EE247
Lecture 8

• Continuous-time filters continued
  – Various Gm-C filter implementations
  – Comparison of continuous-time filter topologies

• Switched-Capacitor Filters
  – “Analog” sampled-data filters:
    • Continuous amplitude
    • Quantized time
  – Applications:
    • First commercial product: Intel 2912 voice-band CODEC chip, 1979
    • Oversampled A/D and D/A converters
    • Stand-alone filters
      E.g. National Semiconductor LMF100 (x2 biquads)

Summary Last Lecture

• Automatic on-chip filter tuning (continued from previous lecture)
  – Continuous tuning
    • Reference integrator locked to a reference frequency
    • DC tuning of resistive timing element
  – Periodic digitally assisted tuning
    • Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

• Continuous-time filters
  – Highpass filters- 1st order → integrator in the feedback path
  – Bandpass filters
    • Cascade of LP and HP for $Q_{filter} < 5$
    • Direct implementation for narrow-band filter via LP to BP transformation
Simplest Form of CMOS Gm-Cell Nonidealities

- DC gain (integrator Q)
  \[
  a = \frac{g_{M1,2}}{g_{M1,2} + g_{load}}
  \]
  \[
  a = \frac{2L}{\theta(V_{gs} - V_{th})_{M1,2}}
  \]
  
- Where \( a \) denotes DC gain & \( \theta \) is related to channel length modulation by:
  \[
  \lambda = \frac{\theta}{L}
  \]
- Seems no extra poles!

CMOS Gm-Cell High-Frequency Poles

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles
CMOS Gm-Cell High-Frequency Poles

\[
p^\text{effective}_2 = \frac{1}{2} \sum_{i=2}^{\infty} \frac{1}{P_i}
\]

\[
p^\text{effective}_2 = 2.5 \alpha_{1 M1,2}
\]

\[
\alpha_{1 M1,2} = \frac{\alpha_{M1,2}}{2(C_{ox})WL} = \frac{1}{2} \frac{\mu (V_{gs} - V_{th})^2}{L^2}
\]

- Distributed nature of gate capacitance & channel resistance results in an effective pole at 2.5 times input device cut-off frequency

Simple Gm-Cell Quality Factor

\[
a = \frac{2L}{\theta(V_{gs} - V_{th})_{M1,2}}
\]

\[
p^\text{effective}_2 = \frac{15}{4} \frac{\mu (V_{gs} - V_{th})^2_{M1,2}}{L^2}
\]

\[
Q_{\text{real}}^{\text{intg.}} = \frac{1}{\mu a - \epsilon b \sum_{n=2}^{\infty} \frac{1}{n}}
\]

\[
Q_{\text{intg.}}^{\text{real}} = \frac{1}{\mu a - \epsilon b \sum_{n=2}^{\infty} \frac{1}{n}}
\]

- Note that phase lead associated with DC gain is inversely prop. to L
- Phase lag due to high-freq. poles directly prop. to L
  \(\rightarrow\) For a given \(Q_0\) there exists an optimum \(L\) which cancel the lead/lag phase error resulting in high integrator \(Q\)
Simple Gm-Cell Channel Length for Optimum Integrator Quality Factor

\[ L_{opt} = \left[ \frac{15}{4} \frac{\theta \mu (V_{gs} - V_{th})^2 M_{1,2}}{\alpha_b} \right]^{1/3} \]

- Optimum channel length computed based on process parameters (could vary from process to process)

Source-Coupled Pair CMOS Gm-Cell Transconductance

For a source-coupled pair the differential output current \( \Delta I_d \) as a function of the input voltage \( \Delta V_i \):

\[ \Delta I_d = I_{ss} \left[ \frac{\Delta V_i}{(V_{gs} - V_{th})_{M1,2}} \right] \left[ 1 - \frac{\Delta V_i}{(V_{gs} - V_{th})_{M1,2}} \right]^{1/2} \]

Note: For small \( \frac{\Delta V_i}{V_{gs} - V_{th}} \) \( M1,2 \) to \( g_{m1M2} \)

Note: As \( \Delta V_i \) increases \( \frac{\Delta I_d}{\Delta V_i} \) or the effective transconductance decreases

\[ \Delta V_i = V_{i1} - V_{i2} \]

\[ \Delta I_d = I_{d1} - I_{d2} \]
Source-Coupled Pair CMOS Gm-Cell Linearity

- Large signal $G_m$ drops as input voltage increases
  $\Rightarrow$ Gives rise to nonlinearity

Measure of Linearity

$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots$

$HD_3 = \frac{\text{amplitude 3rd harmonic dist. comp.}}{\text{amplitude fundamental}} = \frac{1}{4} \frac{\alpha_2}{\alpha_1} V_{in}^2 + \ldots$

$IM_3 = \frac{\text{amplitude 3rd order IM comp.}}{\text{amplitude fundamental}} = \frac{3}{4} \frac{\alpha_3}{\alpha_1} V_{in}^3 + \frac{25}{8} \frac{\alpha_4}{\alpha_1} V_{in}^4 + \ldots$
Source-Coupled Pair Gm-Cell Linearity

\[
\Delta I_d = I_{ss} \left( \frac{\Delta V_{th}}{V_{gs} - V_{th}} \right)_{M1,2} \left[ 1 + \frac{\Delta V_{th}}{V_{gs} - V_{th}} \right]^{1/2} \quad (1)
\]

\[
\Delta I_d = a_1 \Delta V_{th} + a_2 \Delta V_{th}^2 + a_3 \Delta V_{th}^3 + \ldots
\]

Series expansion used in (1)

\[
a_1 = \frac{I_{ss}}{V_{gs} - V_{th}}_{M1,2} \quad \& \quad a_2 = 0
\]

\[
a_3 = \frac{8I_{ss}}{(V_{gs} - V_{th})^3}_{M1,2} \quad \& \quad a_4 = 0
\]

\[
a_5 = \frac{128I_{ss}}{(V_{gs} - V_{th})^5}_{M1,2} \quad \& \quad a_6 = 0
\]

Linearity of the Source-Coupled Pair CMOS Gm-Cell

\[
IM_3 = \frac{3a_3}{4a_1} \Delta V_{th}^3 + \frac{25a_5}{8a_1} \Delta V_{th}^5 \quad \ldots...........
\]

Substituting for \(a_1, a_2, \ldots\)

\[
IM_3 = \frac{3}{32} \left( \frac{\Delta V_{th}}{V_{GS} - V_{th}} \right)^2 + \frac{25}{1024} \left( \frac{\Delta V_{th}}{V_{GS} - V_{th}} \right)^4 \quad \ldots...........
\]

\[
\hat{V}_{i_{\text{max}}} = 4(V_{GS} - V_{th}) \times \sqrt{\frac{2}{3} \times IM_3}
\]

\[
IM_3 = 1\% \ \& \ (V_{GS} - V_{th}) = IV \Rightarrow \hat{V}_{i_{\text{rms}}} = 230mV
\]

• Key point: Max. signal handling capability function of gate-overdrive voltage
Simplest Form of CMOS Gm Cell

Disadvantages

• Max. signal handling capability function of gate-overdrive
  \[ IM_{3} \propto (V_{GS} - V_{th})^{2} \]

• Critical freq. is also a function of gate-overdrive
  \[ \omega_{c} \propto \frac{g_{m}^{1.2}}{2 \times C_{int} g} \]
  since \( g_{m} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{th}) \)
  then \( \omega_{c} \propto (V_{GS} - V_{th}) \)

→ Filter tuning affects max. signal handling capability!

Simplest Form of CMOS Gm Cell
Removing Dependence of Maximum Signal Handling Capability on Tuning

• Can overcome problem of max. signal handling capability being a function of tuning by providing tuning through:
  – Coarse tuning via switching in/out binary-weighted cross-coupled pairs
  → Try to keep gate overdrive voltage constant
  – Fine tuning through varying current sources

→ Dynamic range dependence on tuning removed (to 1st order)

Dynamic Range for Source-Coupled Pair Based Filter

\[ IM_S = 1\% \text{ & } (V_{GSO} - V_{in}) = W \Rightarrow V_{in}^{\text{rms}} = 230mV \]

- Minimum detectable signal determined by total noise voltage
- It can be shown for the 6th order Butterworth bandpass filter fundamental noise contribution is given by:

\[ \sqrt{\frac{v_{rms}^2}{v_{noise}^2}} = \sqrt{3 \cdot \frac{Q \cdot kT}{C_{intg}}} \]

Assuming \( Q = 10 \), \( C_{intg} = 5pF \)
\( v_{rms}^2 = 160\mu V \)
Since \( v_{max} = 230mV \)

Dynamic Range \( = 20 \log \left( \frac{230 \times 10^{-3}}{160 \times 10^{-6}} \right) \approx 63dB \)

Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm-Cell

- 2nd source-coupled pair added to subtract current proportional to nonlinear component associated with the main SCP

\[ \frac{I_{ss2}}{I_{ss3}} = b \] and \( \frac{(V_{gs} - V_{th})_{M12}}{(V_{gs} - V_{th})_{M34}} = a \]

\[ \frac{W}{T}_{M12} = b \] and \( \frac{W}{T}_{M34} = a \]
Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm


- Improves maximum signal handling capability by about 12dB
- Dynamic range theoretically improved to 63+12=75dB
Simplest Form of CMOS Gm-Cell

**Pros**
- Capable of very high frequency performance (highest?)
- Simple design

**Cons**
- Tuning affects power dissipation
- Tuning affects max. signal handling capability (can overcome)
- Limited linearity (possible to improve)


Gm-Cell
Source-Coupled Pair with Degeneration

\[
I_d = \frac{\mu C_{ox} W}{2} \left[ \frac{2(V_{gs} - V_{th})}{L} \right] \left[ V_{ds} - V_{th}^2 \right]
\]

\[
g_{ds} = \frac{\partial I_d}{\partial V_{ds}} = \frac{\mu C_{ox} W}{2} \left( \frac{V_{gs} - V_{th}}{L} \right) \left( V_{ds} - V_{th} \right)
\]

\[
g_{eff} = \frac{1}{g_{ds} M^2} + \frac{2}{g_{m}}
\]

\[
g_{m} \ll g_{d3}
\]

M3 operating in triode mode → source degeneration → determines overall gm
Provides tuning through varying $V_c$
Gm-Cell
Source-Coupled Pair with Degeneration

• Pros
  – Moderate linearity
  – Continuous tuning provided by $V_c$
  – Tuning does not affect power dissipation

• Cons
  – Extra poles associated with the source of M1,2,3
  → Low frequency applications only


BiCMOS Gm-Cell
Example

• MOSFET in triode mode:
  \[ I_d = \frac{\mu C_{ox} W}{2L} \left( 2(V_{gs} - V_{th}) V_{ds} - V_{ds}^2 \right) \]

• Note that if $V_{ds}$ is kept constant:
  \[ s_{M1} = \frac{\partial I_d}{\partial V_{gs}} = \frac{\mu C_{ox} W}{L} V_{ds} \]

• Linearity performance → keep $gm$ constant
  → function of how constant $V_{ds}$ can be held
  – Need to minimize Gain @ Node X
  \[ A_x = s_{M1} / s_{B1} \]

• Since for a given current, $gm$ of BJT is larger compared to MOS- preferable to use BJT

• Extra pole at node X
Alternative Fully CMOS Gm-Cell Example

- BJT replaced by a MOS transistor with boosted gm

- Lower frequency of operation compared to the BiCMOS version due to more parasitic capacitance at nodes A & B

BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt

- Freq.tuned by varying Vb

- Design tradeoffs:
  - Extra poles at the input device drain junctions
  - Input devices have to be small to minimize parasitic poles
  - Results in high input-referred offset voltage → could drive ckt into non-linear region
  - Small devices → high 1/f noise
7th Order Elliptic Gm-C LPF
For CDMA RX Baseband Application

- Gm-Cell in previous page used to build a 7th order elliptic filter for CDMA baseband applications (650kHz corner frequency)
- In-band dynamic range of <50dB achieved

Comparison of 7th Order Gm-C versus Opamp-RC LPF

- Gm-C filter requires 4 times less intg. cap. area compared to Opamp-RC
  ➔ For low-noise applications where filter area is dominated by Cs, could make a significant difference in the total area
- Opamp-RC linearity superior compared to Gm-C
- Power dissipation tends to be lower for Gm-C since OTA load is C and thus no need for buffering
• Used to build filter for disk-drive applications
• Since high frequency of operation, time-constant sensitivity to parasitic caps significant.
  → Opamp used
• M2 & M3 added to compensate for phase lag (provides phase lead)


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• Gm-C-opamp of the previous page used to build a 6th order filter for Disk Drive
• Filter consists of 3 Biquad with max. Q of 2 each
• Performance in the order of 40dB SNDR achieved for up to 20MHz corner frequency

Gm-Cell
Source-Coupled Pair with Degeneration

- Gm-cell intended for low Q disk drive filter


Gm-Cell
Source-Coupled Pair with Degeneration

- M7,8 operating in triode mode determine the overall gm of the cell
- Feedback provided by M5,6 maintains the gate-source voltage of M1,2 constant by forcing their current to be constant → helps linearize $r_{ds}$ of M7,8
- Current mirrored to the output via M9,10 with a factor of $k$
- Performance level of about 50dB SNDR at fcorner of 25MHz achieved
• Needs higher supply voltage compared to the previous design since quite a few devices are stacked vertically

• \( M_{1,2} \rightarrow \) triode mode

• \( Q_{1,2} \rightarrow \) hold \( V_{ds} \) of \( M_{1,2} \) constant

• Current \( I_D \) used to tune filter critical frequency by varying \( V_{ds} \) of \( M_{1,2} \) and thus controlling \( g_m \) of \( M_{1,2} \)

• \( M_{3,4} \) operate in triode mode and added to provide common-mode feedback

BiCMOS Gm-C Filter For Disk-Drive Application

- Using the integrators shown in the previous page
- Biquad filter for disk drives
  - \( \text{gm1} = \text{gm2} = \text{gm4} = 2 \text{gm3} \)
  - \( Q = 2 \)
  - Tunable from 8MHz to 32MHz


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**Summary**

*Continuous-Time Filters*

- **Opamp RC filters**
  - Good linearity \( \rightarrow \) High dynamic range (60-90dB)
  - Only discrete tuning possible
  - Medium usable signal bandwidth (<10MHz)

- **Opamp MOSFET-C**
  - Linearity compromised (typical dynamic range 40-60dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)

- **Opamp MOSFET-RC**
  - Improved linearity compared to Opamp MOSFET-C (D.R. 50-90dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)

- **Gm-C**
  - Highest frequency performance (at least an order of magnitude higher compared to the rest <100MHz)
  - Dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (40-70dB)
Switched-Capacitor Filters

Today

- Emulating resistor via switched-capacitor network
- 1st order switched-capacitor filter
- Switch-capacitor filter considerations:
  - Issue of aliasing and how to avoid it
  - Tradeoffs in choosing sampling rate
  - Effect of sample and hold
  - Switched-capacitor filter electronic noise

Switched-Capacitor Resistor

- Capacitor C is the “switched capacitor”
- Non-overlapping clocks $\phi_1$ and $\phi_2$ control switches S1 and S2, respectively
- $v_{IN}$ is sampled at the falling edge of $\phi_1$
  - Sampling frequency $f_S$
- Next, $\phi_2$ rises and the voltage across C is transferred to $v_{OUT}$
- Why does this behave as a resistor?

\[ T = \frac{1}{f_s} \]
Switched-Capacitor Resistors

- Charge transferred from $v_{IN}$ to $v_{OUT}$ during each clock cycle is:
  \[ Q = C(v_{IN} - v_{OUT}) \]

- Average current flowing from $v_{IN}$ to $v_{OUT}$ is:
  \[ i = \frac{Q}{t} = Q \cdot f_s \]

Substituting for $Q$:
\[ i = f_s C(v_{IN} - v_{OUT}) \]

With the current through the switched-capacitor resistor proportional to the voltage across it, the equivalent "switched capacitor resistance" is:

\[ R_{eq} = \frac{v_{IN} - v_{OUT}}{i} = \frac{1}{f_s C} \]

**Example:**
\[ f_s = 100\text{KHz}, C = 0.1pF \]
\[ R_{eq} = 100\text{Mega}\Omega \]

Note: Can build large time-constant in small area
Switched-Capacitor Filter

• Let’s build a “switched-capacitor” filter …

• Start with a simple RC LPF

• Replace the physical resistor by an equivalent switched-capacitor resistor

• 3-dB bandwidth:

\[
\omega_{-3dB} = \frac{1}{R_{eq}C_2} = f_s \times \frac{C_1}{C_2}
\]

\[
f_{-3dB} = \frac{1}{2\pi f_s} \times \frac{C_1}{C_2}
\]

Switched-Capacitor Filter Advantage versus Continuous-Time Filter

• Corner freq. proportional to:
  - System clock (accurate to few ppm)
  - C ratio accurate \( \Rightarrow < 0.1\% \)

• Corner freq. proportional to:
  - Absolute value of Rs & Cs
  - Poor accuracy \( \Rightarrow 20 \text{ to } 50\% \)

Main advantage of SC filters \( \Rightarrow \) inherent corner frequency accuracy
Typical Sampling Process
Continuous-Time(CT) ⇒ Sampled Data (SD)

Uniform Sampling

Nomenclature:
- Continuous time signal \( x_c(t) \)
- Sampling interval \( T \)
- Sampling frequency \( f_s = 1/T \)
- Sampled signal \( x_s(kT) = x(k) \)

- Problem: Multiple continuous time signals can yield exactly the same discrete time signal
- Let's look at samples taken at 1μs intervals of several sinusoidal waveforms …

Note: Samples are the waveform values at \( kT \) instances and undefined in between
Sampling Sine Waves

Sampling Sine Waves

\[ v(t) = \cos (2\pi f_{in} t) \]

Sampled-data domain \( \rightarrow t \rightarrow nT \) or \( t \rightarrow n/f_s \)

\[ v(n) = \cos \left( 2\pi \frac{f_{in}}{f_s} n \right) = \cos \left( 2\pi \frac{101kHz}{1MHz} n \right) \]

\[ v(n) = \cos \left( 2\pi \frac{899kHz}{1MHz} n \right) = \cos \left( 2\pi \frac{(1000kHz-101kHz)}{1MHz} n \right) = \cos \left( 2\pi \frac{-101kHz}{1MHz} n \right) = \cos \left( 2\pi \frac{101kHz}{1MHz} n \right) \]
Sampling Sine Waves

Aliasing

Problem:

Identical samples for:

\[ v(t) = \cos \left[ 2\pi f_{in} t \right] \]
\[ v(t) = \cos \left[ 2\pi (f_{in} + n f_s) t \right] \]
\[ v(t) = \cos \left[ 2\pi (f_{in} - n f_s) t \right] \quad (*) \text{ (n-integer)} \]

\[ \rightarrow \text{Multiple continuous time signals can yield exactly the same discrete time signal} \]
Aliasing

- Multiple continuous time signals can produce identical series of samples

- The folding back of signals from $n f_s \pm f_{\text{sig}}$ (n integer) down to $f_{\text{in}}$ is called aliasing
  - Sampling theorem: $f_s > 2f_{\text{max Signal}}$

- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal
How to Avoid Aliasing?

• Must obey sampling theorem:
  \[ f_{\text{max-Signal}} < \frac{f_s}{2} \]

*Note:
Minimum sampling rate of \( f_s = 2xf_{\text{max-Signal}} \) is called Nyquist rate

• Two possibilities:
  1. Sample fast enough to cover all spectral components, including "parasitic" ones outside band of interest
  2. Limit \( f_{\text{max-Signal}} \) through filtering \( \Rightarrow \) attenuate out-of-band components prior to sampling

How to Avoid Aliasing?

1-Sample Fast

Push sampling frequency to \( x2 \) of the highest frequency signal to cover all unwanted signals as well as wanted signals

\( \Rightarrow \) In vast majority of cases not practical
How to Avoid Aliasing?
2-Filter Out-of-Band Signal Prior to Sampling

How to Avoid Aliasing?
2-Filter Out-of-Band Signal Prior to Sampling

Anti-Aliasing Filter Considerations

Case 1: \( B = f_{sig}^{max} = f_s / 2 \)
- Non-practical since an extremely high order anti-aliasing filter (close to an ideal brickwall filter) is required
- Practical anti-aliasing filter \( \rightarrow \) Non-zero filter "transition band"
- In order to make this work, we need to sample much faster than 2x the signal bandwidth
  \( \rightarrow \) "Oversampling"
Practical Anti-Aliasing Filter

Case 2 - \( B = f_{\text{max}_{\text{Signal}}} \ll f_s/2 \)

- More practical anti-aliasing filter
- Preferable to have an anti-aliasing filter with:
  - The lowest order possible
  - No frequency tuning required
  (if frequency tuning is required then why use switched-capacitor filter, just use the prefilter!?)

Tradeoff
Oversampling Ratio versus Anti-Aliasing Filter Order

* Assumption → anti-aliasing filter is Butterworth type (not a necessary requirement)

→ Tradeoff: Sampling speed versus anti-aliasing filter order

Effect of Sample & Hold

- Using the Fourier transform of a rectangular impulse:

\[ |H(f)| = \frac{T_p}{T_s} \frac{\sin(\pi T_p f)}{\pi T_p f} \]

Effect of Sample & Hold on Frequency Response

- More practical:

\[ T_p = 0.5T_s \]

More practical
Sample & Hold Effect
(Reconstruction of Analog Signals)

Magnitude droop due to $\sin x/x$ effect:

Case 1) $f_{\text{sig}} = f_s/4$

Droop = -1dB
Sample & Hold Effect
(Reconstruction of Analog Signals)

Magnitude droop due to \( \sin x/x \) effect:

Case 2)
\[ f_{\text{sig}} = f_s/32 \]

\[ \text{Droop} = -0.0035 \text{dB} \]

\( \Rightarrow \text{High oversampling ratio desirable} \)

Sampling Process Including S/H