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## EE C247B - ME C218 Introduction to MEMS Design Spring 2017

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Lecture Module 8: Microstructural Elements

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## Outline

- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - ↪ Bending of beams
  - ↪ Cantilever beam under small deflections
  - ↪ Combining cantilevers in series and parallel
  - ↪ Folded suspensions
  - ↪ Design implications of residual stress and stress gradients

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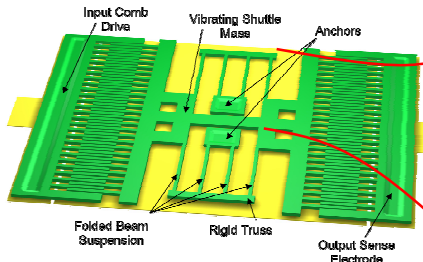
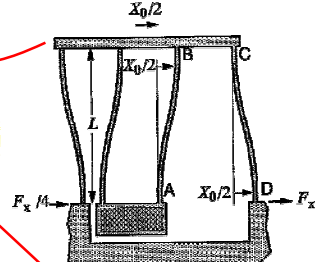
## Bending of Beams

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## Beams: The Springs of Most MEMS

- Springs and suspensions very common in MEMS
  - ↪ Coils are popular in the macro-world; but not easy to make in the micro-world
  - ↪ Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS

Comb-Driven Folded Beam Actuator

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Bending a Cantilever Beam

Clamped end condition:  
At  $x=0$ :  
 $y=0$   
 $dy/dx = 0$

Free end condition

$F$

$x'$

$L$

$x$

• Objective: Find relation between tip deflection  $y(x=L_c)$  and applied load  $F$

• Assumptions:

1. Tip deflection is small compared with beam length
2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
3. Shear stresses are negligible

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Reaction Forces and Moments

Reaction Moment  $M_R = M_1$

Point Load  $F$

Moment due to  $F$ , here:  $M_1 = FL$

Moment due to  $F$ , here:  $M_2 = F(L-x)$

Reaction Force  $F_R = F$

split

$M_{x,l}$   $M_{x,r}$

$V_{x,l}$   $V_{x,r}$

Reactions (Senturia gives expressions)

For equilibrium:  $M_{x,r} = M_3 = F(L-x)$   
 $V_{x,r} = F$

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Sign Conventions for Moments & Shear Forces

Positive

Negative

(+) moment leads to deformation with a (+) radius of curvature (i.e., upwards)

(-) moment leads to deformation with a (-) radius of curvature (i.e., downwards)

(+) shear forces produce clockwise rotation

(-) shear forces produce counter-clockwise rotation

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