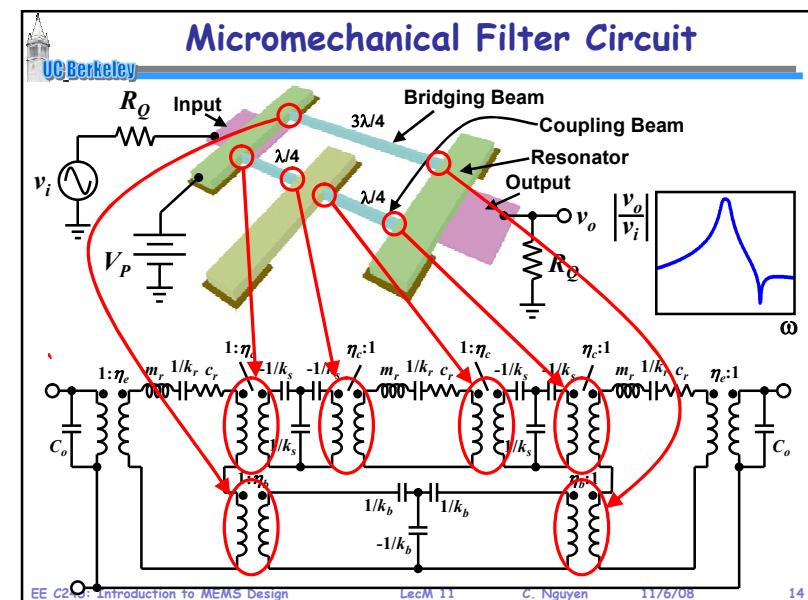
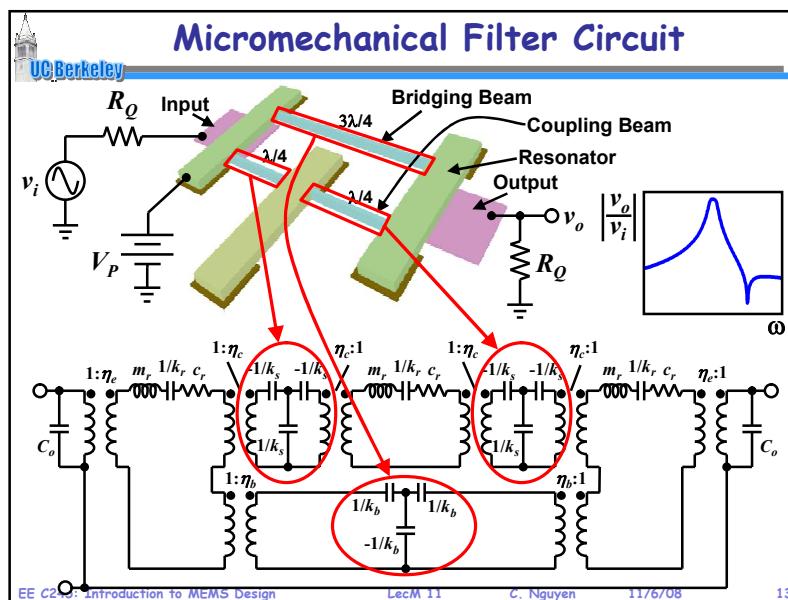
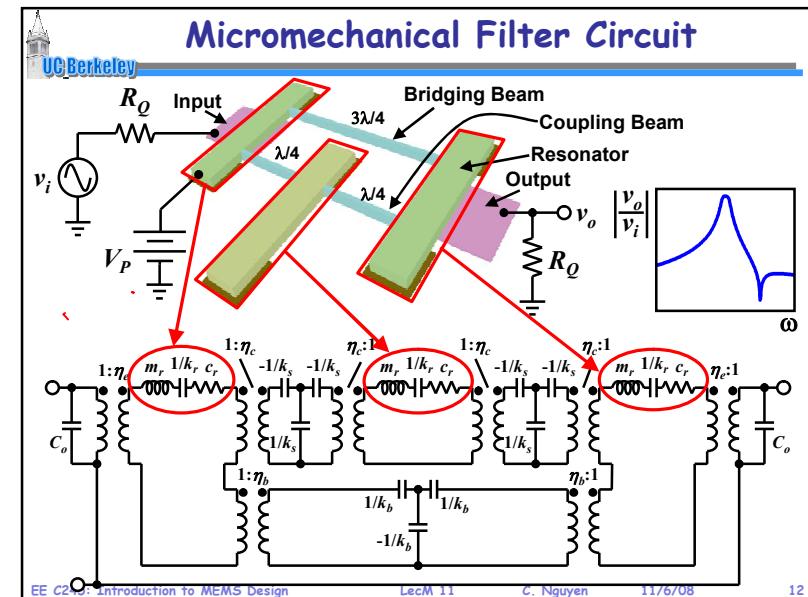
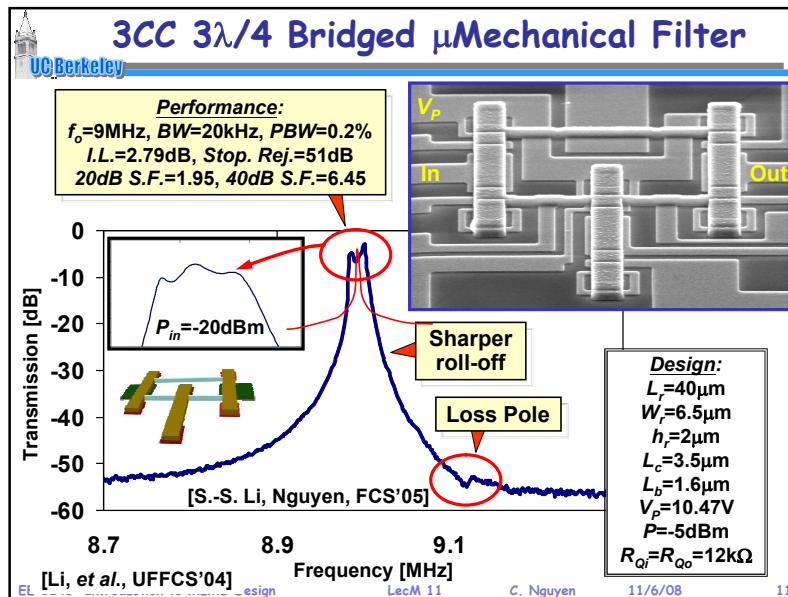
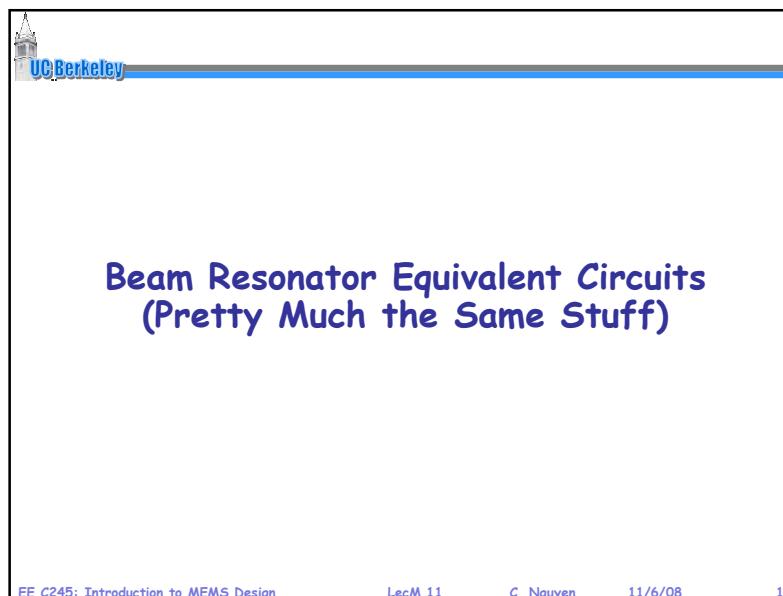
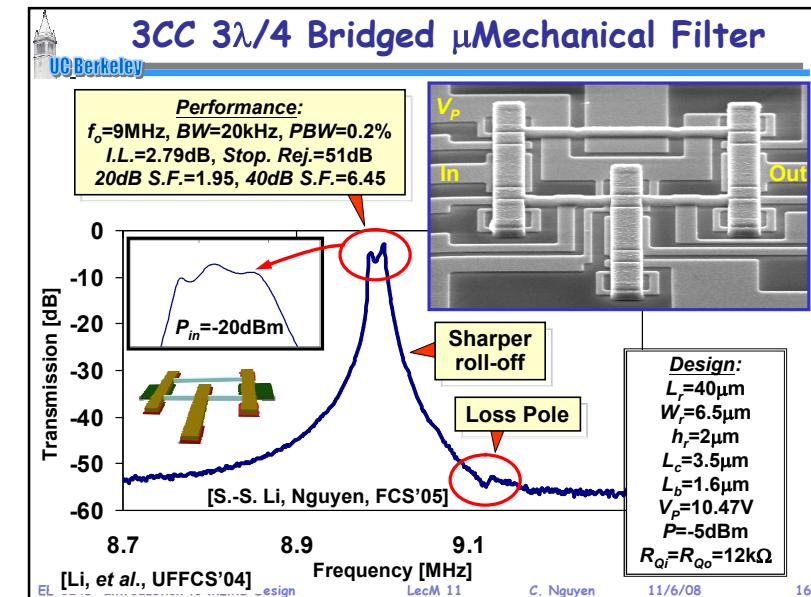
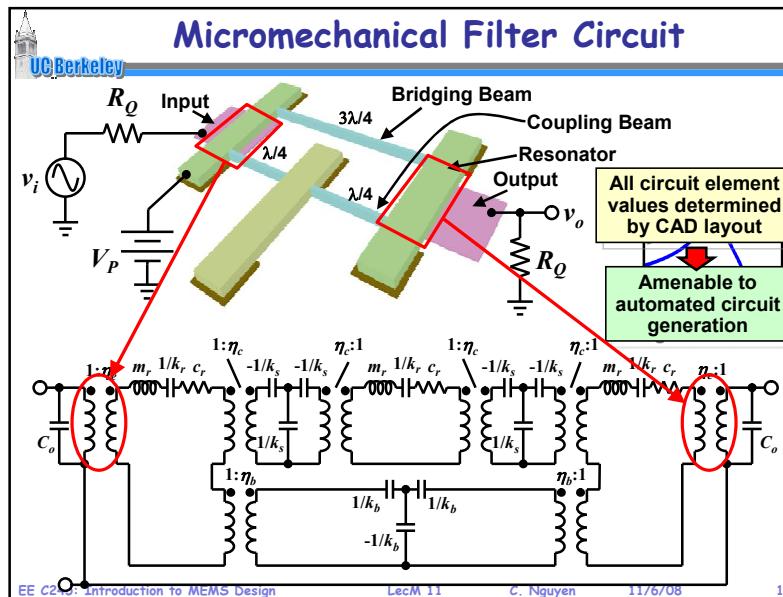


## Lecture 18m2: Equivalent Circuits I



## Lecture 18m2: Equivalent Circuits I



**Equivalent Dynamic Mass**

UC Berkeley

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location  $x$  using knowledge of kinetic energy and velocity

Location  $x$ , Maximum Kinetic Energy, Equivalent Mass  $M_{eq,x}$ , Maximum Velocity @ location  $x$ , Maximum Velocity Function.

$$\text{Equivalent Mass} = M_{eq,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^l V^2(x) dx}{\frac{1}{2}V_x^2}$$

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## Lecture 18m2: Equivalent Circuits I

### Equivalent Dynamic Mass

We know the mode shape, so we can write expressions for displacement and velocity at resonance.

Displacement:  $u(x) = B \left[ S(\cosh kx + \cos kx') + (\sinh kx + \sin kx') \right], S = \frac{A}{B}$

$[V(x) = \omega u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{KPA}{\frac{1}{2}\rho A \int_0^l \omega^2 [u(x')]^2 dx'} = \frac{KPA}{\frac{1}{2}\rho A [u(x)]^2}$

$M_{eq}(x) = \rho A \int_0^l \left[ S(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx') \right]^2 dx' = \frac{B^2 [S(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}{\rho A}$

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### Equivalent Dynamic Stiffness & Damping

Stiffness then follows directly from knowledge of mass and resonance frequency.

 $\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_o^2 M_{eq}(x)$ 

And damping also follows readily

 $Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$ 

And damping

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### Equivalent Lumped Mechanical Circuit

$K_{eq}(x) = \omega_o^2 M_{eq}(x)$

$M_{eq}(x) = \frac{\rho A \int_0^l [u(x')]^2 dx'}{[u(x)]^2}$

$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$

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### Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/  $l=14.9\mu\text{m}$ ,  $W=6\mu\text{m}$ ,  $h=2\mu\text{m} \rightarrow 70 \text{ MHz}$

$K_{eq}(0) = 19,927 \text{ N/m}$   
 $M_{eq}(0) = 1.03 \times 10^{-13} \text{ kg}$   
 $C_{eq}(0) = 5.66 \times 10^{-9} \text{ kg/s}$

$K_{eq}(l/2) = 53,938 \text{ N/m}$   
 $M_{eq}(l/2) = 2.78 \times 10^{-13} \text{ kg}$   
 $C_{eq}(l/2) = 1.53 \times 10^{-8} \text{ kg/s}$

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