EE247
Lecture 4

• Ladder type filters
  – For simplicity, will start with all pole ladder type filters
    • Convert to integrator based form- example shown
  – Then will attend to high order ladder type filters incorporating zeros
    • Implement the same 7th order elliptic filter in the form of ladder RLC with zeros
      – Find level of sensitivity to component mismatch
      – Compare with cascade of biquads
    • Convert to integrator based form utilizing SFG techniques
  – Effect of Integrator Non-Idealities on Filter Frequency Characteristics

LC Ladder Filters

• Design:
  – Filter tables
  – CAD tools
    • Matlab
    • Spice
LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

- \( f_{-3dB} = 10\text{MHz} \)
- \( f_{\text{stop}} = 20\text{MHz} \)
- \( R_s > 27\text{dB} \)

- Maximally flat passband \( \Leftrightarrow \) Butterworth
- Find minimum filter order:
  - Use of Matlab
  - Or Tables
- Here tables used

\[
\frac{f_{\text{stop}}}{f_{-3dB}} = 2
\]
\[
R_s > 27\text{dB}
\]

Minimum Filter Order \( \Rightarrow \) 5th order Butterworth

From: Williams and Taylor, p. 2-37

Find values for \( L \) & \( C \) from Table:

Note \( L \) & \( C \) values normalized to \( \omega_{-3dB} = 1 \)

Denormalization:

- Multiply all \( L_{\text{Norm}}, C_{\text{Norm}} \) by:
  - \( L_r = R/\omega_{3dB} \)
  - \( C_r = 1/(RX\omega_{3dB}) \)

\( R \) is the value of the source and termination resistor

(choose both 1\( \Omega \) for now)

Then:

\[
\begin{align*}
L &= L_r \times L_{\text{Norm}} \\
C &= C_r \times C_{\text{Norm}}
\end{align*}
\]

From: Williams and Taylor, p. 11.3
LC Ladder Filter Design Example

Find values for L & C from Table:

Normalized values:
- C1\textsubscript{Norm} = C5\textsubscript{Norm} = 0.618
- C3\textsubscript{Norm} = 2.0
- L2\textsubscript{Norm} = L4\textsubscript{Norm} = 1.618

Denormalization:

Since $\omega_{3\text{dB}} = 2\pi \times 10\text{MHz}$
- $L_r = \frac{R}{\omega_{3\text{dB}}} = 15.9 \text{ nH}$
- $C_r = \frac{1}{(RX_{3\text{dB}})} = 15.9 \text{ nF}$
- $R = 1$
- $C_1 = C_5 = 9.836 \text{ nF}$, $C_3 = 31.83 \text{ nF}$
- $L_2 = L_4 = 25.75 \text{ nH}$

From: Williams and Taylor, p. 11.3

Last Lecture:

Example: 5th Order Butterworth Filter

Specifications:
- $f_{3\text{dB}} = 10\text{MHz}$,
- $f_{\text{stop}} = 20\text{MHz}$
- $R_s > 27\text{dB}$

Used filter tables to obtain $L_s \& C_s$
Low-Pass RLC Ladder Filter
Conversion to Integrator Based Active Filter

\[ V_1 = V_{in} - V_2, \quad V_2 = \frac{I_2}{sC_1}, \quad V_3 = V_2 - V_4 \]
\[ V_4 = \frac{I_4}{sC_3}, \quad V_5 = V_4 - V_6 \]
\[ V_6 = \frac{I_6}{sC_5}, \quad V_o = V_6 \]
\[ I_1 = \frac{V_1}{R_s}, \quad I_2 = I_1 - I_3 \]
\[ I_4 = I_3 - I_5 \]

Low-Pass RLC Ladder Filter
Signal Flowgraph

\[ V_1 = V_{in} - V_2, \quad V_2 = \frac{I_2}{sC_1}, \quad V_3 = V_2 - V_4 \]
\[ V_4 = \frac{I_4}{sC_3}, \quad V_5 = V_4 - V_6 \]
\[ V_6 = \frac{I_6}{sC_5}, \quad V_o = V_6 \]
\[ I_1 = \frac{V_1}{R_s}, \quad I_2 = I_1 - I_3 \]
\[ I_4 = I_3 - I_5 \]
\[ I_5 = \frac{V_5}{sL_3}, \quad I_6 = I_5 - I_7 \]
\[ I_7 = \frac{V_6}{R_L} \]
Low-Pass RLC Ladder Filter

Signal Flowgraph

\[ V_{in} \]

\[ V_{in} \]

\[ \frac{1}{R_s} \]

\[ \frac{1}{sC_1} \]

\[ I_2 \]

\[ I_3 \]

\[ I_4 \]

\[ I_5 \]

\[ I_6 \]

\[ I_7 \]

\[ \frac{1}{R_L} \]

\[ V_1 \]

\[ V_2 \]

\[ V_3 \]

\[ V_4 \]

\[ V_5 \]

\[ V_6 \]

\[ V_7 \]

\[ I_1 \]

\[ I_2 \]

\[ I_3 \]

\[ I_4 \]

\[ I_5 \]

\[ I_6 \]

\[ I_7 \]

\[ \frac{1}{R_s} \frac{1}{sC_1} \]

\[ \frac{1}{sL_2} \]

\[ \frac{1}{sC_3} \]

\[ \frac{1}{sL_4} \]

\[ \frac{1}{sC_5} \]

\[ \frac{1}{R_L} \]

\[ V_{in} \]

\[ V_{in} \]

\[ \frac{1}{R_s} \]

\[ \frac{1}{sC_1} \]

\[ I_2 \]

\[ I_3 \]

\[ I_4 \]

\[ I_5 \]

\[ I_6 \]

\[ I_7 \]

\[ \frac{1}{R_L} \]

\[ V_1 \]

\[ V_2 \]

\[ V_3 \]

\[ V_4 \]

\[ V_5 \]

\[ V_6 \]

\[ V_7 \]

\[ I_1 \]

\[ I_2 \]

\[ I_3 \]

\[ I_4 \]

\[ I_5 \]

\[ I_6 \]

\[ I_7 \]

\[ \frac{1}{R_s} \frac{1}{sC_1} \]

\[ \frac{1}{sL_2} \]

\[ \frac{1}{sC_3} \]

\[ \frac{1}{sL_4} \]

\[ \frac{1}{sC_5} \]

\[ \frac{1}{R_L} \]

\[ V_{in} \]

\[ V_{in} \]

\[ \frac{1}{R_s} \]

\[ \frac{1}{sC_1} \]

\[ I_2 \]

\[ I_3 \]

\[ I_4 \]

\[ I_5 \]

\[ I_6 \]

\[ I_7 \]

\[ \frac{1}{R_L} \]

Low-Pass RLC Ladder Filter

Normalize
Low-Pass RLC Ladder Filter

Synthesize

\[ V_{in} \rightarrow V_2 \rightarrow V_4 \rightarrow V_6 \rightarrow V_0 \]

\[ V_1 = \frac{R}{R_s} \]
\[ V_2 = \frac{1}{sC_1R} \]
\[ V_3 = \frac{R_s}{sL_2} \]
\[ V_4 = \frac{1}{sC_3R} \]
\[ V_5 = \frac{R_s}{sL_4} \]
\[ V_6 = \frac{1}{sC_5R} \]
\[ V_7 = \frac{R_s}{R_L} \]

Integrator Based Implementation

\[ \tau_1 = C_1R^* \]
\[ \tau_2 = \frac{L_2}{R_s^*} \]
\[ \tau_3 = C_3R^* \]
\[ \tau_4 = \frac{L_4}{R^*} \]
\[ \tau_5 = C_5R^* \]

Building Block:
RC Integrator

\[ \tau = \frac{1}{sRC} \]
Negative Resistors

Integrator Based Implementation of LP Ladder Filter
Synthesize
Frequency Response

Scale Node Voltages

Scale $V_o$ by factor "s"
Node Scaling

Maximizing Signal Handling by Node Voltage Scaling

Before Node Scaling

After Node Scaling

Scale $V_o$ by factor $s$
Filter Noise

Total noise @ the output: 1.4 μV rms (noiseless opamps)

That’s excellent, but the capacitors are very large (and the resistors small → high power dissipation). Not possible to integrate.

Suppose our application allows higher noise in the order of 140 μV rms ...

Scale to Meet Noise Target

Scale capacitors and resistors to meet noise objective

s = 10⁻⁴
Noise: 141 μV rms (noiseless opamps)
Completed Design

5th order ladder filter
Final design utilizing:
- Node scaling
- Final R & C scaling based on noise considerations

Sensitivity

- $C_1$ made (arbitrarily) 50% (!) larger than its nominal value
- 0.5 dB error at band edge
- 3.5 dB error in stopband
- Looks like very low sensitivity
Differential 5th Order Lowpass Filter

- Since each signal and its inverse readily available, eliminates the need for negative resistors!
- Differential design has the advantage of even order harmonic distortion components and common mode spurious pickup automatically cancels
- Disadvantage: Double resistor and capacitor area!

RLC Ladder Filters
Including Transmission Zeros

All poles

Poles & Zeros
RLC Ladder Filter Design Example

- Design a baseband filter for CDMA IS95 cellular phone receive path with the following specs.
  - Filter frequency mask shown on the next page
  - Allow enough margin for manufacturing variations
    - Assume pass-band magnitude variation of 1.8dB
    - Assume the -3dB frequency can vary by +/-8% due to manufacturing tolerances & circuit inaccuracies
  - Assume any phase impairment can be compensated in the digital domain

* Note this is the same example as for cascade of biquad while the specifications are given closer to a real product case
RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Since phase impairment can be corrected for, use filter type with max. roll-off slope/rod
  → Filter type: Elliptic
- Design filter freq. response to fall well within the freq. mask
  - Allow margin for component variations & mismatches
- For the passband ripple, allow enough margin for ripple change due to component & temperature variations
  → Design nominal passband ripple of 0.2dB
- For stopband rejection add a few dB margin: 44 + 5 = 49dB
- Final design specifications:
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 49 dB
- Use Matlab or filter tables to decide the min. order for the filter (same as cascaded biquad example)
  - 7th Order Elliptic

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RLC Low-Pass Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Use filter tables to determine LC values
RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Specifications
  - $f_{pass} = 650$ kHz  $R_{pass} = 0.2$ dB
  - $f_{stop} = 750$ kHz  $R_{stop} = 49$ dB
- Use filter tables to determine LC values
  - Table from: A. Zverev, Handbook of filter synthesis, Wiley, 1967
  - Elliptic filters tabulated wrt “reflection coefficient $\rho$”

$$R_{pass} = -10 \times \log \left( 1 - \rho^2 \right)$$

- Since $R_{pass} = 0.2\text{dB} \rightarrow \rho = 20\%$
- Use table accordingly

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RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Table from Zverev book page #281 & 282:
- Since our spec. is $A_{min} = 44$dB add 5dB margin & design for $A_{min} = 49$dB
Table from Zverev page #281 & 282:

Normalized component values:

- \( C_1 = 1.17677 \)
- \( C_2 = 0.19393 \)
- \( L_2 = 1.19467 \)
- \( C_3 = 1.51134 \)
- \( C_4 = 1.01098 \)
- \( L_4 = 0.72398 \)
- \( C_5 = 1.27776 \)
- \( C_6 = 0.71211 \)
- \( L_6 = 0.80165 \)
- \( C_7 = 0.83597 \)

RLC Filter Frequency Response

- Frequency mask superimposed
- Frequency response well within spec.
Passband Detail

- Passband well within spec.

![Passband Detail Graph]

RLC Ladder Filter Sensitivity

- The design has the same specifications as the previous example implemented with cascaded biquads.

- To compare the sensitivity of RLC ladder versus cascaded-biquads:
  - Changed all Ls & Cs one by one by 2% in order to change the pole/zeroes by 1% (similar test as for cascaded biquad)
  - Found frequency response most sensitive to L4 variations
  - Note that by varying L4 both poles & zeros are varied
RCL Ladder Filter Sensitivity

Component mismatch in RLC filter:
- Increase L4 from its nominal value by 2%
- Decrease L4 by 2%
Sensitivity of Cascade of Biquads

Component mismatch in Biquad 4 (highest Q pole):
- Increase $\omega_{p4}$ by 1%
- Decrease $\omega_{z4}$ by 1%

High Q poles $\Rightarrow$ High sensitivity in Biquad realizations

Sensitivity Comparison for Cascaded-Biquads versus RLC Ladder

- 7th Order elliptic filter
  - 1% change in pole & zero pair

<table>
<thead>
<tr>
<th></th>
<th>Cascaded Biquad</th>
<th>RLC Ladder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passband deviation</td>
<td>2.2dB (29%)</td>
<td>0.2dB (2%)</td>
</tr>
<tr>
<td>Stopband deviation</td>
<td>3dB (40%)</td>
<td>1.7dB (21%)</td>
</tr>
</tbody>
</table>

Doubly terminated LC ladder filters $\Rightarrow$ Significantly lower sensitivity compared to cascaded-biquads particularly within the passband
RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

7th order Elliptic

- Previously learned to design integrator based ladder filters without transmission zeros
  - Question:
    - How do we implement the transmission zeros in the integrator-based version?
    - Preferred method → no extra power dissipation → no active elements

Integrator Based Ladder Filters
How Do to Implement Transmission zeros?

- Use KCL & KVL to derive:
  \[ I_2 = I_1 - I_3 - I_{C_a} \]
  \[ I_{C_a} = (V_2 - V_4) s C_a \]
  \[ V_2 = \frac{I_2}{s C_1} \]
  \[ V_2 = \frac{I_1 - I_3 - I_{C_a}}{s C_1} \]

Substituting for \( I_{C_a} \) and rearranging:

\[ V_2 = \frac{I_1 - I_3}{s (C_1 + C_a)} + V_4 \times \frac{C_a}{C_1 + C_a} \]
Integrator Based Ladder Filters
How Do to Implement Transmission zeros?

- Use KCL & KVL to derive:

\[ V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \times \frac{C_a}{C_1 + C_a} \]

\[ V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \times \frac{C_a}{C_3 + C_a} \]

- Frequency independent constants can be substituted by Voltage-Controlled Voltage Source

Replace shunt capacitors with voltage controlled voltage sources:

\[ V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a} \]

\[ V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a} \]
3rd Order Lowpass Filter
All Poles & No Zeros

Transmission Zero Implementation
W/O Use of Active Elements
Integrator Based Ladder Filters

Higher Order Transmission zeros

Convert zero generating Cs in C loops to voltage-controlled voltage sources
Example: 5th Order Chebyshev II Filter

- 5th order Chebyshev II
- Table from: Williams & Taylor book, p. 11.112
- 50dB stopband attenuation
- \( f_{3dB} = 10 \text{MHz} \)

Realization with Integrator

\[
V_1 = \frac{I}{s(C_a + C_f)} \left[ V_i - V_i \frac{V_i}{R_s} - \frac{V_2}{R_s} \right] + \frac{C_a}{C_a + C_f} V_3
\]
5th Order Butterworth Filter

From:
Lecture 4
page 14

5th Order Chebyshev II Filter
Opamp-RC Simulation
7th Order Differential Lowpass Filter Including Transmission Zeros

Transmission zeros implemented with pair of coupling capacitors

Effect of Integrator Non-Idealities on Filter Frequency Characteristics

• In the passive filter design (RLC filters) section:
  – Reactive element (L & C) non-idealities \(\rightarrow\) expressed in the form of Quality Factor (\(Q\))
  – Filter impairments due to component non-idealities explained in terms of component \(Q\)

• In the context of active filter design (integrator-based filters)
  – Integrator non-idealities \(\rightarrow\) Translated to have form of Quality Factor (\(Q\))
  – Filter impairments due to integrator non-idealities explained in terms of integrator \(Q\)
Effect of Integrator Non-Idealities on Filter Performance

- Ideal integrator characteristics

- Real integrator characteristics:
  - Effect of opamp finite DC gain
  - Effect of integrator non-dominant poles

\[ H(s) = -\frac{\omega_0}{s} \]
\[ \omega_0 = \frac{1}{RC} \]
Ideal Integrator Quality Factor

**Ideal intg. transfer function:**

\[ H(s) = \frac{-\omega_o}{s} = \frac{-\omega_o}{j\omega} = -\frac{j}{\omega_o} \]

Since component Q is defined as:

\[ H(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \]

\[ Q = \frac{X(\omega)}{R(\omega)} \]

Then:

\[ Q_{\text{ideal}} = \infty \]

---

Real Integrator Non-Idealities

**Ideal Intg.**

\[ H(s) = \frac{-\omega_o}{s} \]

**Real Intg.**

\[ H(s) = \frac{-a}{(1 + \frac{s}{a})(1 + \frac{s}{p_2})(1 + \frac{s}{p_3})} \]
**Effect of Integrator Finite DC Gain on Q**

Example: $P / \omega_0 = 1/100$  
→ phase error ≅ +0.5 degree

**Effect of Integrator Finite DC Gain on Q**

- Phase lead @ $\omega_0$  
  → Droop in the passband

Normalized Frequency
Effect of Integrator Non-Dominant Poles

Example: $\omega_0 P_2 = 1/100$

$\Rightarrow$ phase error $\equiv -0.5\text{ degree}$

Effect of Integrator Non-Dominant Poles

- Phase lag @ $\omega_0$
- Peaking in the passband
- In extreme cases could result in oscillation!
Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q

$$\omega_o \approx -\frac{\pi}{2} + \arctan \frac{P_1}{Q_0}$$

Note that the two terms have different signs → Can cancel each other’s effect!

Integrator Quality Factor

Real intg. transfer function: $H(s) = \frac{-a}{(1 + s \frac{a}{Q_0})(1 + \frac{s}{p_2})(1 + \frac{s}{p_3})\ldots}$

Based on the definition of $Q$ and assuming that:

$$\frac{a}{p_2,3,\ldots} \ll 1 \quad \text{&} \quad a >> 1$$

It can be shown that in the vicinity of unity-gain-frequency:

$$Q_{real}^{intg} = \frac{1}{a} \sum_{i=2}^{\infty} \frac{1}{p_i}$$

Phase lead @ $\omega_0$  Phase lag @ $\omega_0$
Example:
Effect of Integrator Finite Q on Bandpass Filter Behavior

Integrator DC gain = 100

Integrator P2 @ 100 \( \omega_o \)

Example:
Effect of Integrator Q on Filter Behavior

\( (0.5^\circ \phi_{\text{lead}} - 0.5^\circ \phi_{\text{excess}}) @ \omega_o^{\text{intg}} \)

\( \Rightarrow \phi_{\text{error}} @ \omega_o^{\text{intg}} \sim 0 \)

Integrator DC gain = 100 & P2 @ 100 \( \omega_o \)
Summary
Effect of Integrator Non-Idealities on Q

\[ Q_{\text{ideal}}^{\text{intg}} = \infty \]

\[ Q_{\text{real}}^{\text{intg}} = \frac{1}{\frac{1}{a} - \omega_0 \sum_{n=2}^{\infty} \frac{1}{n} \beta^n} \]

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements.
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
  - If non-dominant poles close to unity-gain freq. \( \rightarrow \) Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!