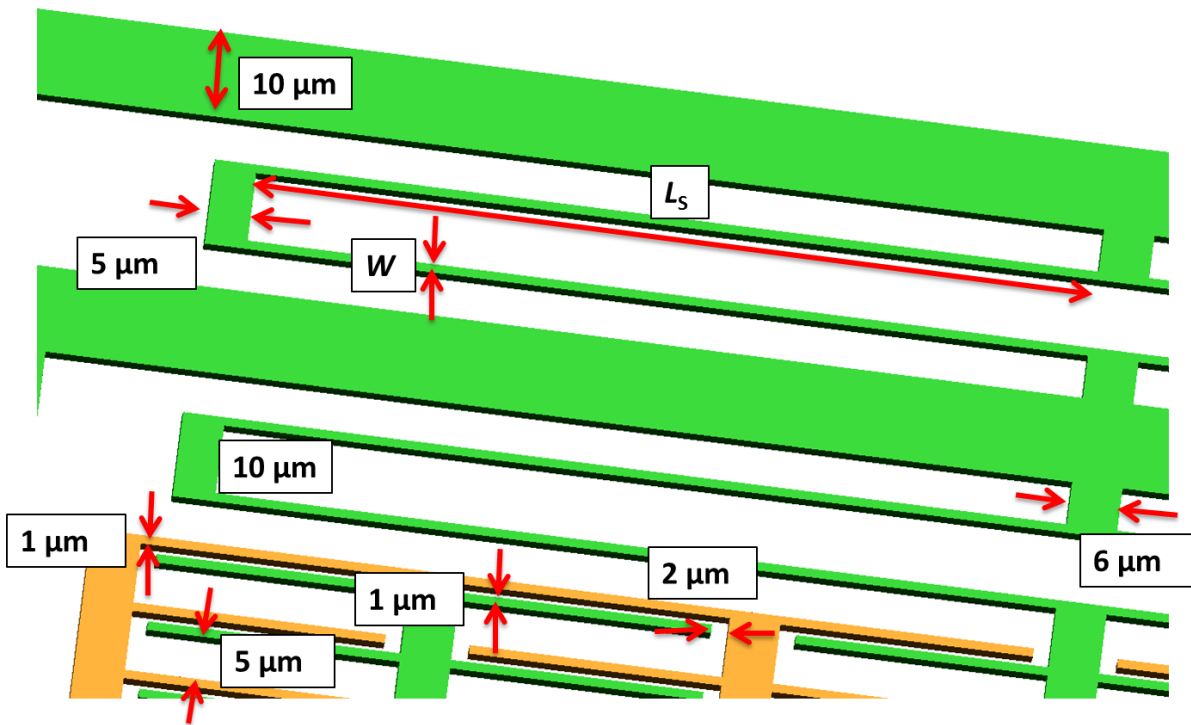


Figure 4: Sense Axis Folded-Flexures

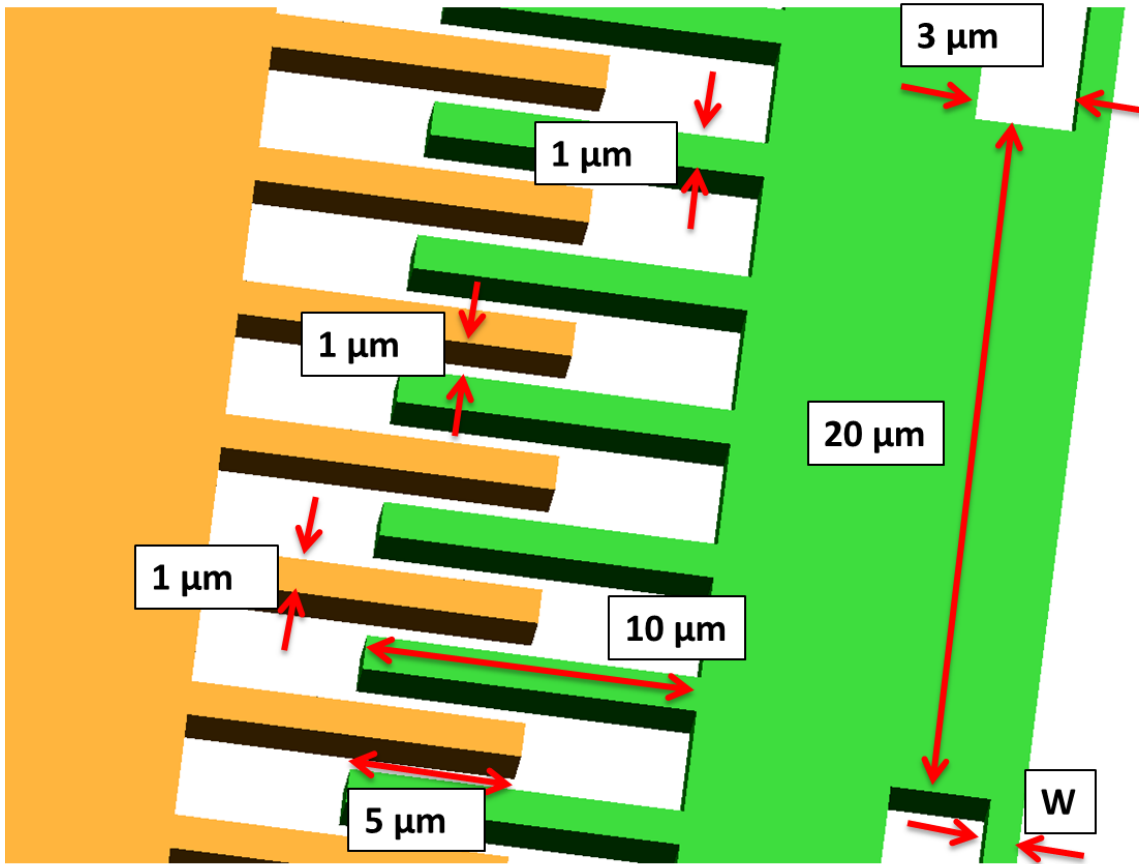


Figure 5: Drive Axis comb-Fingers

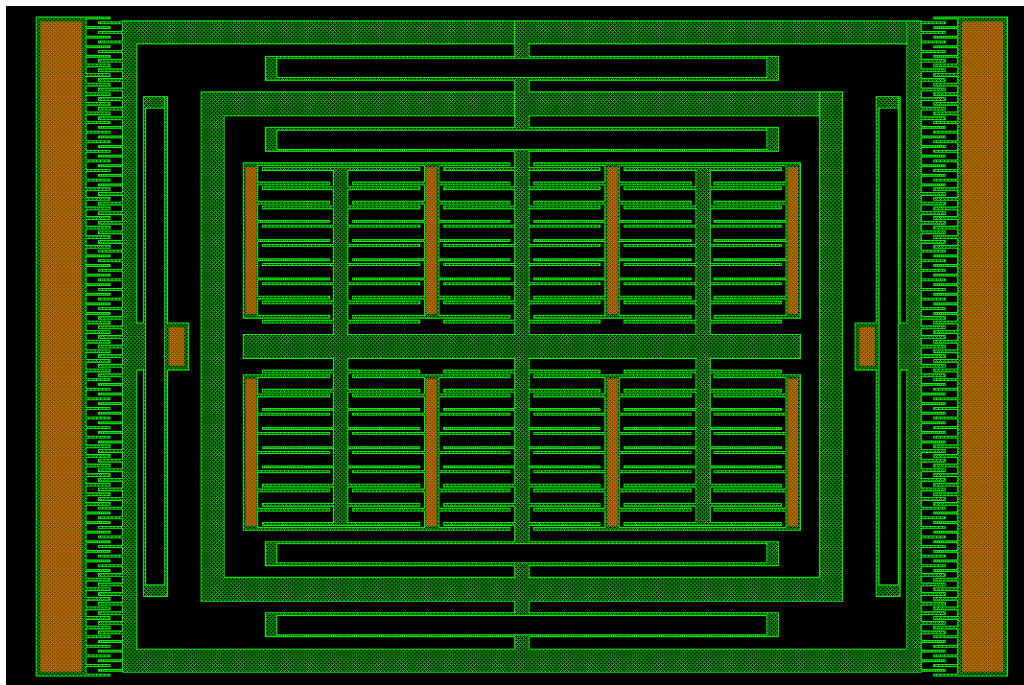


Figure 6: Gyroscope Layout