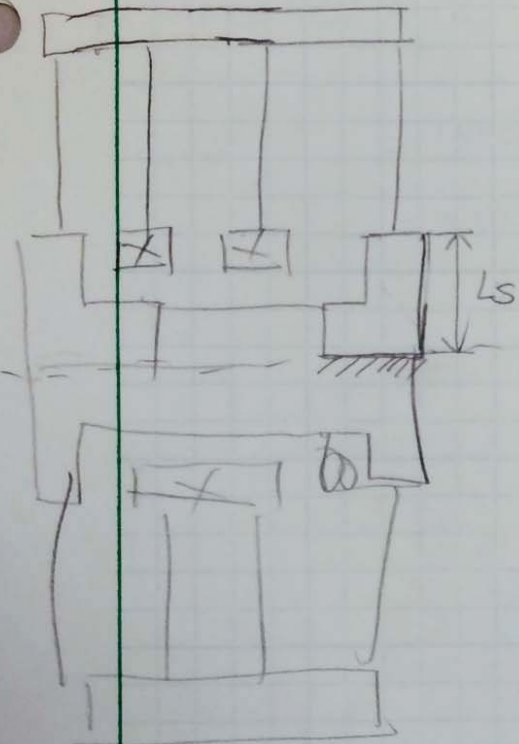


$$L_s = \frac{L_i}{2} = \frac{L_o}{2}$$

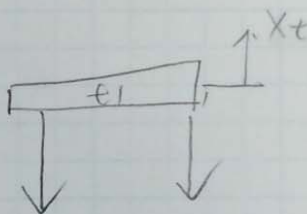


2.8 poly 0.5 → substrate
 $\sigma_f - \alpha_s \cdot \Delta T$
 $= (2.8 - 0.5) (650^\circ - 25^\circ C) \cdot \left(\frac{E_{poly}}{1-\nu} \right) =$
 160 GPa
 \downarrow
 $= 287.5 \text{ MPa}$

the amount of expansion or contraction when beams are free to move

$$\Delta L_x = (\alpha_s - \alpha_f) \cdot \Delta T \cdot L_x$$

$$\Delta L_s = (\alpha_s - \alpha_f) \cdot \Delta T \cdot L_s$$



$$E \cdot A \cdot \frac{(X_t - L_i')}{L_i} + EA \cdot \frac{(X_t - L_o' - \Delta L_s)}{L_o} = 0$$

$$\frac{X_t - L_i'}{L_i} + \frac{X_t - L_o' - \Delta L_s}{L_o} = 0$$

$$L_i' = L_o$$

$$2X_t = L_o' + L_i' + \Delta L_s$$

$$X_t = \frac{L_o' + L_i'}{2} + \frac{\Delta L_s}{2}$$

$$= L_o' + \frac{\Delta L_s}{2}$$

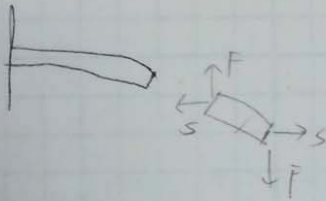
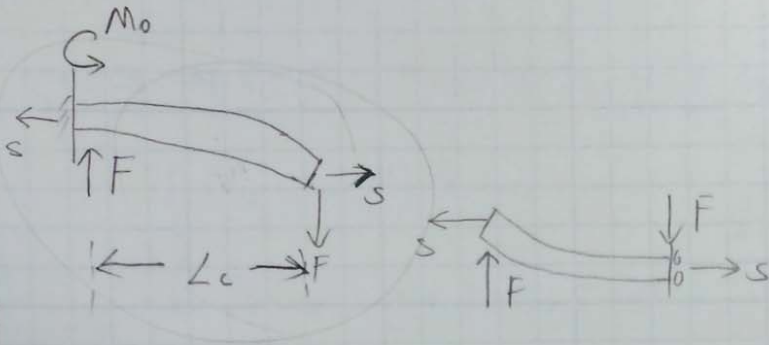
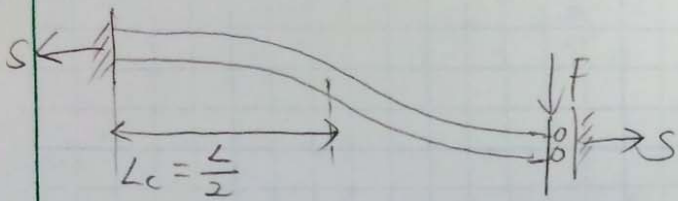
stress in outside beam

$$\sigma_o = E \cdot \epsilon_o = E \cdot \frac{L_o' + \frac{\Delta L_s}{2} - L_o' - \Delta L_s}{L_o} = E \cdot \left(-\frac{\Delta L_s}{2L_o} \right) = -E \cdot \frac{(\alpha_s - \alpha_f) \Delta T \cdot L_s}{2L_o}$$

$$= -160 \text{ GPa} \cdot \frac{(0.5 - 2.8) \times 10^{-6} \times (650 - 25)}{4} = 57.5 \text{ MPa}$$

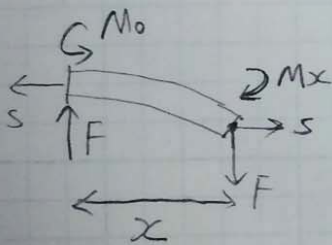
inside beam

$$\sigma_i = E \cdot \frac{L_o' + \frac{\Delta L_s}{2} - L_i'}{L_i} = E \cdot \frac{\Delta L_s}{2L_i} = 160 \times \frac{(2.5 - 2.8) \times 10^{-6} \times (650 - 25)}{4} = -57.5 \text{ MPa}$$



$$F \cdot L_c - S \cdot y(L_c) - M_0 = 0$$

$$\Rightarrow M_0 = F \cdot L_c - S \cdot y(L_c)$$



$$M_x + F \cdot x - S \cdot y(x) - M_0 = 0$$

$$\Rightarrow M_x = -F \cdot x + S \cdot y(x) + F \cdot L_c - S \cdot y(L_c)$$

$$= S \cdot y(x) - F \cdot x + F \cdot L_c - S \cdot y(L_c)$$

$$EI \frac{d^2 y}{dx^2} = S \cdot y(x) - F \cdot x + F \cdot L_c - S \cdot y(L_c)$$

$$EI y'' = S y - F \cdot x + [F L_c - S y(L_c)] \quad y'' = \frac{S}{EI} y - \frac{F}{EI} x + \frac{[F L_c - S y(L_c)]}{EI}$$

$$y = C_1 \cos p x + C_2 \sin p x + C_3 x + C_4$$

$$y'' = -C_1 \cdot p^2 \cos p x - C_2 p^2 \sin p x = \frac{S}{EI} C_1 \cos p x + \frac{S}{EI} C_2 \sin p x$$

$$\Rightarrow p = \sqrt{\frac{S}{EI}}$$

$$y = c_1 \cos px + c_2 \sin px + c_3 x + c_4$$

$$EI y'' = c_1 \cdot s \cdot \cos px + c_2 \cdot s \cdot \sin px$$

$$= c_1 s \cdot \cos px + c_2 s \sin px + \underline{c_3 \cdot s \cdot x} + \underline{c_4 \cdot s} - \underline{F x} + \underline{F \cdot L_c} - \underline{s \cdot y \cdot (L_c)}$$

$$\Rightarrow \begin{cases} (c_3 \cdot s - F) x = 0 \\ F \cdot L_c - s \cdot y(L_c) + c_4 \cdot s = 0 \end{cases} \Rightarrow \begin{cases} c_3 = \frac{F}{s} & B1 \\ c_4 = -\frac{F L_c}{s} + y(L_c) & (4) \end{cases}$$

Boundary condition

$$y(0) = 0 \Rightarrow c_1 + c_4 = 0 \quad (5)$$

$$\frac{dy}{dx} \Big|_{x=0} = 0 \Rightarrow -c_1 p \cdot \sin px + c_2 p \cos px + c_3 = 0 \quad (6)$$

$$c_2 p + c_3 = 0$$

$$\Rightarrow \begin{cases} c_2 = -\frac{c_3}{p} = -\frac{F}{s \cdot p} \\ c_1 = -c_4 = -\left[-\frac{F L_c}{s} + y(L_c)\right] \\ c_3 = \frac{F}{s} \\ c_4 = -\frac{F L_c}{s} + y(L_c) \end{cases}$$

$$y(L_c) = c_1 \cdot \cos(p \cdot L_c) + c_2 \sin(p \cdot L_c) + c_3 \cdot L_c + c_4$$

$$y(L_c) = \left[\frac{F L_c}{s} - y(L_c)\right] \cos(p L_c) - \frac{F}{s p} \sin(p L_c) + \frac{F}{s} \cdot L_c - \frac{F L_c}{s}$$

$$\frac{F L_c}{s} \cos(p L_c) - y(L_c) \cos(p L_c) - \frac{F}{s p} \sin(p L_c) = 0$$

$$\Rightarrow y(L_c) = \frac{F L_c}{s} \frac{\cos(p L_c)}{\cos(p L_c)} - \frac{F}{s p} \frac{\sin(p L_c)}{\cos(p L_c)} = \frac{F L_c}{s} - \frac{F}{s p} \tan$$

$$K_c = \frac{F}{y(L_c)} = \frac{F}{\frac{FL_c}{S} - \frac{F}{SP} \tan(PL_c)} = \frac{p \cdot S}{p \cdot L_c - \tan(PL_c)}$$

$$S \cdot P = \sqrt{\frac{-S}{EI}} = \sqrt{P} \cdot i \cdot \sqrt{\frac{S}{EI}}$$

$$K_c = \frac{i \cdot \sqrt{\frac{S}{EI}} \cdot S}{\sqrt{\frac{S}{EI}} \cdot i \cdot L_c - \tan\left(\sqrt{\frac{S}{EI}} \cdot i \cdot L_c\right)}$$

$$= \frac{\sqrt{\frac{S}{EI}}}{\sqrt{\frac{S}{EI}} \cdot L_c + i \cdot \tan\left[i \sqrt{\frac{S}{EI}} \cdot L_c\right]}$$

$$\tanh(x) = -i \tan(ix)$$

$$= \frac{\sqrt{\frac{S}{EI}} \cdot S}{\sqrt{\frac{S}{EI}} \cdot L_c - \tanh\left[\sqrt{\frac{S}{EI}} \cdot L_c\right]}$$